Exercise 23.7

1. Solution:

Given,

position vector of A = Position vector of $A = \vec{a} - 2\vec{b} + 3\vec{c}$

position vector of B = Position vector of $B = 2\vec{a} + 3\vec{b} - 4\vec{c}$

position vector of C = Position vector of $C = -7\vec{b} + 10\vec{c}$

Now,

$\overrightarrow{AB}$ = position vector of B – position vector of A

$= \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right) - \left(\vec{a} - 2\vec{b} + 3\vec{c}\right)$

$= 2\vec{a} + 3\vec{b} - 4\vec{c} - \vec{a} + 2\vec{b} - 3\vec{c}$

$\overrightarrow{AB} = \vec{a} + 5\vec{b} - 7\vec{c}$

And,

$\overrightarrow{BC}$ = position vector of C – position vector of B

$= \left(-7\vec{b} + 10\vec{c}\right) - \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right)$

$= -7\vec{b} + 10\vec{c} - 2\vec{a} - 3\vec{b} + 4\vec{c}$

$\overrightarrow{BC} = -2\vec{a} - 10\vec{b} + 14\vec{c}$

From $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$\overrightarrow{BC} = -2\left(\overrightarrow{AB}\right)$

So, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel but $\vec{B}$ is a common vector. Hence, A, B, C are collinear.

2. (i)
Solution:

Let the points be A, B and C
So,

Position vector of $A = \vec{a}$

Position vector of $B = \vec{b}$

Position vector of $C = 3\vec{a} - 2\vec{b}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \vec{b} - \vec{a}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= 3\vec{a} - 2\vec{b} - \vec{b}$

$= 3\vec{a} - 3\vec{b}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

Let $\overrightarrow{BC} = \lambda\left(\overrightarrow{AB}\right)$ [where λ is and scalar]

$3\vec{a} - 3\vec{b} = \lambda\left(\vec{b} - \vec{a}\right)$

$3\vec{a} - 3\vec{b} = \lambda\vec{b} - \lambda\vec{a}$

$3\vec{a} - 3\vec{b} = \lambda\vec{a} + \lambda\vec{b}$

On comparing the coefficients of LHS and RHS, we get
- λ = 3, λ = -3 and λ = 3
As the value of λ are different we can conclude that A, B and C are not collinear.

2. (ii)
Solution:

Let the points be A, B and C

Position vector of $A = \vec{a} + \vec{b} + \vec{c}$

Position vector of $B = 4\vec{a} + 3\vec{b}$

Position vector of $C = 10\vec{a} + 7\vec{b} - 2\vec{c}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(4\vec{a} + 3\vec{b}\right) - \left(\vec{a} + \vec{b} + \vec{c}\right)$

$= 4\vec{a} + 3\vec{b} - \vec{a} - \vec{b} - \vec{c}$

$\overrightarrow{AB} = 3\vec{a} + 2\vec{b} - \vec{c}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(10\vec{a} + 7\vec{b} - 2\vec{c}\right) - \left(4\vec{a} + 3\vec{b}\right)$

$= 10\vec{a} + 7\vec{b} - 2\vec{c} - 4\vec{a} - 3\vec{b}$

$\overrightarrow{BC} = 6\vec{a} + 4\vec{b} - 2\vec{c}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

$\overrightarrow{BC} = 2\left(\overrightarrow{AB}\right)$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, the points A, B C are collinear.

3. Solution:

Let the points be A, B and C
We have,

Position vector of $A = \hat{i} + 2\hat{j} + 3\hat{k}$

Position vector of $B = 3\hat{i} + 4\hat{j} + 7\hat{k}$

Position vector of $C = -3\hat{i} - 2\hat{j} - 5\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{AB} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

And,

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right) - \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right)$

$= -3\hat{i} - 2\hat{j} - 5\hat{k} - 3\hat{i} - 4\hat{j} - 7\hat{k}$

$\overrightarrow{BC} = -6\hat{i} - 6\hat{j} - 12\hat{k}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$ we get

$\overrightarrow{BC} = -3\left(\overrightarrow{AB}\right)$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a comm on vector. Hence, A, B, C are collinear.

4. Solution:

Let the points be A, B and C

Position vector of $A = 10\hat{i} + 3\hat{j}$

Position vector of $B = 12\hat{i} - 5\hat{j}$

Position vector of $C = a\hat{i} + 11\hat{j}$

Given that, A,B,C are collinear

$\Rightarrow \overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear

$\Rightarrow \overrightarrow{AB} = \lambda\left(\overrightarrow{BC}\right)$ (Where λ is same scalar)

So,

Position vector of B – Position vector of A = λ – (Position vector of C – Position vector of B)

$$\left(12\hat{i} - 5\hat{j}\right) - \left(10\hat{i} + 3\hat{j}\right) = \lambda\left[\left(a\hat{i} + 11\hat{j}\right) - \left(12\hat{i} - 5\hat{j}\right)\right]$$

$$12\hat{i} - 5\hat{j} - 10\hat{i} - 3\hat{j} = \lambda\left(a\hat{i} + 11\hat{j} - 12\hat{i} + 5\hat{j}\right)$$

$$2\hat{i} - 8\hat{j} = \left(\lambda a - 12\lambda\right)\hat{i} = \left(11\lambda + 5\lambda\right)\hat{j}$$

On comparing the coefficients of L.H.S and R.H.S, we get

$\lambda a - 12\lambda = 2$ … (i)

$-8 = 11\lambda + 5\lambda$ … (ii)

$-8 = 16\lambda$

$\lambda = -8/16$

$\Rightarrow \lambda = -1/2$

Now, putting the value of λ in (i) we get

$\lambda a - 12\lambda = 2$

$(-1/2)a - 12(-1/2) = 2$

$-1/2a + 12/2 = 2$

$-1/2a + 6 = 2$

$-1/2a = -4$

$a = (-4) \times (-2)$

$\therefore a = 8$

5. Solution:

Let A, B and C be the points then

Position vector of $A = \vec{a} + \vec{b}$

Position vector of $B = \vec{a} - \vec{b}$

Position vector of $C = \vec{a} + \lambda\vec{b}$

Now,

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (\vec{a} - \vec{b}) - (\vec{a} + \vec{b})$

$= \vec{a} - \vec{b} - \vec{a} - \vec{b}$

$\overrightarrow{AB} = -2\vec{b}$

And,

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= (\vec{a} + \lambda\vec{b}) - (\vec{a} - \vec{b})$

$= \vec{a} + \lambda\vec{b} - \vec{a} + \vec{b}$

$= \lambda\vec{b} + \vec{b}$

$\overrightarrow{BC} = (\lambda + 1)\vec{b}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$$\overrightarrow{AB} = \left[\frac{(\lambda+1)}{-2}\right](\overrightarrow{BC})$$

Let $\left(\frac{\lambda+1}{-2}\right) = \mu$

Since λ is a real number. So, μ is also a real no.

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$, but $\vec{B}$ is a common vector. Hence, A, B, C are collinear.