EXERCISE 33.2

Question. 1

Solution:

Now, let us assume x be a binomial variate with parameters n and p,

So, Mean = np

and Variance = npq

Then, Mean – Variance = np – npq

= np(1 - q)

= np(p)

= np^2

Mean – Variance > 0

Mean > Variance

Therefore, mean can never be less than variance.

Question. 2

Solution:

From the question it is given that, mean = 9 and variance = 9/4

Now, let us assume x be a binomial variate with parameters n and p,

P + Q = 1

Q = 1 – P

So, Mean = np = 9 … [equation (i)]

Variance = npq = 9/4 … [equation (ii)]

By dividing equation (i) by equation (ii) we get,

npq/np = (9/4)/9

q = ¼

Then, P = 1 – Q

Substituting the value of Q we get,

P = 1 – ¼

P = (4 - 1)/4

P = ¾

Now, substitute the value of p in equation (i) we get,

n (¾) = 9

n = 36/3

n = 12

Then, the distribution is given by = ${}^nC_rP^rQ^{n-r}$

$P(X = r) = {}^{12}C_r (3/4)^r (1/4)^{12-r}$

Therefore, r = 0, 1, 2, …. 12

Question. 3

Solution:

From the question it is given that, mean = 9 and variance = 6
Now, let us assume x be a binomial variate with parameters n and p,
P + Q = 1
Q = 1 – P
So, Mean = np = 9 … [equation (i)]
Variance = npq = 6 … [equation (ii)]
By dividing equation (i) by equation (ii) we get,
npq/np = 6/9
q = 2/3
Then, P = 1 – Q
Substituting the value of Q we get,
P = 1 – 2/3
P = (3 - 2)/3
P = 1/3
Now, substitute the value of p in equation (i) we get,
n (1/3) = 9
n = 27
Then, the distribution is given by = $^nC_r P^r Q^{n-r}$
$P(X = r) = {}^{27}C_r (1/3)^r (2/3)^{27-r}$
Therefore, r = 0, 1, 2, …. 27

Question. 4

Solution:

From the question it is given that, the sum of mean and variance for 5 trials is 4.8
So, n = 5
Then, Mean + Variance = 4.8
np + npq = 4.8
By taking out common terms we get,
np(1 + q) = 4.8
Substitute the value of n,
5p (1 + q) = 4.8
We know that, P + Q = 1
5 (1 + q) (1 - q) = 4.8

$5(1 - q^2) = 4.8$
$1 - q^2 = 4.8/5$
$q^2 = 1 - 4.8/5$
$q^2 = (5 - 4.8)/5$
$q^2 = 0.2/5$
$q^2 = 2/50$
$q^2 = 1/25$
$q = \surd(1/25)$
$q = 1/5$

Then, P = 1- q
$P = 1 - 1/5$
$P = (5 -1)/5$
$P = 4/5$

Therefore, n = 5, p = 4/5 and q = 1/5

So, binomial distribution is

$P(X = r) = {}^nC_r P^r Q^{n-r}$

$P(X = r) = \ = {}^5C_r(4/5)^r (1/5)^{5-r}$

r = 0, 1, 2, 3, … 5

Question. 5

Solution:

From the question it is given that, mean = 20 and variance = 16

Now, let us assume parameters n and p of distribution,

So, Mean = np = 20 … [equation (i)]

Variance = npq = 16 … [equation (ii)]

By dividing equation (i) by equation (ii) we get,

npq/np = 16/20

q = 4/5

We know that, P + Q = 1

Then, P = 1 – Q

Substituting the value of Q we get,

$P = 1 - 4/5$
$P = (5 - 4)/5$
$P = 1/5$

Now, substitute the value of p in equation (i) we get,

$n (1/5) = 20$
$n = 100$

Then, the distribution is given by = $^nC_rP^rQ^{n-r}$
$P(X = r) = {}^{100}C_r (1/5)^r (4/5)^{100-r}$
Therefore, r = 0, 1, 2, …. 100

Question. 6
Solution:
From the question it is given that, the sum and product of the mean and the variance are 25/3 and 50/3 respectively.
Now, let us assume parameters n and p of binomial distribution,
We know that, P + Q = 1
Q = 1 – P
Then, Mean + Variance = 25/3
np + npq = 25/3
By taking common terms outside we get,
np (1 + q) = 25/3
np = 25/3(1 + q) … [equation (i)]
Now, Mean × Variance = 50/3
np × npq = 50/3
$n^2p^2q = 50/q$
From equation (i)
$(25/3(1 + q))^2 q = 50/3$
$625q = 50/3 (9(1 + q)^2)$
$625q = 150 (1 + q)^2$
$25q = 6(1 + q)^2$
$6 + 6q^2 + 12q – 25q = 0$
$6q^2 – 13q + 6 = 0$
$6q^2 – 9q – 4q + 6 = 0$
By taking common terms outside we get,
3q (2q - 3) – 2 (2q - 3) = 0
(2q - 3) (3q - 2) = 0
2q – 3 = 0, 3q – 2 = 0
q = 3/2, q = 2/3
Since q ≤ 1
q = 2/3
P = 1 – q
Substitute the value of q,
P = 1 – (2/3)

P = (3 - 2)/3

P = 1/3

Now, substitute the value of p and q in equation (i) we get,

np = 25/3(1 + q)

By cross multiplication,

3np (1 + q) = 25

3n (1/3) (1 + 2/3) = 25

3n (1/3) ((3 + 2)/3) = 25

3n (1/3) (5/3) = 25

n (5/3) = 25

n = 75/5

n = 15

Then, the binomial distribution is given by = $^{n}C_{r}P^{r}Q^{n-r}$

$P(X = r) = {}^{15}C_{r}\,(1/3)^{r}\,(2/3)^{15-r}$

Therefore, r = 0, 1, 2, …. 15