

## EXERCISE 21.1

### Question. 1

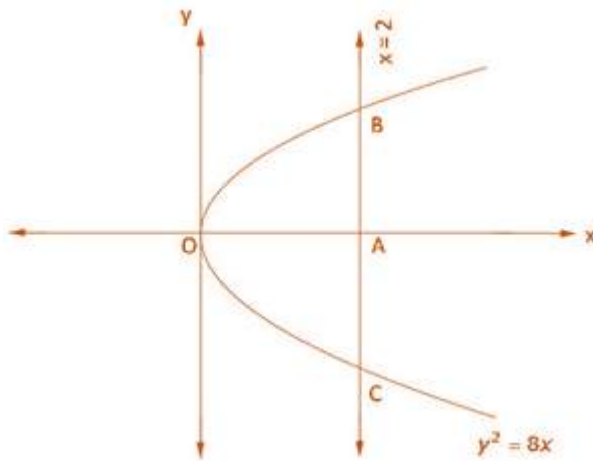
#### Solution:

From the question it is given that two equations,

$$x = 2 \quad \dots \text{ [equation (i)]}$$

$$y^2 = 8x \quad \dots \text{ [equation (ii)]}$$

So, equation (i) represents a line parallel to  $y$  – axis and equation (ii) represents a parabola with vertex at origin and  $x$  – axis as its axis, as shown in the rough sketch below,



Now, we have to find the area of OCBO,

Then, the area can be found by taking a small slice in each region of width  $\Delta x$ ,

And length =  $(y - 0) = y$

The area of sliced part will be as it is a rectangle =  $y \Delta x$

So, this rectangle can move horizontal from  $x = 0$  to  $x = 2$

The required area of the region bounded between the lines = Region OCBO

$$= 2 \text{ (region OABO)}$$

$$= 2 \int_0^2 y \, dx$$

Given,  $y^2 = 8x$

$$y = \sqrt{8x}$$

$$= 2 \int_0^2 \sqrt{8x} \, dx$$

$$= 2 \cdot 2\sqrt{2} \int_0^2 \sqrt{x} \, dx$$

On integrating we get,

$$= 4\sqrt{2} \left[ \frac{2}{3} x \sqrt{x} \right]_0^2$$

Now, applying limits we get,

$$= 4\sqrt{2}[(\frac{2}{3}2\sqrt{2}) - ((\frac{2}{3})0\sqrt{0})]$$

$$= 4\sqrt{2}(4\sqrt{2}/3)$$

Therefore, the required area =  $32/3$  square units.

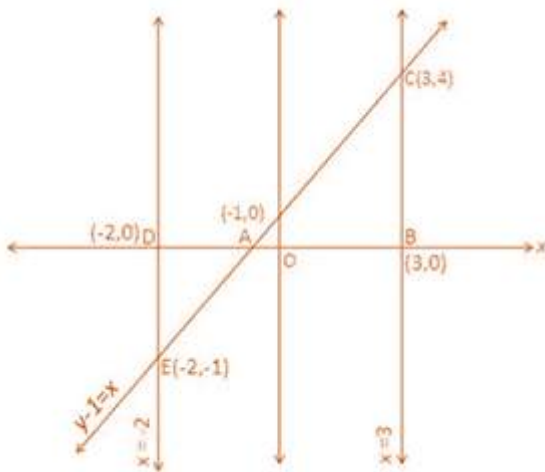
### Question. 2

#### Solution:

From the question it is given that equation,

$$y - 1 = x \quad \dots \text{ [equation (i)]}$$

So, equation (i) represents a line that meets at (0, 1) and (-1, 0), is as shown in the rough sketch below,



Now, we have to find the area of the region bounded by the line  $y - 1 = x$ ,

So, Required area = Region ABCA + Region ADEA

$$A = \int_{-1}^3 y dx + \left| \int_{-2}^{-1} y dx \right|$$

$$= \int_{-1}^3 (x + 1) dx + \left| \int_{-2}^{-1} (x + 1) dx \right|$$

On integrating we get,

$$= \left( \frac{x^2}{2} + x \right)_{-1}^3 + \left( \frac{x^2}{2} + x \right)_{-2}^{-1}$$

Now applying limits we get,

$$= [(9/2) + 3] - [(1/2) - 1] + [(1/2) - 1] - (2 - 2)$$

$$= [(15/2) + (1/2)] + [-1/2]$$

On simplification we get,

$$= 8 + \frac{1}{2}$$

A =  $17/2$  square units

Therefore, the area of the region bounded by the line  $y - 1 = x$  is  $17/2$  square units.

**Question. 3**

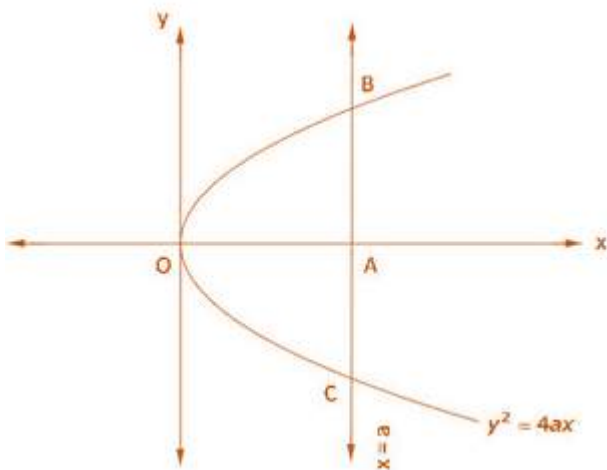
**Solution:**

From the question it is given that two equations,

$x = a$  ... [equation (i)]

$y^2 = 4ax$  ... [equation (ii)]

So, equation (i) represents a line parallel to  $y - axis$  and equation (ii) represents a parabola with vertex at origin and  $x - axis$  as its axis, as shown in the rough sketch below,



Now, we have to find the area of OCBO,

Then, the area can be found by taking a small slice in each region of width  $\Delta x$ ,

And length  $= (y - 0) = y$

The area of sliced part will be as it is a rectangle  $= y \Delta x$

So, this rectangle can move horizontal from  $x = 0$  to  $x = a$

The required area of the region bounded between the lines = Region OCBO

$$= 2(\text{Region OABO})$$

$$= 2 \int_0^a y \, dx$$

Given,  $y^2 = 4ax$

$$y = \sqrt{4ax}$$

$$= 2 \int_0^a \sqrt{4ax} \, dx$$

$$= 2 \cdot 2\sqrt{a} \int_0^a \sqrt{x} \, dx$$

On integrating we get,

$$= 4\sqrt{a} \left[ \frac{2}{3} x \sqrt{x} \right]_0^a$$

Now, applying limits we get,

$$= 4\sqrt{a} \left[ \frac{2}{3} a \sqrt{a} \right]$$

Therefore, the required area =  $(8/3)a^2$  square units.

#### Question. 4

##### Solution:

From the question it is given that equation,

$$y = 4x - x^2$$

Adding 4 on both side,

$$y + 4 = 4x - x^2 + 4$$

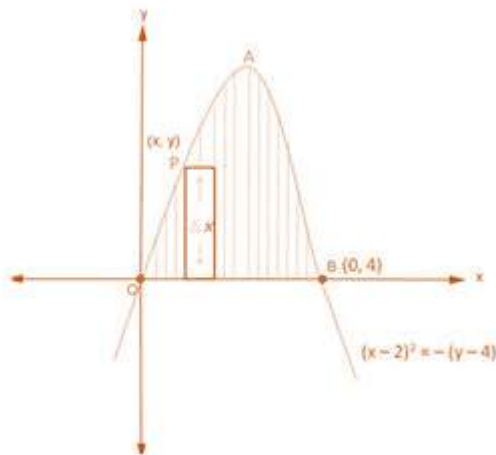
Transposing we get,

$$x^2 - 4x + 4 = -y + 4$$

We know that,  $(a - b)^2 = a^2 - 2ab + b^2$

$$(x - 2)^2 = -(y - 4) \quad \dots \text{[equation (i)]}$$

So, equation (i) represents a downward parabola with vertex (2, 4) and passing through (0, 0) and (0, 4), is as shown in the rough sketch below,



Then, the area can be found by taking a small slice in each region of width  $\Delta x$ ,

And length =  $(y - 0) = y$

The area of sliced part will be as it is a rectangle =  $y \Delta x$

So, this rectangle can move horizontal from  $x = 0$  to  $x = a$

The required area of the region bounded between the lines = Region OABO

Required area = region OABO

$$= \int_0^4 (4x - x^2) dx$$

On integrating we get,

$$= \left( 4\frac{x^2}{2} - \frac{x^3}{3} \right)_0^4$$

Now we have to apply limits,

$$= [(4 \times 16)/2 - (64/3)] - [0 - 0]$$

On simplification we get,

$$= 64/6$$

Divide both numerator and denominator by 2 we get,

$$= 32/3$$

Therefore, the required area is 32/3 square units.

### Question. 5

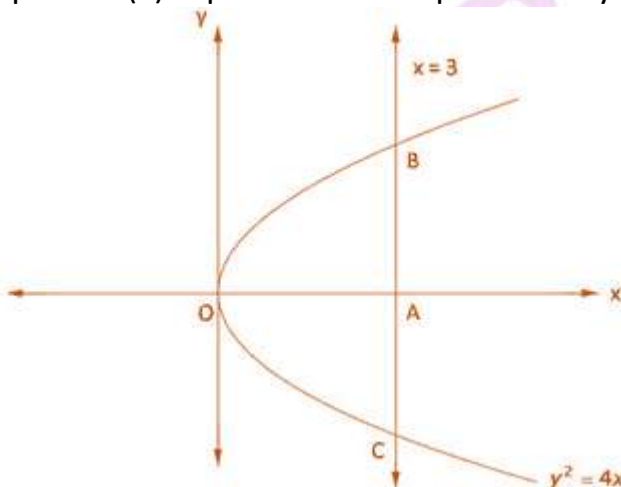
#### Solution:

From the question it is given that two equations,

$$y^2 = 4x \quad \dots \text{[equation (i)]}$$

$$x = 3 \quad \dots \text{[equation (ii)]}$$

So, equation (i) represents a parabola with vertex at origin and x – axis as its axis and equation (ii) represents a line parallel to y – axis, as shown in the rough sketch below,



Now, we have to find the area of OCBO,

Then, the area can be found by taking a small slice in each region of width  $\Delta x$ ,

And length =  $(y - 0) = y$

The area of sliced part will be as it is a rectangle =  $y \Delta x$

So, this rectangle can move horizontal from  $x = 0$  to  $x = 3$

The required area of the region bounded between the lines = Region OCBO

$$= 2 \text{ (region OABO)}$$

$$= 2 \int_0^3 y dx$$

Given,  $y^2 = 4x$

$$y = \sqrt{4x}$$

$$= 2 \int_0^3 \sqrt{4x} dx$$

$$= 4 \int_0^3 \sqrt{x} dx$$

On integrating we get,

$$= 4 \left( \frac{2}{3} x \sqrt{x} \right)_0^3$$

Now, applying limits we get,

$$= 8\sqrt{3}$$

Therefore, the required area is  $8\sqrt{3}$  square units.

