EXERCISE 20.2

Question. 1
Solution:

Let us assume that I = $\int_2^4 \frac{x}{x^2+1}dx$

Then, $\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{2x}{x^2+1}dx$

½ log (1 + x^2) = F(x)

We know that, by the second fundamental theorem of calculus, we get

I = F (4) – F (2)

= ½ [log (1 + 4^2) – log (1 + 2^2)]

= ½ [log 17 – log 5]

We know that, log a – log b = log (a/b)

So,

= ½ log (17/5)

Question. 2
Solution:

Let us assume 1 + log x = t

Then differentiating w.r.t. x, we get

(1/x) dx = dt

Now substitute x = 1

t = 1

Again substitute x = 2

t = 1 + log 2

Then given question becomes,

$$\int_1^2 \frac{1}{x(1+\log x)^2}dx = \int_1^{1+\log 2}\frac{dt}{t^2}$$

$$= \left[\frac{-1}{t}\right]_1^{1+\log 2}$$

$$= \left[\frac{-1}{1+\log 2}+1\right]$$

Now applying limits, we get,

= [(-1 + 1 + log 2)/(1 + log 2)]

= [log 2/(log e + log 2)] ... [because log e = 1]

We know that, log a + log b = log ab

= log 2/log 2e

Therefore, $\int_1^2 \frac{1}{x(1+\log x)^2} dx = \frac{\log 2}{\log 2e}$

Question. 3

Solution:

Let us assume that $9x^2 - 1 = t$

Then, differentiating w.r.t. x, we get,

18x dx = dt

Dividing both side by 6, we get

3x dx = dt/6

So, substitute x = 1

t = 8

Again substitute x = 2

t = 35

Then, given question becomes,

$$\int_1^2 \frac{3x}{9x^2 - 1} dx = \int_8^{35} \frac{dt}{6t}$$

$$= \frac{1}{6}\left[\log t\right]_8^{35}$$

Now applying limits, we get,

= 1/6 (log 35 – log 8)

We know that, log a – log b = log (a/b)

So,

$\int_1^2 \frac{3x}{9x^2 - 1} dx$ = 1/6 log (35/8)

Question. 6

Solution:

Let us assume that, $e^x = t$

Then, differentiating w.r.t. x, we get,

e^x dx = dt

So, substitute x = 0

t = 1

Again substitute x = 1

t = e

Then, the given question becomes,

$$\int_0^1 \frac{e^x}{1+e^{2x}}dx = \int_1^e \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_1^e$$

By applying limits, we get,

$= [\tan^{-1} e - \tan^{-1} 1]$

We know that, $\tan\frac{\pi}{4} = 1$

Then,

$= \tan^{-1} e - \pi/4$

Therefore, $\int_0^1 \frac{e^x}{1+e^{2x}}dx = \tan^{-1} e - \frac{\pi}{4}$

Question. 7

Solution:

Let us assume that, $x^2 = t$

Then, differentiating w.r.t. x, we get,

$2x\,dx = dt$

So, substitute $x = 0$

$t = 0$

Again substitute $x = 1$

$t = 1$

Then, the given question becomes,

$$\int_0^1 xe^{x^2}dx = \int_0^1 \frac{e^t dt}{2}$$

$$= \frac{1}{2}\int_0^1 e^t dt$$

$$= \frac{1}{2}\left[e^t\right]_0^1$$

By applying limits, we get,

$= ½ [e^1 - e^0]$

$= ½ [e - 1]$

Therefore, $\int_0^1 xe^{x^2}dx = \frac{1}{2}(e-1)$