Exercise 23.6

1. Solution:

We know that,

Magnitude of a vector $x\hat{i} + y\hat{j} + z\hat{k}$ is given by $\sqrt{(x)^2 + y^2 + z^2}$.

So,

$$|\vec{a}| = \sqrt{(2)^2 + (3)^2 + (-6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$
$$= 7$$
$$\therefore |\vec{a}| = 7$$

2. Solution:

We know that,

Unit vector of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

Unit vector of $3\hat{i} + 4\hat{j} - 12\hat{k} = \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}$

$$= \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{9 + 16 + 144}}$$
$$= \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{169}}$$

Hence,

Unit vector of $(3\hat{i} + 4\hat{j} - 12\hat{k}) = \frac{1}{13}(3\hat{i} + 4\hat{j} - 12\hat{k})$

3. Solution:

Let

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Let d be the resultant vector of $\vec{a}, \vec{b},$ and $\vec{c}$,

$$\vec{d} = \vec{a} + \vec{b} + \vec{c}$$
$$= (\hat{i} - \hat{j} + 3\hat{k}) + (2\hat{i} + \hat{j} - 2\hat{k}) + (\hat{i} + 2\hat{j} - 2\hat{k})$$
$$\vec{d} = 4\hat{i} + 2\hat{j} - \hat{k}$$

Now,

Unit vector $\vec{d} = \frac{\vec{d}}{|\vec{d}|}$

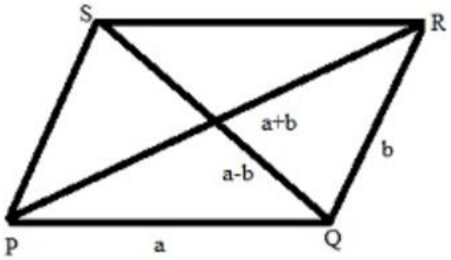
$$= \frac{4\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{(4)^2 + (2)^2 + (-1)^2}}$$

$$\vec{d} = \frac{4\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{16 + 4 + 1}}$$

4. Solution:

Let PQRS be a parallelogram such that

$PQ = \vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $QR = \vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$



In $\triangle PQR$

$$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$$

$$\overrightarrow{PR} = \vec{a} + \vec{b} = \hat{i} + \hat{j} - \hat{k} + \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{PR} = -\hat{i} + 2\hat{j} + \hat{k}$$

In $\triangle PSQ$

$$\overrightarrow{PS} + \overrightarrow{SQ} = \overrightarrow{PQ}$$

$$\overrightarrow{SQ} = \vec{a} - \vec{b} = \hat{i} + \hat{j} - \hat{k} - \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{SQ} = 3\hat{i} + 0\hat{j} - 3\hat{k}$$

Hence,

The unit vector along $\overrightarrow{PR} = \frac{\overrightarrow{PR}}{|\overrightarrow{PR}|} = \frac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{1+4+1}} = \frac{1}{\sqrt{6}}\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

The unit vector along $\overrightarrow{SQ} = \frac{\overrightarrow{SQ}}{|\overrightarrow{SQ}|} = \frac{3\hat{i} + 0\hat{j} - 3\hat{k}}{\sqrt{9+0+9}} = \frac{1}{\sqrt{2}}\left(\hat{i} - \hat{k}\right)$

5. Solution:

We have,

$$3\vec{a} - 2\vec{b} + 4\vec{c} = 3\left(3\hat{i} - \hat{j} - 4\hat{k}\right) - 2\left(-2\hat{i} + 4\hat{j} - 3\hat{k}\right) + 4\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$

$$= 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} - 8\hat{j} + 6\hat{k} + 4\hat{i} + 8\hat{j} - 4\hat{k}$$

$$= 17\hat{i} - 3\hat{j} - 10\hat{k}$$

Now,

$$\left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| = \sqrt{(17)^2 + (-3)^2 + (10)^2}$$

$$= \sqrt{289 + 9 + 100}$$

$$= \sqrt{398}$$

$$\therefore \left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| = \sqrt{398}$$