Exercise 23.8

1. (i)
Solution:

Let P, Q and R be the points whose position vectors are $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively.

Now,

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$= \left(3\hat{i}-2\hat{j}+\hat{k}\right)-\left(2\hat{i}+\hat{j}-\hat{k}\right)$

$= 3\hat{i}-2\hat{j}+\hat{k}-2\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{PQ} = \hat{i}-3\hat{j}+2\hat{k}$

$\overrightarrow{QR}$ = Position vector of R – Position vector of Q

$= \left(\hat{i}+4\hat{j}-3\hat{k}\right)-\left(3\hat{i}-2\hat{j}+\hat{k}\right)$

$= \hat{i}+4\hat{j}-3\hat{k}-3\hat{i}+2\hat{j}-\hat{k}$

$= -2\hat{i}+6\hat{j}-4\hat{k}$

$\overrightarrow{QR} = -2\overrightarrow{PQ}$

Therefore, $\overrightarrow{QR}$ is parallel to $\overrightarrow{PQ}$ but there is a common point Q. So, P, Q, R are collinear.

1. (ii)
Solution:

Let P, Q and R be the points represented by the vectors
$3\hat{i}-2\hat{j}+4\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+4\hat{j}-2\hat{k}$ respectively.

Now,

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$= \left(-\hat{i}+4\hat{j}-2\hat{k}\right)-\left(3\hat{i}-2\hat{j}+4\hat{k}\right)$

$= \hat{i}+\hat{j}+\hat{k}-3\hat{i}+2\hat{j}-4\hat{k}$

$= -2\hat{i}-3\hat{j}-3\hat{k}$

And,

$\overrightarrow{QR}$ = Position vector of R – Position vector of Q

$= \left(-\hat{i}+4\hat{j}-2\hat{k}\right)-\left(\hat{i}+\hat{j}+\hat{k}\right)$

$= -\hat{i}+4\hat{j}-2\hat{k}-\hat{i}-\hat{j}-\hat{k}$

$= -2\hat{i}+3\hat{j}-3\hat{k}$

$\overrightarrow{PQ} = \overrightarrow{QR}$

So, $\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ but Q is the common point Q.

Thus, P, Q and R are collinear.

2. (i)
Solution:

Given,

$\vec{A} = 6\hat{i} - 7\hat{j} - \hat{k}$

$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{C} = 4\hat{i} - 5\hat{j} - 0 \times \hat{k}$

Now,

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(2\hat{i} - 3\hat{j} + \hat{k}\right) - \left(6\hat{i} - 7\hat{j} - \hat{k}\right)$

$= 2\hat{i} - 3\hat{j} + \hat{k} - 6\hat{i} + 7\hat{j} + \hat{k}$

$\overrightarrow{AB} = -4\hat{i} + 4\hat{j} + 2\hat{k}$

And,

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(4\hat{i} - 5\hat{j} - 0 \times \hat{k}\right) - \left(2\hat{i} - 3\hat{j} + \hat{k}\right)$

$= 4\hat{i} - 5\hat{j} - 0 \times \hat{k} - 2\hat{i} + 3\hat{j} - \hat{k}$

$\overrightarrow{BC} = 2\hat{i} - 2\hat{j} - \hat{k}$

Thus, its seen that

$\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but B is the common point. Hence, A, B and C are collinear.

2. (ii)
Solution:

$\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\vec{C} = 3\hat{i} + \hat{j} + 2\hat{k}$

Now,

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(4\hat{i} + 3\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= 4\hat{i} + 3\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

And,

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(3\hat{i} + \hat{j} + 2\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + \hat{k}\right)$

$= 3\hat{i} + \hat{j} + 2\hat{k} - 4\hat{i} - 3\hat{j} - \hat{k}$

$\overrightarrow{BC} = -\hat{i} - 2\hat{j} + \hat{k}$

So, $\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B, C are collinear.

2. (iii)
Solution:

Given,

$\vec{A} = \hat{i} + 2\hat{j} + 7\hat{k}$

$\vec{B} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{C} = 3\hat{i} + 10\hat{j} - \hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 7\hat{k}\right)$

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k}$

$\overrightarrow{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$

And,

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(3\hat{i} + 10\hat{j} - \hat{k}\right) - \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} + \hat{j} + 2\hat{k} - 2\hat{i} - 6\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = \hat{i} + 4\hat{j} - 4\hat{k}$

Thus, it's seen that

$\overrightarrow{AB} = \overrightarrow{BC}$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B and C are collinear.

2. (iv)
Solution:

Given,

$\vec{A} = -3\hat{i} - 2\hat{j} - 5\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{C} = 3\hat{i} + 4\hat{j} + 7\hat{k}$

Now,

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right)$

$= \hat{i} + 2\hat{j} + 3\hat{k} + 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = 4\hat{i} + 4\hat{j} + 8\hat{k}$

And,

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So, $\overrightarrow{AB} = 2\overrightarrow{BC}$

Hence, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B and C are collinear.

2. (v)
Solution:

Given,

$\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$

$\vec{C} = -\hat{i} + 11\hat{j} + 9\hat{k}$

Now,

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \left(3\hat{i} - 5\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= 3\hat{i} - 5\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{AB} = \hat{i} - 4\hat{j} - 2\hat{k}$

And,

$\overrightarrow{BC} = \vec{C} - \vec{B}$

$= \left(-\hat{i} + 11\hat{j} + 9\hat{k}\right) - \left(3\hat{i} - 5\hat{j} + \hat{k}\right)$

$= -\hat{i} + 11\hat{j} + 9\hat{k} - 3\hat{i} + 5\hat{j} - \hat{k}$

$= -4\hat{i} + 16\hat{j} + 8\hat{k}$

So, $\overrightarrow{AB} = -4\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to vector $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

Therefore, A, B and C are collinear.