Exercise 23.3

1. Solution:

When,
Point R divides the line joining the two points P and Q in the ratio 1: 2 internally.

So, position vector of point R = $\frac{1(\vec{a}-2\vec{b})+2(2\vec{a}+\vec{b})}{1+2} = \frac{5\vec{a}}{3}$

And when,
Point R divides the line joining the two points P and Q in the ratio 1: 2 externally.

So, position vector of point R = $\frac{1(\vec{a}-2\vec{b})-2(2\vec{a}+\vec{b})}{1-2} = \frac{-3\vec{a}-4\vec{b}}{-1} = 3\vec{a}+4\vec{b}$

2. Solution:

It's given that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be the position vectors of the four distinct points A, B, C and D such that
$\vec{b}-\vec{a}=\vec{c}-\vec{d}$
Given that,
$\vec{b}-\vec{a}=\vec{c}-\vec{d}$
$\overrightarrow{AB}=\overrightarrow{DC}$
So, AB is parallel and equal to DC (in magnitude).
Therefore, ABCD is a parallelogram.

3. Solution:

It's given that $\vec{a}$ and $\vec{b}$ are position vectors of A and B respectively.
Let C be a part in AB produced such that AC = 3AB
Thus, it's clear that point C divides the line AB in ratio 3: 2 externally
So,
Position vector of point C is given by

$$\vec{c}=\frac{m\vec{b}-n\vec{a}}{m-n}$$
$$=\frac{3\vec{b}-2\vec{a}}{3-2}$$
$$\vec{c}=3\vec{b}-2\vec{a}$$

Again, let D be a point in BA produced such that BD = 2BA

Let $\vec{d}$ be the position vector of D. It is clear that point D divides the line AB in 1: 2 externally. So, position vector of D is given by

$\vec{d} = \frac{m\vec{a} - n\vec{b}}{m - n}$

$= \frac{2\vec{a} - \vec{b}}{2 - 1}$

$\vec{d} = 2\vec{a} - \vec{b}$

Therefore,

$\vec{c} = 3\vec{b} - 2\vec{a}$

$\vec{d} = 2\vec{a} - \vec{b}$

4. Solution:

Given that,

$3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$

$3\vec{a} + 5\vec{c} = 2\vec{b} + 6\vec{d} \qquad \text{(i)}$

It's seen that the sum of the coefficients on both the sides of the equation (i) is 8, so divide equation (i) by 8 on both the sides.

$\frac{3\vec{a} + 5\vec{c}}{8} = \frac{2\vec{b} + 6\vec{d}}{8}$

$\frac{3\vec{a} + 5\vec{c}}{3 + 5} = \frac{2\vec{b} + 6\vec{d}}{2 + 6}$

It shows that the position vector of a point P dividing AC in the ratio 3: 5, is same as that of point dividing BD in the ratio of 2: 6.
Hence, point P is the common point to AC and BD and P is the point of intersection of AC and BD.
So, A, B, C and D are coplanar.

Therefore, the position vector of point P is given by

$\frac{3\vec{a} + 5\vec{c}}{8}$ or $\frac{2\vec{b} + 6\vec{d}}{8}$

5. Solution:

Given that,

$5\vec{p} - 2\vec{q} + 6\vec{r} - 9\vec{s} = \vec{0}$

Where $\vec{p}, \vec{q}, \vec{r}$ and $\vec{s}$ are the position vectors of point P, Q, R and S.

$5\vec{p} + 6\vec{r} = 2\vec{q} + 9\vec{s} \qquad \text{(i)}$

It's seen that the sum of the coefficients on both the sides of the equation (i) is 11, so divide equation (i) by 11 on both the sides.

$$\frac{5\vec{p}+6\vec{r}}{11}=\frac{2\vec{q}+9\vec{s}}{11}$$
$$\frac{5\vec{p}+6\vec{r}}{5+6}=\frac{2\vec{q}+9\vec{s}}{2+9}$$

It shows that the position vector of a point A dividing PR in the ratio 6: 5 and QS in the ratio of 9: 2. Hence, point A is the common point to PR and QS and it is also point of intersection of PQ and QS. So, P, Q, R and S are coplanar.

Therefore, the position vector of point A is given by

$$\frac{5p+6q}{11} \quad \text{or} \quad \frac{2\vec{q}+9\vec{s}}{11}$$