EXERCISE 27.1

Q1.
Solution:
Let us consider l, m, and n be the direction cosines of a line.
So,
$l = \cos 90^o = 0$
$m = \cos 60^o = ½$
$n = \cos 30^o = \sqrt{3}/2$
Hence,
The direction cosines of the line are 0, ½, √3/2

Q2.
Solution:
Let us consider l, m, and n be the direction cosines of a line.
Then,
a = 2, b = -1, c = -2 will be the direction ratios of the line.
So by using the formula,

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

We get,

$$l = \frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, m = \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, n = \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

$$l = \frac{2}{\sqrt{9}}, m = \frac{-1}{\sqrt{9}}, n = \frac{-2}{\sqrt{9}}$$

$$l = \frac{2}{3}, m = -\frac{1}{3}, n = -\frac{2}{3}$$

Hence,
The direction ratios of the line are 2/3, -1/3, -2/3

Q3.
Solution:
We know that the direction ratios of the line joining (-2, 4, -5) and (1, 2, 3) are
(1+2, 2-4, 3+5) = (3, -2, 8)
Then,
a = 3, b = -2, c = 8
So let us find the direction cosines:

$$\frac{3}{\sqrt{3^2+(-2)^2+8^2}}, \frac{-2}{\sqrt{3^2+(-2)^2+8^2}}, \frac{8}{\sqrt{3^2+(-2)^2+8^2}}$$

$$= \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

Hence,

The direction cosines are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

Q4.

Solution:

We know that,
A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11)
So,
The direction ratios of AB = (1-2, -2-3, 3+4) = (-1, -5, 7)
The direction ratios of BC = (3-1, 8+2, -11-3) = (2, 10, -14)
Then,
The direction cosines of AB and AC
-1/2 = -5/10 = 7/-14 are proportional.
We also know that,
B is a common point between two lines,
Hence, the points A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11) are collinear.

Q5.

Solution:

We know that,
A(3, 5, -4), B(-1, 1, 2) and C(-5, -5, -2)
So,
The direction ratios of side AB = (-1-3, 1-5, 2+4) = (-4, -4, 6)
So,
The direction cosines of AB is

$$\frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{6}{\sqrt{(-4)^2+(-4)^2+6^2}}$$

$$= \frac{-4}{\sqrt{68}}, \frac{-4}{\sqrt{68}}, \frac{6}{\sqrt{68}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

Now,
The direction ratios of side BC = (-5+1, -5-1, -2-2) = (-4, -6, -4)
So,
The direction cosines of BC is

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$
$$= \frac{-4}{\sqrt{68}}, \frac{-6}{\sqrt{68}}, \frac{-4}{\sqrt{68}}$$
$$= \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

Now,
The direction ratios of side AC = (-5-3, -5-5, -2+4) = (-8, -10, 2)
So,
The direction cosines of AC is

$$\frac{-8}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{-10}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{2}{\sqrt{(-8)^2+(-10)^2+2^2}}$$
$$= \frac{-8}{\sqrt{168}}, \frac{-10}{\sqrt{168}}, \frac{2}{\sqrt{168}}$$
$$= \frac{-4}{\sqrt{42}}, \frac{-5}{\sqrt{42}}, \frac{1}{\sqrt{42}}$$