Exercise 24.1

1.

(i) Solution:

Given,

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

$\vec{a}.\vec{b} = 1 \times 4 + (-2) \times (-4) + 1 \times 7$

$\vec{a}.\vec{b} = 19$

Hence, the dot product is 19.

(ii) Solution:

Given,

$\vec{a} = \hat{j} + 2\hat{k}$

$\vec{b} = 2\hat{i} + \hat{k}$

$\vec{a}.\vec{b} = 0 \times 2 + 0 \times 2 + 1 \times 2$

$\vec{a}.\vec{b} = 2$

Hence, the dot product is 2.

(iii) Solution:

Given,

$\vec{a} = \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{a}.\vec{b} = 0 \times 2 + 1 \times 3 + (-1) \times (-2)$

$\vec{a}.\vec{b} = 5$

Hence, the dot product is 5.

2.

(i) Solution:

Given,

$\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$

If $\vec{a}$ and $\vec{b}$ are $\perp$ to each other then $\vec{a}.\vec{b} = 0$

Now

$\vec{a}.\vec{b} = 0$

$\lambda (4) + 2 (-9) + 1 (2) = 0$
$4\lambda - 18 + 2 = 0$
$4\lambda - 16 = 0$
$4\lambda = 16$
$\lambda = 16/4$
$\therefore \lambda = 4$

(ii) Solution:

Given,

$\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 5\hat{i} - 9\hat{j} + 2\hat{k}$

If $\vec{a}$ and $\vec{b}$ are $\perp$ to each other then $\vec{a}.\vec{b} = 0$

Now,

$\vec{a}.\vec{b} = 0$

$\lambda (5) + 2 (-9) + 1 (2) = 0$
$5\lambda - 18 + 2 = 0$
$5\lambda - 16 = 0$
$5\lambda = 16$
$\therefore \lambda = 16/5$

(iii) Solution:

Given,

$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$

If $\vec{a}$ and $\vec{b}$ are $\perp$ to each other then $\vec{a}.\vec{b} = 0$

Now,

$\vec{a}.\vec{b} = 0$

2 (3) + 3 (2) + 4 (-λ) = 0
-4λ + 6 + 6 = 0
-4λ + 12 = 0
-4λ = -12
λ = -12/-4
∴ λ = 3

(iv) Solution:

$\vec{a} = \lambda\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$\vec{b} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$

If $\vec{a}$ and $\vec{b}$ are ⊥ to each other then $\vec{a}.\vec{b} = 0$

Now,

$\vec{a}.\vec{b} = 0$

λ (1) + 3 (-1) + 2 (3) = 0
λ – 3 + 6 = 0
λ + 3 = 0
∴ λ = -3

3. Solution:

Given,

$|\vec{a}| = 4, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 6$

We know that,

$\vec{a}.\vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$

So,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$$

= 6/(4 x 3)
= 6/12
= ½

θ = $\cos^{-1}$ (½)
∴ θ = π/3

4. Solution:

$\vec{a} = \hat{i} - \hat{j}$

$\vec{b} = -\hat{j} + 2\hat{k}$

So,

$\Rightarrow \vec{a} - 2\vec{b} = (\hat{i} - \hat{j}) - 2(-\hat{j} + 2\hat{k})$

$\vec{a} - 2\vec{b} = \hat{i} - \hat{j} + 2\hat{j} - 4\hat{k}$

$\vec{a} - 2\vec{b} = \hat{i} + \hat{j} - 4\hat{k}$

$\Rightarrow \vec{a} + \vec{b} = (\hat{i} - \hat{j}) + (-\hat{j} + 2\hat{k})$

$\vec{a} + \vec{b} = \hat{i} - \hat{j} - \hat{j} + 2\hat{k}$

$\vec{a} + \vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

Now,

$(\vec{a} - 2\vec{b}).(\vec{a} + \vec{b}) = (\hat{i} + \hat{j} - 4\hat{k})(\hat{i} - 2\hat{j} + 2\hat{k})$

$(\vec{a} - 2\vec{b}).(\vec{a} + \vec{b}) = 1 \times 1 + 1 \times (-2) + (-4) \times 2$

$(\vec{a} - 2\vec{b}).(\vec{a} + \vec{b}) = 1 - 2 - 8$

$(\vec{a} - 2\vec{b}).(\vec{a} + \vec{b}) = -9$

5.
(i) Solution:

Given,

$\vec{a} = \hat{i} - \hat{j}$

$\vec{b} = \hat{j} + \hat{k}$

We know that,

$\vec{a}.\vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$

So,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos\theta = \frac{(\hat{i} - \hat{j})(\hat{j} + \hat{k})}{\sqrt{1^2 + 1^2} \times \sqrt{1^2 + 1^2}}$$

$$\cos\theta = \frac{1 \times 0 + (-1) \times 1 + 0 \times 1}{\sqrt{2} \times \sqrt{2}}$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \pi - \frac{\pi}{3}$$

$$\therefore \theta = \frac{2\pi}{3}$$

Therefore, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$.

(ii) Solution:

Given,

$\vec{a} = 3\hat{i} - 2\hat{j} - 6\hat{k}$

$\vec{b} = 4\hat{i} - \hat{j} + 8\hat{k}$

We know that,

$\vec{a}.\vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$

So,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos\theta = \frac{(3\hat{i} - 2\hat{j} - 6\hat{k})(4\hat{i} - \hat{j} + 8\hat{k})}{\sqrt{3^2 + (-2)^2 + (-6)^2} \times \sqrt{4^2 + (-1)^2 + 8^2}}$$

$$\cos\theta = \frac{3 \times 4 + (-2) \times (-1) + (-6) \times 8}{\sqrt{9+4+36} \times \sqrt{16+1+64}}$$

$$\cos\theta = -\frac{34}{\sqrt{49} \times \sqrt{81}}$$

$$\cos\theta = -\frac{34}{7 \times 9}$$

$$\theta = \cos^{-1}\left(-\frac{34}{63}\right)$$

$$\theta = 122.66°$$

Therefore, angle between $\vec{a}$ and $\vec{b}$ is 122.66°.