EXERCISE 28.1

Q1.

Solution:

We know that the vector equation of a line is given as

$\vec{r} = \vec{a} + \lambda\vec{b}$

So, the Cartesian equation of a line is given as

$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{x - x_3}{a_3}$

By using the above formulas,

Hence,

The vector equation of the line is

$\vec{r} = (5\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

The Cartesian equation of a line is

$\frac{x - 5}{3} = \frac{y - 2}{2} = \frac{z + 4}{-8}$

Q2.

Solution:

It is given that,

The direction ratios of the line are: (3+1, 4-0, 6-2) = (4, 4, 4)

Since the given line passes through (-1, 0, 2)

We know that the vector equation of a line is given as

$\vec{r} = \vec{a} + \lambda\vec{b}$

So let us substitute the values, we get

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$

Hence,

The vector equation of the line is

$\vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$

Q3.

Solution:

Let us consider,

Vector equation of line passing through a fixed point vector a and parallel to vector b is given as

$\vec{r} = \vec{a} + \lambda\vec{b}$ Where, λ is scalar.

So here,

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{a} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

The equation of the required line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

Let us substitute the value of r as

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

So we get,

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (5 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (4 + 3\lambda)\hat{k}$

Now, let us compare the coefficients of vector i, j, R

$x = 5 + 2\lambda$, $y = -2 - \lambda$, $z = 4 + 3\lambda$

By equating to λ we get,

$\frac{x-5}{2} = \lambda, \frac{y+2}{-0} = \lambda, \frac{z-4}{3} = \lambda$

Hence,

The Cartesian form of equation of the line is

$\frac{x-5}{2} = \frac{y+2}{-0} = \frac{z-4}{3}$

Q4.

Solution:

Let us consider,

Vector equation of line passing through a fixed point vector a and parallel to vector b is given as

$\vec{r} = \vec{a} + \lambda\vec{b}$ Where, λ is scalar.

So here,

$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

The equation of the required line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$

Let us substitute the value of r as

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

So we get,

$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 3\lambda)\hat{i} + (-3 + 4\lambda)\hat{j} + (4 - 5\lambda)\hat{k}$

Now, let us compare the coefficients of vector i, j, R

$x = 2 + 3\lambda$, $y = -3 + 4\lambda$, $z = 4 - 5\lambda$

By equating to λ we get,

$\frac{x-2}{3} = \lambda, \frac{y+3}{4} = \lambda, \frac{z-4}{-5} = \lambda$

Hence,
The Cartesian form of equation of the line is
$$\frac{x-2}{3}=\frac{y+3}{4}=\frac{z-4}{-5}$$

Q5.
Solution:
It is given that, ABCD is a parallelogram.
Let us consider, AC and BD bisects each other at point O.
So,

$$\text{Position vector of point } O = \frac{\vec{a}+\vec{c}}{2}$$
$$= \frac{\left(4\hat{i}+5\hat{j}-10\hat{k}\right)+\left(-\hat{i}+2\hat{j}+\hat{k}\right)}{2}$$
$$= \frac{3\hat{i}+7\hat{j}-9\hat{k}}{2}$$

Let us consider position vector of point O and B are represented by $\vec{o}$ and $\vec{b}$.
So,
Equation of the line BD is the line passing through O and B is given by
$$\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right) \qquad \left[\begin{array}{l}\text{Since equation of the line passing through}\\ \text{two points } \vec{a} \text{ and } \vec{b}\end{array}\right]$$

$$\vec{r}=\vec{b}+\lambda\left(\vec{o}-\vec{b}\right)$$
$$=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(\frac{3\hat{i}+7\hat{j}-9\hat{k}}{2}-2\hat{i}-3\hat{j}+4\hat{k}\right)$$
$$\vec{r}=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}-9\hat{k}-4\hat{i}+6\hat{j}-8\hat{k}\right)$$
$$\vec{r}=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(-\hat{i}+13\hat{j}-17\hat{k}\right)$$

Now let us substitute the value r as
$$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$
So we get,
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)=(2-\lambda)\hat{i}+(-3+13\lambda)\hat{j}+(4-17\lambda)\hat{k}$$

Now, let us compare the coefficients of vector i, j, R
x = 2 - λ, y = -3 - 13λ, z = 4 - 17λ

By equating to λ we get,

$$\frac{x-2}{-1}=\lambda,\ \frac{y+3}{13}=\lambda,\ \frac{z-4}{-17}=\lambda$$

Hence,

The Cartesian form of equation of the line BD is

$$\frac{x-2}{-1}=\frac{y+3}{13}=\frac{z-4}{-17}$$