Exercise 23.5

1. Solution:

Given,

$\vec{a} = -4\hat{i} - 3\hat{j}$

$|\vec{a}| = \sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

$\therefore |\vec{a}| = 5$

2. Solution:

Given,

$\vec{a} = 12\hat{i} + n\hat{j}$

$|\vec{a}| = \sqrt{(12)^2 + (n)^2}$

$13 = \sqrt{144+n^2}$ $\left[\text{Since } |\vec{a}| = 13\right]$

Now,

Squaring both sides,

$(13)^2 = \left(\sqrt{144+n^2}\right)^2$

$169 = 144 + n^2$

$n^2 = 169 - 144$

$n^2 = 25$

$n = \pm\sqrt{25}$

$n = \pm 5$

3. Solution:

Given,

$\vec{a} = \sqrt{3}\hat{i} + \hat{j}$

Let $\vec{b}$ is any vector parallel to $\vec{a}$

So, $\vec{b} = \lambda\vec{a}$ (where λ is any scalar)

$= \lambda\left(\sqrt{3}\hat{i} + \hat{j}\right)$

$\vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$

$|\vec{b}| = \sqrt{\left(\lambda\sqrt{3}\right)^2 + (\lambda)^2}$

$$= \sqrt{3\lambda^2 + \lambda^2}$$
$$= \sqrt{4\lambda^2}$$
$$|\vec{b}| = 2\lambda$$
$$4 = 2\lambda$$
$$\lambda = \frac{4}{2}$$
$$\therefore \lambda = 2$$

Therefore,

$$\vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$$
$$\vec{b} = 2\sqrt{3}\hat{i} + 2\hat{j}$$

4. (i)
Solution:

Given,
A = (4, -1) and B = (1, 3)
So, the position vector of A and B will be

$$A = 4\hat{i} - \hat{j}$$
$$B = \hat{i} + 3\hat{j}$$

Now, the vector AB is

$$\overrightarrow{AB} = \text{Position vector of } B - \text{Position vector of } A$$
$$= (\hat{i} + 3\hat{j}) - (4\hat{i} - \hat{j})$$
$$= \hat{i} + 3\hat{j} - 4\hat{i} + \hat{j}$$
$$\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$$

And,

$$|\overrightarrow{AB}| = \sqrt{(-3)^2 + (4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$|\overrightarrow{AB}| = 5$$

4. (ii)
Solution:

Given,
A = (-6, 3) and B = (-2, -5)
So, the position vector of A and B will be

$$A = -6\hat{i} + 3\hat{j}$$
$$B = -2\hat{i} - 5\hat{j}$$

Now, the vector AB is

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(-2\hat{i} - 5\hat{j}\right) - \left(-6\hat{i} + 3\hat{j}\right)$

$= -2\hat{i} - 5\hat{j} + 6\hat{i} - 3\hat{j}$

$\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$

And,

$\left|\overrightarrow{AB}\right| = \sqrt{(4)^2 + (-8)^2}$

$= \sqrt{16 + 64}$

$= \sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$

$\left|\overrightarrow{AB}\right| = 4\sqrt{5}$

5. Solution:

Given,
A = (-1, 3) and B = (-2, 1)
So, the position vector of A and B will be

$A = -\hat{i} + 3\hat{j}$

$B = -2\hat{i} + 1\hat{j}$

Now, the vector AB is

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(-2\hat{i} + \hat{j}\right) - \left(-\hat{i} + 3\hat{j}\right)$

$= -2\hat{i} + \hat{j} + \hat{i} - 3\hat{j}$

$= -\hat{i} - 2\hat{j}$

Therefore,

Coordinate of the position vector equivalent to $\overrightarrow{AB} = (-1, -2)$.