EXERCISE 28.2

Q1.

Solution:

Let us consider,

$$l_1 = \frac{12}{13}, m_1 = -\frac{3}{13}, n_1 = -\frac{4}{13}$$
$$l_2 = \frac{4}{13}, m_2 = \frac{12}{13}, n_1 = \frac{3}{13}$$
$$l_3 = \frac{3}{13}, m_3 = -\frac{4}{13}, n_3 = \frac{12}{13}$$

Now let us simplify, we get

$$l_1l_2 + m_1m_2 + n_1n_2$$
$$= \frac{12}{13} \times \frac{4}{13} + (-\frac{3}{13}) \times \frac{12}{13} + (-\frac{4}{13}) \times \frac{3}{13}$$
$$= \frac{48 - 36 - 12}{169} = 0$$
$$l_2l_3 + m_2m_3 + n_2n_3$$
$$= \frac{4}{13} \times \frac{3}{13} + \frac{12}{13} \times (-\frac{4}{13}) + \frac{3}{13} \times \frac{12}{13}$$
$$= \frac{12 - 48 + 36}{169} = 0$$
$$l_1l_3 + m_1m_3 + n_1n_3$$
$$= \frac{12}{13} \times \frac{3}{13} + (-\frac{3}{13}) \times (-\frac{4}{13}) + (-\frac{4}{13}) \times \frac{12}{13}$$
$$= \frac{36 + 12 - 48}{169} = 0$$

Hence, we can say that the lines are mutually perpendicular.

Q2.

Solution:

Given points are (1, -1, 2) and (3, 4, -2)

The direction ratios of a line passing through the points will be

(3-1, 4+1, -2-2) = (2, 5, -4)

Given points are (0, 3, 2) and (3, 5, 6)

The direction ratios of a line passing through the points will be

(3-0, 5-3, 6-2) = (3, 2, 4)

So,

Now the angle between the lines will be

$$\cos\theta = \frac{(a_1a_2 + b_1b_2 + c_1c_2)}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{[2\times 3 + 5\times 2 + (-4)\times 4]}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos\theta = \frac{0}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

Hence, we can say that the lines are mutually perpendicular.

Q3.

Solution:

Given points are (4, 7, 8) and (2, 3, 4)

The direction ratios of a line passing through the points will be

(4-2, 7-3, 8-4) = (2, 4, 4)

Given points are (-1, -2, 1) and (1, 2, 5)

The direction ratios of a line passing through the points will be

(-1-1, -2-2, 1-5) = (-2, -4, -4)

So,

The direction ratios are proportional.

2/-2 = 4/-4 = 4/-4

Hence, we can say that the lines are mutually perpendicular.

Q4.

Solution:

It is given that,

The Cartesian equation of a line passing through (x_1, y_1, z_1) and with direction ratios (a_1, b_1, c_1) is

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

So the Cartesian equation of a line passing through points (-2, 4, -5) and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

Q5.

Solution:

It is given that.

Equation of lines is $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

By solving we get

$7 \times 1 + (-5) \times 2 + 1 \times 3 = 7 - 10 + 3$

$= 0$

Hence,

The lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$. are perpendicular to each other.