

EXERCISE 26.1

Q1.i

Solution:

We know that,

$$[\hat{i} \hat{j} \hat{k}] + [\hat{j} \hat{k} \hat{i}] + [\hat{k} \hat{i} \hat{j}]$$

Now let us simplify the above expression, we get

$$\begin{aligned} [\hat{i} \hat{j} \hat{k}] + [\hat{j} \hat{k} \hat{i}] + [\hat{k} \hat{i} \hat{j}] &= (\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{k} \times \hat{i}) \cdot \hat{j} \\ &= \hat{k} \cdot \hat{k} + \hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{j} \\ &= 1 + 1 + 1 \\ &= 3 \end{aligned}$$

Hence,

$$[\hat{i} \hat{j} \hat{k}] + [\hat{j} \hat{k} \hat{i}] + [\hat{k} \hat{i} \hat{j}] = 3$$

ii.

Solution:

We know that,

$$[2\hat{i} \hat{j} \hat{k}] + [\hat{i} \hat{k} \hat{j}] + [\hat{k} \hat{j} 2\hat{i}]$$

Now let us simplify the above expression, we get

$$\begin{aligned} [2\hat{i} \hat{j} \hat{k}] + [\hat{i} \hat{k} \hat{j}] + [\hat{k} \hat{j} 2\hat{i}] &= (2\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{i} \times \hat{k}) \cdot \hat{j} + (\hat{k} \times \hat{j}) \cdot 2\hat{i} \\ &= 2\hat{k} \cdot \hat{k} + (-\hat{j}) \cdot \hat{j} + (-\hat{i}) \cdot 2\hat{i} \\ &= 2 - 1 - 2 \\ &= -1 \end{aligned}$$

Hence,

$$[2\hat{i} \hat{j} \hat{k}] + [\hat{i} \hat{k} \hat{j}] + [\hat{k} \hat{j} 2\hat{i}] = -1$$

Q2.i

Solution:

We know that,

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

Now let us simplify we get,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} \\ &= 2(-1-0) + 3(-1+3) \\ &= -2 + 6 \\ &= 4 \end{aligned}$$

Hence,

$$[\vec{a} \vec{b} \vec{c}] = 4$$

ii.

Solution:

We know that,

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

Now let us simplify we get,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} \\ &= 1(1+1) + 2(2+0) + 3(2-0) \\ &= 2 + 4 + 6 \\ &= 12 \end{aligned}$$

Hence,

$$[\vec{a} \vec{b} \vec{c}] = 12$$

Q3.i

Solution:

We know that the volume of a parallelepiped whose three adjacent edges is

$$\vec{a}, \vec{b}, \vec{c} = [[\vec{a} \vec{b} \vec{c}]].$$

So we now have,

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

Let us simplify we get,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} \\ &= 2(4-1) - 3(2+3) + 4(-1-6) \\ &= 6 - 15 - 28 \\ &= -9 - 28 \\ &= -37 \end{aligned}$$

Hence,

The volume of the parallelepiped is $[\vec{a} \vec{b} \vec{c}] = |-37| = 37$ cubic unit.

ii.

Solution:

Let us consider,

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$$

We know that the volume of a parallelepiped whose three adjacent edges is

$$\vec{a}, \vec{b}, \vec{c} = [[\vec{a} \ \vec{b} \ \vec{c}]].$$

So we now have,

$$[[\vec{a} \ \vec{b} \ \vec{c}]] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$$

Let us simplify we get,

$$\begin{aligned} [[\vec{a} \ \vec{b} \ \vec{c}]] &= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix} \\ &= 2(-4-1) + 3(-2+3) + 4(-1-6) \\ &= -10 + 3 - 28 \\ &= -10 - 25 \\ &= -35 \end{aligned}$$

Hence,

The volume of the parallelepiped is $[[\vec{a} \ \vec{b} \ \vec{c}]] = |-35| = 35$ cubic unit.

iii.

Solution:

Let us consider,

$$\vec{a} = 11\hat{i}, \vec{b} = 2\hat{j}, \vec{c} = 13\hat{k}$$

We know that the volume of a parallelepiped whose three adjacent edges is

$$\vec{a}, \vec{b}, \vec{c} = [[\vec{a} \ \vec{b} \ \vec{c}]].$$

So we now have,

$$[[\vec{a} \ \vec{b} \ \vec{c}]] = \begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix}$$

Let us simplify we get,

$$\begin{aligned} [[\vec{a} \ \vec{b} \ \vec{c}]] &= \begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix} \\ &= 11(26-0) + 0 + 0 \\ &= 286 \end{aligned}$$

Hence,

The volume of the parallelepiped is $[[\vec{a} \ \vec{b} \ \vec{c}]] = |286| = 286$ cubic unit.

iv.

Solution:

Let us consider,

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$$

We know that the volume of a parallelepiped whose three adjacent edges is

$$\vec{a}, \vec{b}, \vec{c} = [\vec{a} \vec{b} \vec{c}].$$

So we now have,

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

Let us simplify we get,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} \\ &= 1(1-2) - 1(-1-1) + 1(2+1) \\ &= -1 + 2 + 3 \\ &= 4 \end{aligned}$$

Hence,

The volume of the parallelepiped is $[\vec{a} \vec{b} \vec{c}] = |4| = 4$ cubic unit.

Q4.i
Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e., $[\vec{a} \vec{b} \vec{c}] = 0$.

So we now have,

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 2 & 7 \\ 5 & 6 & 5 \end{vmatrix}$$

Let us simplify we get,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 1 & 2 & -1 \\ 3 & 2 & 7 \\ 5 & 6 & 5 \end{vmatrix} \\ &= 1(10-42) - 2(15-35) - 1(18-10) \\ &= -32 + 40 - 8 \\ &= 0 \end{aligned}$$

\therefore The given vectors are coplanar.

ii.
Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e.,

$$[\vec{a} \vec{b} \vec{c}] = 0.$$

So we now have,

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

Let us simplify we get,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} \\ &= -4(12+3) + 6(-3+24) - 2(1+32) \\ &= -60 + 126 - 66 \\ &= 0 \end{aligned}$$

∴ The given vectors are coplanar.

iii.

Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e.,

$$[\vec{a} \vec{b} \vec{c}] = 0.$$

So we now have,

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

Let us simplify we get,

$$\begin{aligned} [\vec{a} \vec{b} \vec{c}] &= \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} \\ &= 1(15-12) + 2(-10+4) + 3(6-3) \\ &= 3-12+9 \\ &= 0 \end{aligned}$$

∴ The given vectors are coplanar.

Q5.i

Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e.,

$$[\vec{a} \vec{b} \vec{c}] = 0.$$

So we now have,

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0$$

Let us simplify we get,

$$\begin{aligned}
 &= 1(\lambda - 1) + 1(2\lambda + \lambda) + 1(-2 - \lambda) \\
 &= \lambda - 1 + 3\lambda - 2 - \lambda \\
 3 &= 3\lambda \\
 1 &= \lambda
 \end{aligned}$$

∴ The value of $\lambda = 1$.

ii.

Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e. $[\vec{a} \vec{b} \vec{c}] = 0$.

So we now have,

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix} = 0$$

Let us simplify we get,

$$\begin{aligned}
 &= 2(10 + 3\lambda) + 1(5 + 3\lambda) + 1(\lambda - 2\lambda) \\
 &= 20 + 6\lambda + 5 + 3\lambda - \lambda \\
 -25 &= 8\lambda \\
 \lambda &= -\frac{25}{8}
 \end{aligned}$$

∴ The value of $\lambda = -25/8$.

iii.

Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e., $[\vec{a} \vec{b} \vec{c}] = 0$.

So we now have,

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix} = 0$$

Let us simplify we get,

$$\begin{aligned}
 &= 1(2\lambda - 2) - 2(6 - 1) - 3(6 - \lambda) \\
 &= 2\lambda - 2 - 12 + 2 - 18 + 3\lambda \\
 &= 5\lambda - 30 \\
 30 &= 5\lambda \\
 \lambda &= 6
 \end{aligned}$$

∴ The value of $\lambda = 6$.

iv.

Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e., $[\vec{a} \vec{b} \vec{c}] = 0$.

So we now have,

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix} = 0$$

Let us simplify we get,

$$\begin{aligned} &= 1(0+5) - 3(0-5\lambda) + 0 \\ &= 5 + 15\lambda \\ -5 &= 15\lambda \\ \lambda &= -\frac{1}{3} \end{aligned}$$

\therefore The value of $\lambda = -1/3$.

