

EXERCISE 21.2

Question. 1

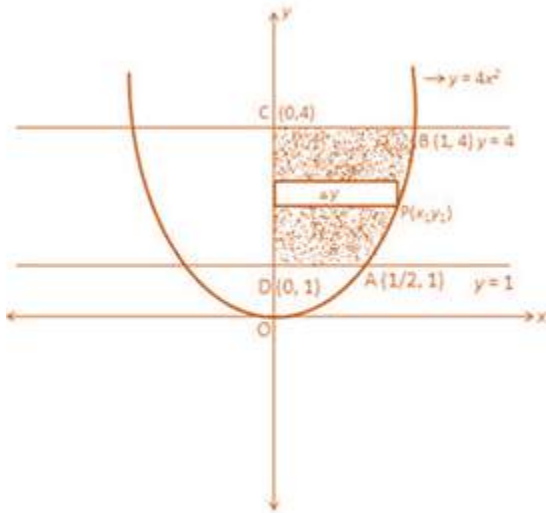
Solution:

From the question it is given that,

Lines, $x = 0$, $y = 1$, $y = 4$

Parabola $y = 4x^2$... [equation (i)]

So, equation (i) represents a parabola with vertex $(0, 0)$ and axis as y – axis. $x = 0$ is y – axis and $y = 1$, $y = 4$ are line parallel to x – axis passing through $(0, 1)$ and $(0, 4)$ respectively, as shown in the rough sketch below,



Now, we have to find the area of ABCDA,

Then, the area can be found by taking a small slice in each region of width Δy ,

And length = x

The area of sliced part will be as it is a rectangle = $x \Delta y$

So, this rectangle can move horizontal from $y = 1$ to $x = 4$

The required area of the region bounded between the lines = Region ABCDA

$$= \int_1^4 x dy$$

Given, $y = 4x^2$

$$x = \sqrt{y/4}$$

$$= \int_1^4 \sqrt{\frac{y}{4}} dy$$

$$= \frac{1}{2} \int_1^4 \sqrt{y} dy$$

On integrating we get,

$$= \frac{1}{2} \left[\frac{2}{3} y \sqrt{y} \right]_1^4$$

Now, applying limits we get,

$$\begin{aligned} &= \frac{1}{2} \left[\left(\frac{2}{3} \right) \times 4 \times \sqrt{4} - \left(\frac{2}{3} \right) \times 1 \times \sqrt{1} \right] \\ &= \frac{1}{2} \left[\left(\frac{16}{3} \right) - \left(\frac{2}{3} \right) \right] \\ &= \frac{1}{2} \left[\frac{16 - 2}{3} \right] \\ &= \frac{1}{2} \left[\frac{14}{3} \right] \\ &= \frac{7}{3} \end{aligned}$$

Therefore, the required area is $\frac{7}{3}$ square units.

Question. 2

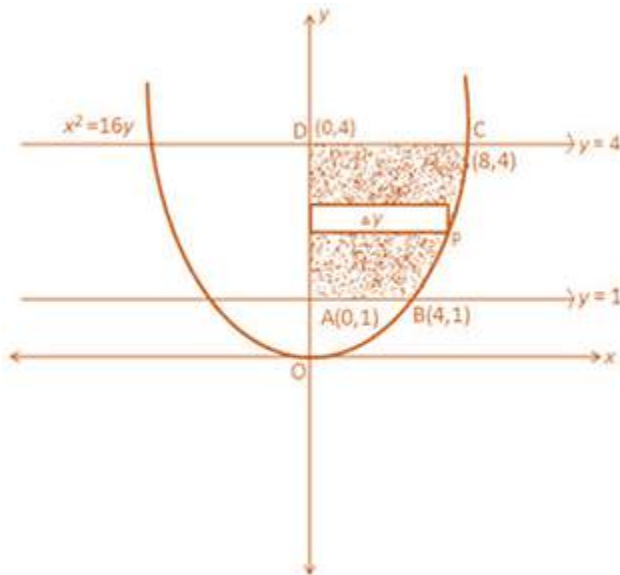
Solution:

From the question it is given that,

Region in first quadrant bounded by $y = 1$, $y = 4$

Parabola $x^2 = 16y$... [equation (i)]

So, equation (i) represents a parabola with vertex $(0, 0)$ and axis as y – axis, as shown in the rough sketch below,



Now, we have to find the area of ABCDA,

Then, the area can be found by taking a small slice in each region of width Δy ,

And length = x

The area of sliced part will be as it is a rectangle = $x \Delta y$

So, this rectangle can move horizontal from $y = 1$ to $x = 4$

The required area of the region bounded between the lines = Region ABCDA

$$= \int_1^4 x dy$$

Given, $x^2 = 16y$

$$x = \sqrt{16y}$$

$$x = 4\sqrt{y}$$

$$= \int_1^4 4\sqrt{y} dy$$

On integrating we get,

$$= 4 \left[\frac{2}{3} y \sqrt{y} \right]_1^4$$

Now, applying limits we get,

$$= 4 \left[\left(\frac{2}{3} \times 4 \times \sqrt{4} \right) - \left(\frac{2}{3} \times 1 \times \sqrt{1} \right) \right]$$

$$= 4 \left[\left(\frac{16}{3} \right) - \left(\frac{2}{3} \right) \right]$$

$$= 4 \left[\frac{16 - 2}{3} \right]$$

$$= 4 \left[\frac{14}{3} \right]$$

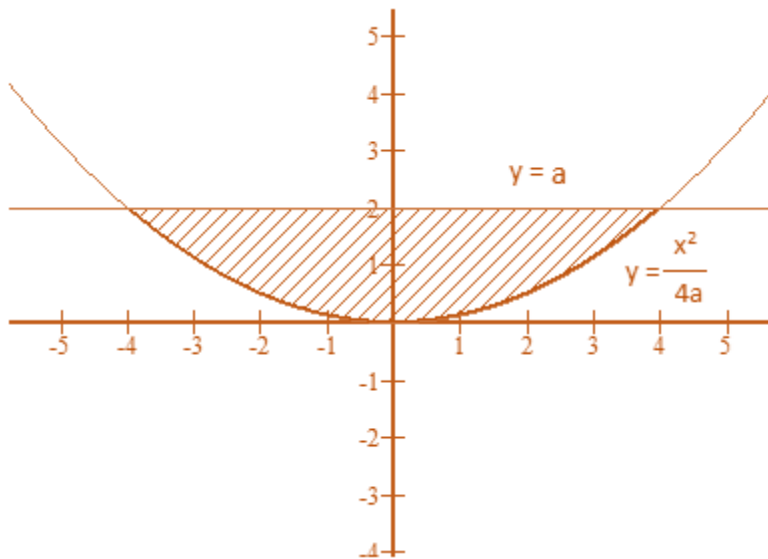
$$= \frac{56}{3}$$

Therefore, the required area is $\frac{56}{3}$ square units.

Question. 3

Solution:

We have to find the area of the region bounded by $x^2 = 4ay$



Then,

$$\text{Area of the region} = 2 \times \int_0^{2a} \left(a - \frac{x^2}{4a} \right) dx$$

On integrating we get,

$$= 2 \times \left[ax - \frac{x^3}{12a} \right]_0^{2a}$$

Now applying limits,

$$= 2 \times \left[\left(a(2a - 0) \right) - \left(\frac{(2a)^3 - 0^3}{12a} \right) \right]$$

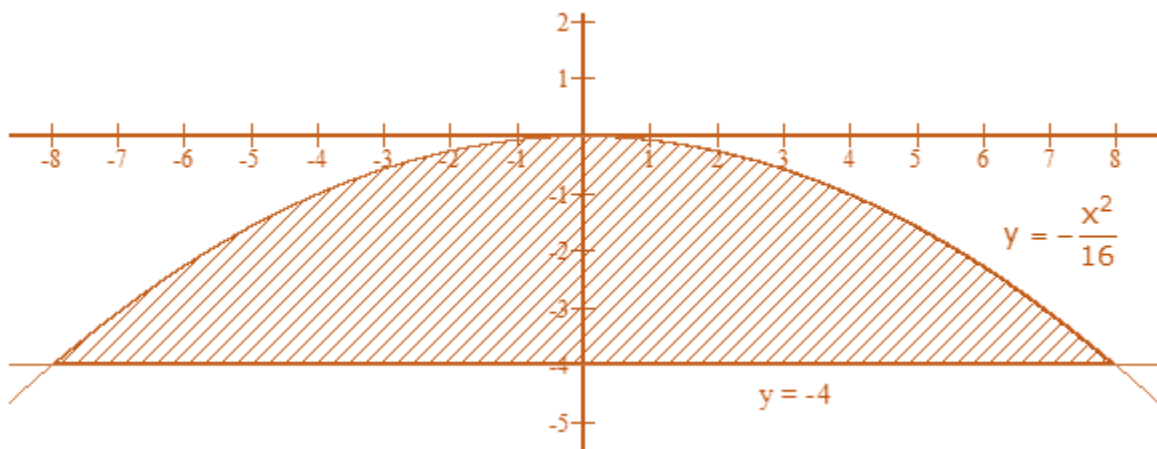
$$\begin{aligned}
 &= 2 \times [(2a^2) - (8a^3/12a)] \\
 &= 2 \times [(24a^3 - 8a^3)/12a] \\
 &= 2 \times [16a^3/12a] \\
 &= 2 \times [4a^2/3] \\
 &= 8a^2/3
 \end{aligned}$$

Therefore, the area of the region is $8a^2/3$ square units.

Question. 4

Solution:

We have to find the area of the region bounded by $x^2 + 16y = 0$



Then,

$$\text{Area of the region} = 2 \times \int_0^8 \left[-\frac{x^2}{16} - (-4) \right] dx$$

On integrating we get,

$$= 2 \times \left[4x - \frac{x^3}{48} \right]_0^8$$

Now applying limits,

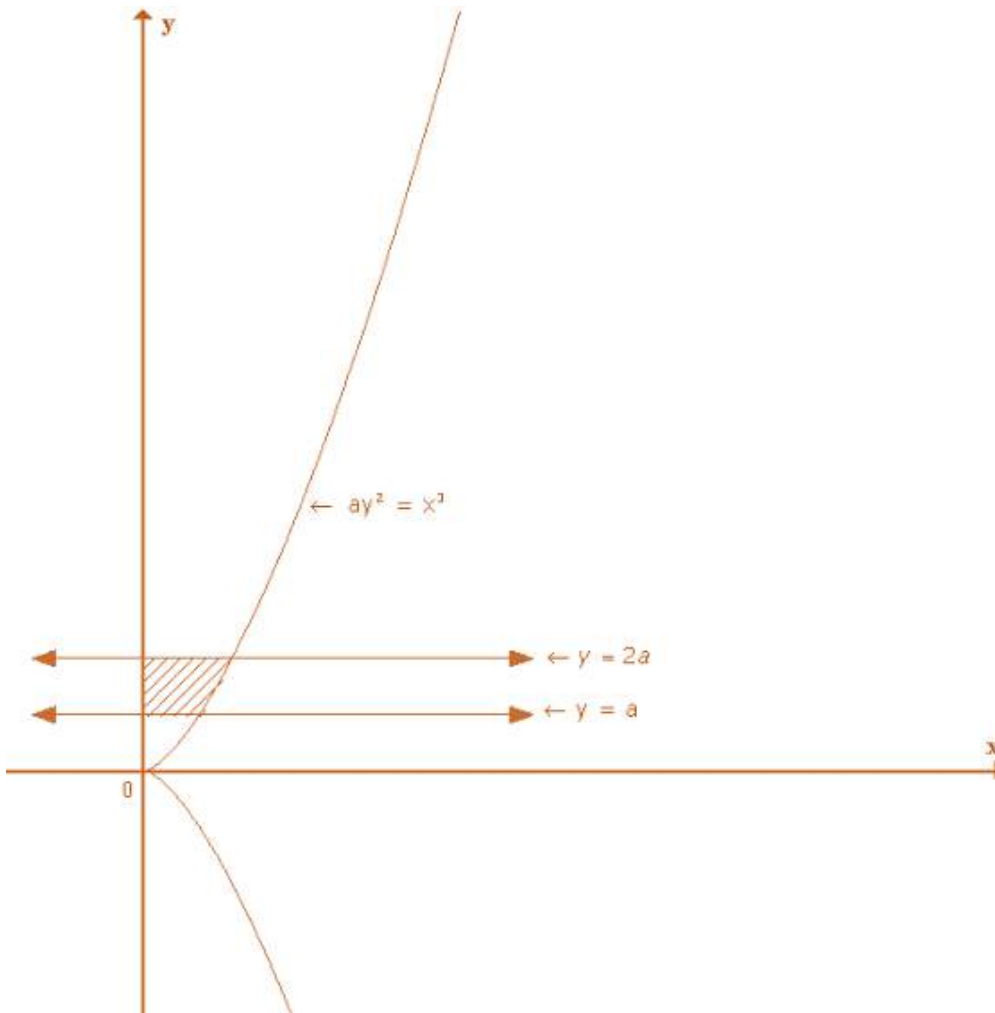
$$\begin{aligned}
 &= 2 \times [(4(8 - 0)) - ((8)^3 - 0^3)/48] \\
 &= 2 \times [(32) - (512/48)] \\
 &= 2 \times [(32) - (32/3)] \\
 &= 2 \times [(96 - 32)/3] \\
 &= 2 \times [64/3] \\
 &= 128/3
 \end{aligned}$$

Therefore, the area of the region is $128/3$ square units.

Question. 5

Solution:

We have to find the area of the region bounded by curve $ay^2 = x^3$, and lines $y = a$ and $y = 2a$.



Then,

$$\begin{aligned} \text{Area of the region} &= \int_a^{2a} (ay^2)^{\frac{1}{3}} dy \\ &= a^{\frac{1}{3}} \int_a^{2a} y^{\frac{2}{3}} dy \end{aligned}$$

On integrating we get,

$$= a^{\frac{1}{3}} \left[\frac{3}{5} y^{\frac{5}{3}} \right]_a^{2a}$$

Now applying limits we get,

$$= \frac{3}{5} \left(2^{\frac{5}{3}} - 1 \right) a^2 \text{ sq units}$$