EXERCISE 28.3

Q1.
Solution:
Let us consider the equation of line,

$$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = \lambda \quad \text{...... (1)}$$

So the general point on line (1) is

$$(\lambda,\ 2\lambda + 2,\ 3\lambda - 3)$$

Now, let us consider another equation of line,

$$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} = \mu \quad \text{....... (2)}$$

So the general point on line (2) is

$$(2\mu + 2,\ 3\mu + 6,\ 4\mu + 3)$$

If lines (1) and (2) intersect, we get a common point
So for same value of λ and μ, must have,

$$\lambda = 2\mu + 2 \quad \Rightarrow \lambda - 2\mu = 2 \quad \text{........ (3)}$$

$$2\lambda + 2 = 3\mu + 6 \Rightarrow 2\lambda - 4\mu = 4 \quad \text{....... (4)}$$

$$3\lambda - 3 = 4\mu + 3 \Rightarrow 3\lambda - 4\mu = 6 \quad \text{...... (5)}$$

Now, let us solve (3) and (4), we get

$$\begin{array}{r} 2\lambda - 4\mu = 4 \\ \underset{(-)}{2\lambda} \underset{(+)}{-} \underset{(-)}{3\mu = 4} \\ \hline -\mu = 0 \end{array}$$

$\mu = 0$
Now, let us substitute $\mu = 0$ in equation (3), we get

$$\lambda - 2\mu = 2$$
$$\lambda - 2(0) = 2$$
$$\lambda = 2$$

By substituting the values λ and μ in equation (5), we get

$$3\lambda - 4\mu = 6$$
$$3(2) - 4(0) = 6$$
$$6 = 6$$

Hence,
LHS = RHS

Q2.

Solution:

Let us consider the equation of line,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \quad \text{....... (1)}$$

So the general point on line (1) is

$(3\lambda + 1,\ 2\lambda - 1,\ 5\lambda + 1)$

Now, let us consider another equation of line,

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \quad \text{...... (2)}$$

So the general point on line (2) is

$(4\mu - 2,\ 3\mu + 1,\ -2\mu - 1)$

If lines (1) and (2) intersect, we get a common point

So for same value of λ and μ, must have,

$3\lambda + 1 = 4\mu - 2 \Rightarrow 3\lambda - 4\mu = -3$ …….. (3)

$2\lambda - 1 = 3\mu + 1 \Rightarrow 2\lambda - 3\mu = 2$ ……… (4)

$5\lambda + 1 = -2\mu - 1 \Rightarrow 5\lambda + 2\mu = -2$ ……. (5)

Now, let us solve (3) and (4), we get

$$\begin{array}{r} 6\lambda - 8\mu = -6 \\ \underset{(-)}{6\lambda} \underset{(+)}{-} \underset{}{9\mu} = \underset{(-)}{6} \\ \hline \mu = -12 \end{array}$$

μ = -12

Now, let us substitute μ = -12 in equation (3), we get

$$3\lambda - 4(-12) = -3$$
$$3\lambda + 48 = -3$$
$$3\lambda = -3 - 48$$
$$3\lambda = -51$$
$$\lambda = \frac{-51}{3}$$
$$\lambda = -17$$

By substituting the values λ and μ in equation (5), we get

$5\lambda + 2\mu = -2$
$5(-17) + 2(-12) = -2$
$-85 - 24 = -2$
$-109 \neq -2$

Hence,
LHS ≠ RHS

Q3.
Solution:

Let us consider the equation of line,

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \quad \text{........ (1)}$$

So the general point on line (1) is

$(3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$

Now, let us consider another equation of line,

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \quad \text{........ (2)}$$

So the general point on line (2) is

$(\mu + 2,\ 3\mu + 4,\ 5\mu + 6)$

If lines (1) and (2) intersect, we get a common point
So for same value of λ and μ, must have,

$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda - \mu = 3$ ..... (3)
$5\lambda - 3 = 3\mu + 4 \Rightarrow 5\lambda - 3\mu = 7$ ...... (4)
$7\lambda - 5 = 5\mu + 6 \Rightarrow 7\lambda - 5\mu = 11$ ..... (5)

Now, let us solve (3) and (4), we get

$$\begin{array}{r} 15\lambda - 5\mu = 15 \\ 15\lambda - 9\mu = 21 \\ (-)\quad (+)\quad (-) \\ \hline 4\mu = -6 \end{array}$$

μ = -6/4

Now, let us substitute μ = -6/4 in equation (3), we get

$3\lambda - \mu = 3$

$3\lambda - \left(-\frac{3}{2}\right) = 3$

$3\lambda = 3 - \frac{3}{2}$

$\lambda = \frac{1}{2}$

By substituting the values λ and μ in equation (5), we get

$$7\lambda - 5\mu = 11$$

$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\frac{7}{2} + \frac{15}{2} = 11$$

$$\frac{22}{2} = 11$$

$$11 = 11$$

LHS = RHS

Since, the value of λ and μ obtained by solving equations (3) and (4) satisfies equation (5).

Hence, the given lines intersect each other.

So,

The point of intersection = $(3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$

$$= \left\{\frac{3}{2} - 1,\ \left(\frac{5}{2} - 3\right),\ \left(\frac{7}{2} - 5\right)\right\}$$

$$= \left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$$

∴ The point of intersection is $\left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$.

Q4.

Solution:

Given points A(0, -1, -1) and B(4, 5, 1)

The equation of line passing through the points is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 0}{4 - 0} = \frac{y + 1}{5 + 1} = \frac{z + 1}{1 + 1}$$

$$\frac{x}{4} = \frac{y + 1}{6} = \frac{z + 1}{2}$$

Let $\lambda = \frac{x}{4} = \frac{y + 1}{6} = \frac{z + 1}{2}$

So, general point on line AB is

(4λ, 4λ, 2λ - 1)

Given points C(3, 9, 4) and D(-4, 4, 4)

The equation of line passing through the points is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$
$$\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{4-4}$$
$$\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}$$

Let $\mu=\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}$

So, general point on line CD is

(-7μ + 3, -5μ + 9, 0μ + 4)

(-7μ + 3, -5μ + 9, 4)

If lines AB and CD intersect, there exists a common point.

So let us find the value of λ and μ.

$4\lambda=-7\mu+3 \quad \Rightarrow 4\lambda+7\mu=3$ ……. (1)

$6\lambda-1=-5\mu+9 \quad \Rightarrow 6\lambda+5\mu=10$ …… (2)

$2\lambda-1=4 \quad \Rightarrow 2\lambda-1=4$ …….. (3)

So from equation (3),

2λ = 4 + 1

2λ = 5

λ = 5/2

By substituting the value of λ = 5/2 in equation (2), we get

6(5/2) + 5μ = 10

5μ = 10 – 15

= -5

μ = -1

Now, by substituting the values of λ and μ in equation (1), we get

4λ + 7μ = 3

4(5/2) + 7(-1) = 3

10 – 7 = 3

3 = 3

LHS = RHS

Since, the value of λ and μ obtained by solving equations (3) and (4) satisfies equation (1).

Hence, the given lines, AB and CD intersect each other.
So,
The point of intersection of AB and CD = (-7μ+3, -5μ+9, 4)
= (-7(-)+3, -5(-1)+9, 4)
= (7+3, 5+9, 4)
= (10, 14, 4)
Hence, the point of intersection of AB and CD is (10, 14, 4).

Q5.
Solution:
Given:
The equations of lines are

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$$
$$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

If these lines intersect, there exists a common point.
So for some value of λ and μ, we must have

$$(\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$
$$(1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$$

So, the equation of coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get
1 + 3λ = 4 + 2μ => 3λ – 2μ = 3 …….. (1)
1 – λ = 0 => λ = 1 …………… (2)
-1 = -1 + 3μ => μ = 0 …………… (3)

By substituting the values of λ and μ in equation (1)
3λ – 2μ = 3
3(1) – 2(0) = 3
3 = 3
LHS = RHS
Since, the value of λ and μ satisfies equation (1).
Hence, the given lines intersect each other.
So,
The point of intersection by substituting the value of λ in equation (1), we get

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + (1)(3\hat{i} - \hat{j})$$
$$= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j}$$
$$= 4\hat{i} - \hat{k}$$

Hence, the point of intersection is (4, 0, -1).