Exercise 23.2

1. Solution:

Given that P, Q and R are collinear
Also, given that

$\overrightarrow{PQ} = \vec{a}$ and $\overrightarrow{QR} = \vec{b}$

Now,

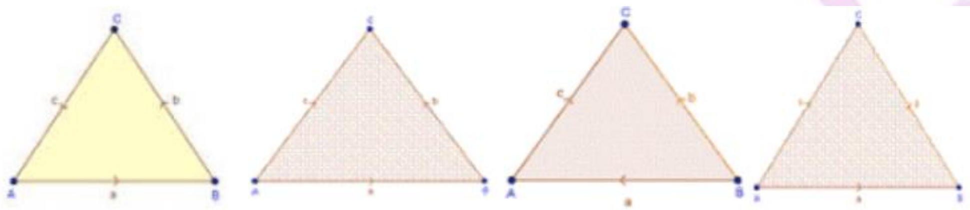
$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= \vec{a} + \vec{b}$

Thus,

$\overrightarrow{PR} = \vec{a} + \vec{b}$

2. Solution:



Given that, $\vec{a}, \vec{b}$, and $\vec{c}$ are the three sides of a triangle
So, we have

$\vec{a} + \vec{b} + \vec{c}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

$= \overrightarrow{AC} + \overrightarrow{CA}$ [Since $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$]

$= \overrightarrow{AC} - \overrightarrow{AC}$ [Since $\overrightarrow{CA} = \overrightarrow{AC}$]

$= \vec{0}$

Thus, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

The triangle law says that, if vectors are represented in magnitude and direction by the two sides of triangle taken in same order, then their sum is represented by the third side taken in reverse order.
Therefore,

$\vec{a} + \vec{b} = \vec{c}$

or

$\vec{a} + \vec{c} = \vec{b}$

$\vec{b} + \vec{c} = \vec{a}$

3. Solution:

Given that $\vec{a}$ and $\vec{b}$ are two non-collinear vectors having the same initial point

Let $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{AD}$, so we can draw a parallelogram ABCD as above
By the properties of parallelogram, we have
$\overrightarrow{BC} = \vec{b}$ and $\overrightarrow{DC} = \vec{a}$

In ΔABC,
Using triangle law, we have
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$
$$\vec{a} + \vec{b} = \overrightarrow{AC} \quad --(i)$$

In ΔABD,
Using triangle law,
$$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$$
$$\vec{b} + \overrightarrow{DB} = \vec{a}$$
$$\overrightarrow{DB} = \vec{a} - \vec{b} \quad --(ii)$$
Thus, from equations (i) and (ii), we get that
$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are diagonals of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

4. Solution:

Given, m is scalar and $\vec{a}$ is a vector such that $m\vec{a} = \vec{0}$
Now,
$$m\left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right) = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k} \quad \left[\text{since let } \vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right]$$
$$ma_1\hat{i} + mb_1\hat{j} + mc_1\hat{k} = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$$
On comparing the coefficients of unit vectors i, j and k of L.H.S and R.H.S, we have
$$ma_1 = 0 \Rightarrow m = 0 \text{ or } a_1 = 0 \quad (i)$$
$$mb_1 = 0 \Rightarrow m = 0 \text{ or } b_1 = 0 \quad (ii)$$
$$mc_1 = 0 \Rightarrow m = 0 \text{ or } c_1 = 0 \quad (iii)$$

From (i), (ii) and (iii)
$$m = 0 \quad \text{or} \quad a_1 = b_1 = c_1 = 0$$
$$\Rightarrow m = 0 \quad \text{or} \quad \vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{j} = 0$$
$$\Rightarrow m = 0 \quad \text{or} \quad \vec{a} = 0$$

5.(i) Solution:

Let
$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$
$$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$
Given that, $a = -b$
$$a_1\hat{i} + b_1\hat{j} + c_1\hat{k} = -a_2\hat{i} - b_2\hat{j} - c_2\hat{k}$$

On comparing the coefficients of unit vectors i, j and k in L.H.S and R.H.S, we get
$a_1 = -a_2$ … (i)
$b_1 = - b_2$ … (ii)
$c_1 = - c_2$ … (iii)
$|\vec{a}| = \sqrt{a_1^2 + b_1^2 + c_1^2}$
Now,
Using (1), (2) and (3),
$|\vec{a}| = \sqrt{(-a_2)^2 + (-b_2)^2 + (-c_2)^2}$
$|\vec{a}| = \sqrt{a_2^2 + b_2^2 + c_2^2}$
$\therefore |\vec{a}| = |\vec{b}|$

(ii) Solution:

Given that a and b are two vectors such that $|\vec{a}| = |\vec{b}|$

It means that the magnitudes of vector $\vec{a}$ are equal to the magnitude of vector $\vec{b}$, but we cannot conclude anything about the direction of vectors.
So, it is false that
$|\vec{a}| = |\vec{b}| \Rightarrow \vec{a} = \pm\vec{b}$

(iii) Solution:

Given that for any vector $\vec{a}$ and $\vec{b}$
$|\vec{a}| = |\vec{b}|$

It means that the magnitudes of vector $\vec{a}$ are equal to the magnitude of vector $\vec{b}$, but we cannot conclude anything about the direction of vectors.
Moreover, we know that only when $\vec{a} = \vec{b}$ means, both the direction and magnitude are the same for both the vectors. Hence, the given statement is false.