EXERCISE 20.5

Question. 1
Solution:

From the question it is given that, $\int_0^3 (x+4)\,dx$

From the formula,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h) + \cdots + f(a+(n-1)h)]$$

Where, h = (b - a)/n

Here, a = 0, b = 3 and f(x) = (x + 4)

Then, h = (3 - 0)/n

By cross multiplication, nh = 3

Let us assume I = $\int_0^3 (x+4)\,dx$

Now, substitute the value of a in formula,

I = $\lim_{h\to 0} h$ [f(0) + f(0 + h) + f(0 + 2h) + … + f(0 + (n - 1)h)]

I = $\lim_{h\to 0} h$ [f(0) + f(h) + f(2h) + … + f((n - 1)h)]

So, f(h) = h + 4 and f(0) = 0 + 4 = 4

I = $\lim_{h\to 0} h$ [4 + (h + 4) + (2h + 4) + … + ((n – 1) h + 4)]

Then,

I = $\lim_{h\to 0} h$[4n + h (1 + 2 + 3 + … + (n - 1))

I = $\lim_{h\to 0} h$[4n + h((n(n - 1))/2)]

Since, $h \to 0 \text{ and } h = \frac{3}{n} \Rightarrow n \to \infty$

I = $\lim_{n\to\infty}$ 3/n [4n + (3/n) n(n - 1)/2]

I = $\lim_{n\to\infty}$ 3/n [4n + 3(n - 1)/2]

I = $\lim_{n\to\infty}$ [12 + 9(n - 1)/2n]

I = $\lim_{n\to\infty}$ 12 + 9/2 (1 – (1/n))

Then,

I = 12 + 9/2 (1 – 1/∞)

I = 12 + (9/2) (1 - 0)

I = 12 + (9/2)

I = (24 + 9)/2

I = 33/2

Therefore, $\int_0^3 (x+4)\,dx = \frac{33}{2}$

Question. 2

Solution:

From the question it is given that, $\int_0^2 (x+3)\,dx$

From the formula,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h) + \cdots + f(a+(n-1)h)]$$

Where, h = (b - a)/n

Here, a = 0, b = 2 and f(x) = (x + 3)

Then, h = (2 - 0)/n = 2/n

By cross multiplication, nh = 2

Let us assume I = $\int_0^2 (x+3)\,dx$

Now, substitute the value of a in formula,

I = $\lim_{h\to 0} h$ [f(0) + f(0 + h) + f(0 + 2h) + … + f(0 + (n - 1)h)]

I = $\lim_{h\to 0} h$ [f(0) + f(h) + f(2h) + … + f((n - 1)h)]

So, f(h) = h + 3 and f(0) = 0 + 3 = 3

I = $\lim_{h\to 0} h$ [3 + (h + 3) + (2h + 3) + … + ((n – 1) h + 3)]

Then,

I = $\lim_{h\to 0} h$[3n + h (1 + 2 + 3 + … + (n - 1))

I = $\lim_{h\to 0} h$[3n + h((n(n - 1))/2)]

Since, $h \to 0$ and $h = \frac{2}{n} \Rightarrow n \to \infty$

I = $\lim_{n\to\infty}$ 2/n [3n + (2/n) (n(n - 1)/2)]

I = $\lim_{n\to\infty}$ 2/n [3n + (n - 1)]

I = $\lim_{n\to\infty}$ [6 + 2(n - 1)/n]

I = $\lim_{n\to\infty}$ [6 + 2 (1 – (1/n))]

Then,

I = 6 + 2 (1 – 1/∞)

I = 6 + 2 (1 - 0)

I = 6 + 2

I = 8

Therefore, $\int_0^2 (x+3)\,dx = 8$

Question. 3

Solution:

From the question it is given that, $\int_1^3 (3x-2)\,dx$

From the formula,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h) + \cdots + f(a+(n-1)h)]$$

Where, h = (b - a)/n

Here, a = 1, b = 3 and f(x) = (3x - 2)

Then, h = (3 - 1)/n = 2/n

By cross multiplication, nh = 2

Let us assume I = $\int_1^3 (3x-2)\,dx$

Now, substitute the value of a in formula

I = $\lim_{h\to 0} h$ [f(1) + f(1 + h) + f(1 + 2h) + … + f(1 + (n - 1)h)]

So, f(1 + h) = 3(1 +h) – 2 = 3 + 3h – 2 = 3h + 1 and f(1) = 3(1) - 2 = 3 – 2 = 1

I = $\lim_{h\to 0} h$ [1 + {3(1 + h) -2} + {3(1 + 2h) – 2} + … + {3(1 + (n – 1) h – 2}]

Then,

I = $\lim_{h\to 0} h$[n + 3h (1 + 2 + 3 + … + (n - 1))

I = $\lim_{h\to 0} h$[n + 3h((n(n - 1))/2)]

Since, $h \to 0$ and $h = \frac{2}{n} \Rightarrow n \to \infty$

I = $\lim_{n\to\infty}$ 2/n [n + (3(2)/n)) (n(n - 1)/2)]

I = $\lim_{n\to\infty}$ 2/n [n + (6(n - 1))/2]

I = $\lim_{n\to\infty}$ 2/n [n + (3(n - 1))]

I = $\lim_{n\to\infty}$ [2 + (6(n - 1))/n]

I = $\lim_{n\to\infty}$ [2 + 6 (1 – (1/n))]

Then,

I = 2 + 6 (1 – 1/∞)

I = 2 + 6 (1 - 0)

I = 2 + 6

I = 8

Therefore, $\int_1^3 (3x - 2)\,dx = 8$

Question. 4

Solution:

From the question it is given that, $\int_{-1}^{1} (x + 3)\,dx$

From the formula,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a) + f(a+h) + f(a+2h) + \cdots + f(a+(n-1)h)]$$

Where, h = (b - a)/n

Here, a = -1, b = 1 and f(x) = (3x - 2)

Then, h = (1 – (-1))/n = 2/n

By cross multiplication, nh = 2

Let us assume I = $\int_{-1}^{1} (x + 3)\,dx$

Now, substitute the value of a in formula

I = $\lim_{h\to 0} h$ [f(-1) + f(-1 + h) + f(-1 + 2h) + … + f(-1 + (n - 1)h)]

So, f(-1 + h) = -1 + h + 3 = h + 2 and f(-1) = - 1 + 3 = 2

I = $\lim_{h\to 0} h$ [2 + (2 + h) + (2 + 2h) + … + ((n – 1) h + 2)]

Then,

I = $\lim_{h\to 0} h$[2n + h (1 + 2 + 3 + … + (n - 1))

I = $\lim_{h\to 0} h$[2n + h((n(n - 1))/2)]

Since, $h \to 0$ and $h = \frac{2}{n} \Rightarrow n \to \infty$

I = $\lim_{n\to\infty}$ 2/n [2n + (2/n) (n(n - 1)/2)]

I = $\lim_{n\to\infty}$ 2/n [2n + (n - 1)]

I = $\lim_{n\to\infty}$ [4 + (2(n - 1))/n]

I = $\lim_{n\to\infty}$ [4 + 2(1 – (1/n))]

Then,

I = 4 + 2 (1 – 1/∞)

I = 4 + 2 (1 - 0)

I = 4 + 2

I = 6

Therefore, $\int_{-1}^{1}(x+3)\,dx = 6$

Question. 5

Solution:

From the question it is given that, $\int_1^3 (3x-2)\,dx$

From the formula,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+\cdots+f(a+(n-1)h)]$$

Where, h = (b - a)/n

Here, a = 1, b = 3 and f(x) = (3x - 2)

Then, h = (3 - 1)/n = 2/n

By cross multiplication, nh = 2

Let us assume I = $\int_1^3 (3x-2)\,dx$

Now, substitute the value of a in formula

I = $\lim_{h\to 0}$ h[f(1) + f(1 + h) + f(1 + 2h) + ... + f(1 + (n - 1)h)]

So, f(1 + h) = 3(1 +h) – 2 = 3 + 3h – 2 = 3h + 1 and f(1) = 3(1) - 2 = 3 – 2 = 1

I = $\lim_{h\to 0}$ h [1 + (h + 1) + (2h + 1) + ... + ((n – 1)h + 1}]

Then,

I = $\lim_{h\to 0}$ h[n + h (1 + 2 + 3 + ... + (n - 1))

I = $\lim_{h\to 0}$ h[n + h((n(n - 1))/2)]

Since, $h \to 0$ and $h = \frac{5}{n} \Rightarrow n \to \infty$

I = $\lim_{n\to\infty}$ 5/n [n + (5/n) (n(n - 1)/2)]

I = $\lim_{n\to\infty}$ 5/n [n + (5(n - 1))/2]

I = $\lim_{n\to\infty}$ [5 + (25(n - 1))/2n]

I = $\lim_{n\to\infty}$ [5 + (25/2) (1 – (1/n))]

Then,

I = 5 + (25/2) (1 – 1/∞)

I = 5 + (25/2) (1 - 0)

I = 5 + 25/2

I = (10 + 25)/2

I = 35/2

Therefore, $\int_0^5 (x+1)\,dx = \frac{35}{2}$