Exercise 23.9

1. Solution:

We know that, if l, m and n are the direction cosines of a vector and α, β, γ be the direction angle
Then,
$l = \cos\alpha$
$m = \cos\beta$
$n = \cos\gamma$

And, $l^2 + m^2 + n^2 = 1$ … (i)
Here,
$l = 1/\sqrt{2}$
$m = ½$
$n = -½$

Putting the value of l, m and n in (i), we get
$(1/\sqrt{2})^2 + (1/2)^2 + (-1/2)^2 = 1$
$½ + ¼ + ¼ = 1$
$(2 + 1 + 1)/4 = 1$
$1 = 1$
L.H.S = R.H.S

Therefore, a vector can have direction angles as 45^o, 60^o and 120^o.

2. Solution:

Given,
$l = 1$, $m = 1$ and $n = 1$
We know that,
$l^2 + m^2 + n^2 = 1$
$(1)^2 + (1)^2 + (1)^2 = 1$
$1 + 1 + 1 = 1$
$3 \neq 1$
L.H.S ≠ R.H.S

Therefore, 1,1,1 cannot be the direction cosines of a straight line.

3. Solution:

We have,
$\alpha = \pi/4$ and $\beta = \pi/4$
So,
$l = \cos\alpha = \cos\pi/4 = 1/\sqrt{2}$
$m = \cos\beta = \cos\pi/4 = 1/\sqrt{2}$
And,

$n = \cos \gamma$
We know that,
$l^2 + m^2 + n^2 = 1$
$(1/\sqrt{2})^2 + (1/\sqrt{2})^2 + \cos^2 \gamma = 1$
$½ + ½ + \cos^2 \gamma = 1$
$1 + \cos^2 \gamma = 1$
$\cos^2 \gamma = 0$
Taking square root on both sides, we get
$\cos \gamma = 0$
$\gamma = \cos^{-1} 0$
$\gamma = \pi/2$

Therefore, the angle made by the vector with the z-axis is $\pi/2$.

4. Solution:

We have,
$\alpha = \beta = \gamma$
So,
$\cos \alpha = \cos \beta = \cos \gamma$
Also,
$l = m = n = k$ (say)
We know that,
$l^2 + m^2 + n^2 = 1$
$k^2 + k^2 + k^2 = 1$
$3k^2 = 1$
$k^2 = 1/3$
Taking square root on both sides, we get
$k = \pm 1/\sqrt{3}$

So,
$l = \pm 1/\sqrt{3}$, $m = \pm 1/\sqrt{3}$ and $n = \pm 1/\sqrt{3}$
Hence, the direction cosines of vector r are $\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3}$
Now,

$$\text{Vector } \vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$$

$$= 6\left(\pm\frac{1}{\sqrt{3}}\hat{i} + \pm\frac{1}{\sqrt{3}}\hat{j} + \pm\frac{1}{\sqrt{3}}\hat{k}\right)$$

$$= \frac{\pm 6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) \quad \text{[Rationalizing the denominator]}$$

$$= \frac{\pm 6\sqrt{3}}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\vec{r} = \pm 2\sqrt{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

5. Solution:

We have,
$\alpha = 45^o$, $\beta = 60^o$ and $\gamma = \theta$ (say)
So,
$l = \cos \alpha = \cos 45^o = 1/\sqrt{2}$
$m = \cos \beta = \cos 60^o = ½$
And,
$n = \cos \gamma = \cos \theta$
We know that,
$l^2 + m^2 + n^2 = 1$
Putting values of l, m and n
$(1/\sqrt{2})^2 + (½)^2 + \cos^2 \theta = 1$
$½ + ¼ + \cos^2 \theta = 1$
$¾ + \cos^2 \theta = 1$
$\cos^2 \theta = 1 – ¾ = (4 - 3)/4$
$\cos^2 \theta = ¼$
Taking square root on both the sides we get,
$\cos \theta = \pm 1/2$

So, the direction cosines of the vector r are ± ½, ± ½, ± ½

Thus, the required vector is given by

$$\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$$

$$= 8\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} \pm \frac{1}{2}\hat{k}\right)$$

$$= 8\,\frac{\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}}{2}$$

$$\vec{r} = 4\left(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}\right)$$