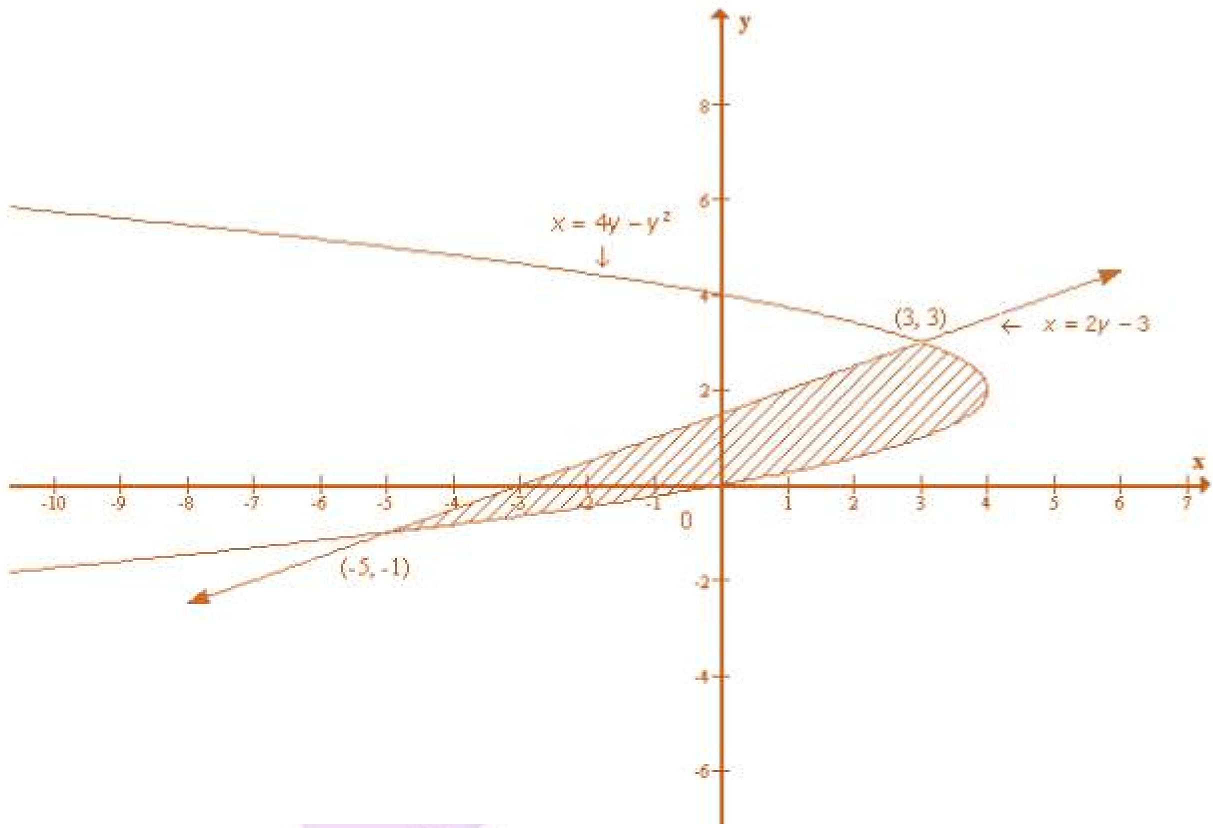
EXERCISE 21.4

Question. 1
Solution:
From the question it is given that, parabola $x = 4y - y^2$ and the line $x = 2y - 3$,
As shown in the figure,



$x_1 = 4y - y^2$
$x_2 = 2y - 3$
So,
$2y - 3 = 4y - y^2$
$y^2 + 2y - 4y - 3 = 0$
$y^2 - 2y - 3 = 0$
$y^2 - 3y + y - 3 = 0$
$y(y - 3) + 1(y - 3) = 0$
$(y - 3)(y + 1) = 0$
$y = -1, 3$
Now, we have to find the area of the bounded region,

$$A = \int_{-1}^{3} |x_1 - x_2| dy$$

Now substitute the values of x_1 and x_2,

$$= \int_{-1}^{3} [(4y - y^2) - (2y - 3)] dy$$

Then,

$$= \int_{-1}^{3} (4y - y^2 - 2y + 3) dy$$

Like terms = 4y – 2y = 2y

$$= \int_{-1}^{3} (-y^2 + 2y + 3) dy$$

On intigrating we get,

$$= \left[-\frac{y^3}{3} + \frac{2y^2}{2} + 3y\right]_{-1}^{3}$$

Applying limits, we get,

$= [- (3^3/3) + 2(3^2)/2 + 3(3)] - [- ((-1)^3/3) + 2(-1^2)/2 + 3(-1)]$

$= [- 3^2 + 3^2 + 9] – [(1/3) + 1 - 3]$

= [9] – [(1/3) – 1 + 3]

= 9 – (1/3) + 2

= 11 – (1/3)

= (33 - 1)/3

= 32/3 square units

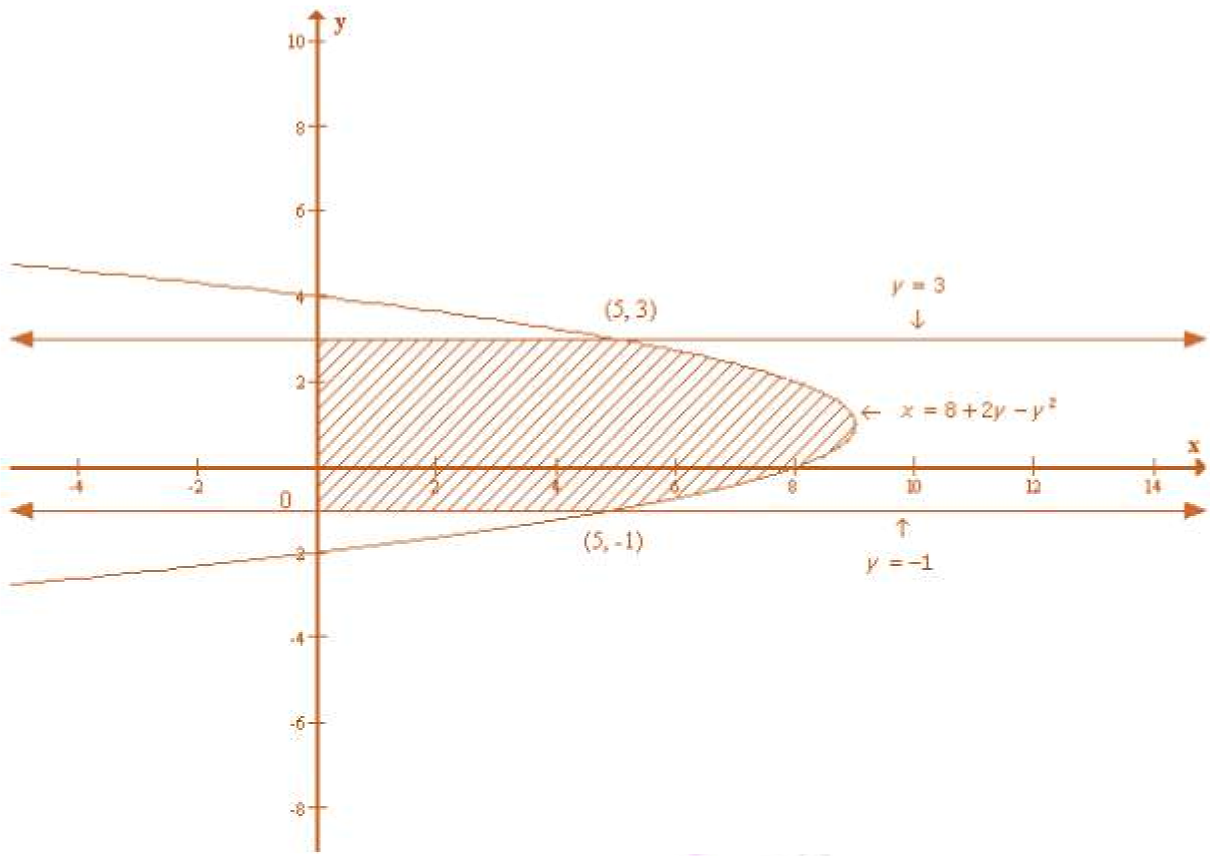
Therefore, the required area is 32/3 square units.

Question. 2

Solution:

From the question it is given that, parabola $x = 8 + 2y - y^2$ and the line y = – 1, y = 3

As shown in the figure,



$$A = \int_{-1}^{3} x dy$$

Now substitute the values of x_1 and x_2,

$$= \int_{-1}^{3} (8 + 2y - y^2) dy$$

On intigrating we get,

$$= \left[8y + y^2 - \frac{y^3}{3}\right]_{-1}^{3}$$

Applying limits, we get,

= [8(3) + (3^2) - $(3)^3$/3] - [8(-1) + (-1^2) - $(-1)^3$/3]

= [24 + 9 - 9] – [-8 + 1 + (1/3)]

= [24] – [-7 + 1/3]

= 24 + 7 - (1/3)

= 31 – (1/3)

= (93 - 1)/3

= 92/3 square units

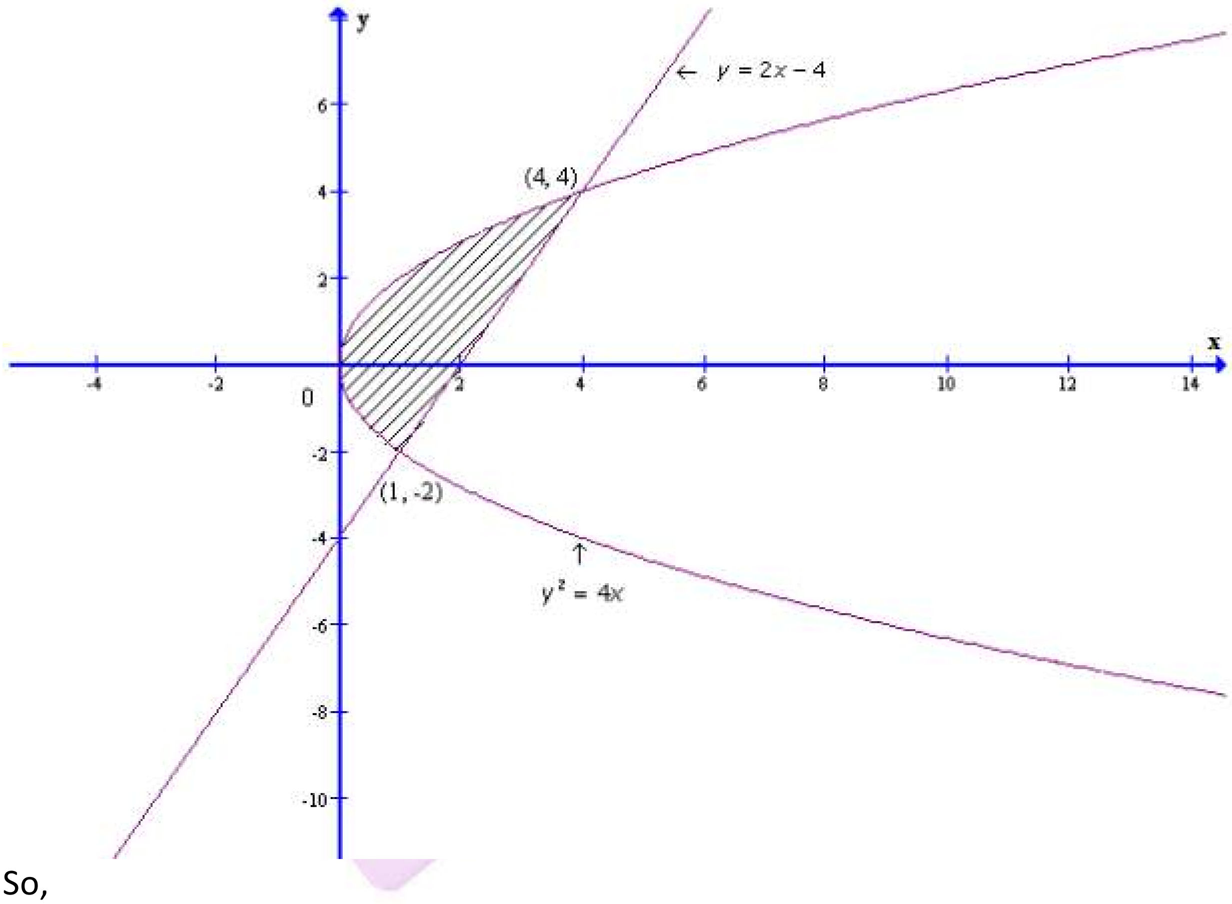
Therefore, the required area is 92/3 square units.

Question. 3

Solution:

From the question it is given that, parabola $y^2 = 4x$ and the line y = 2x - 4,

As shown in the figure,



So,

Now, we have to find the points of intersection,

2x – 4 = √(4x)

Squaring on both side,

$(2x - 4)^2 = (\sqrt{(4x)})^2$

$4x^2 + 16 – 16x = 4x$

$4x^2 + 16 – 16x – 4x = 0$

$4x^2 + 16 – 20x = 0$

Dividing both side by 4 we get,

$x^2 – 5x + 4 = 0$

$x^2 – 4x – x + 4 = 0$

x(x - 4) – 1(x - 4) = 0
(x – 4) (x - 1) = 0
x = 4, 1

$$A = \int_{-2}^{4} (x_1 - x_2) dy$$

Given, y = 2x – 4
$x_1 = (y + 4)/2$
$y^2 = 4x$
$x_2 = y^2/4$
Now substitute the values of x_1 and x_2,

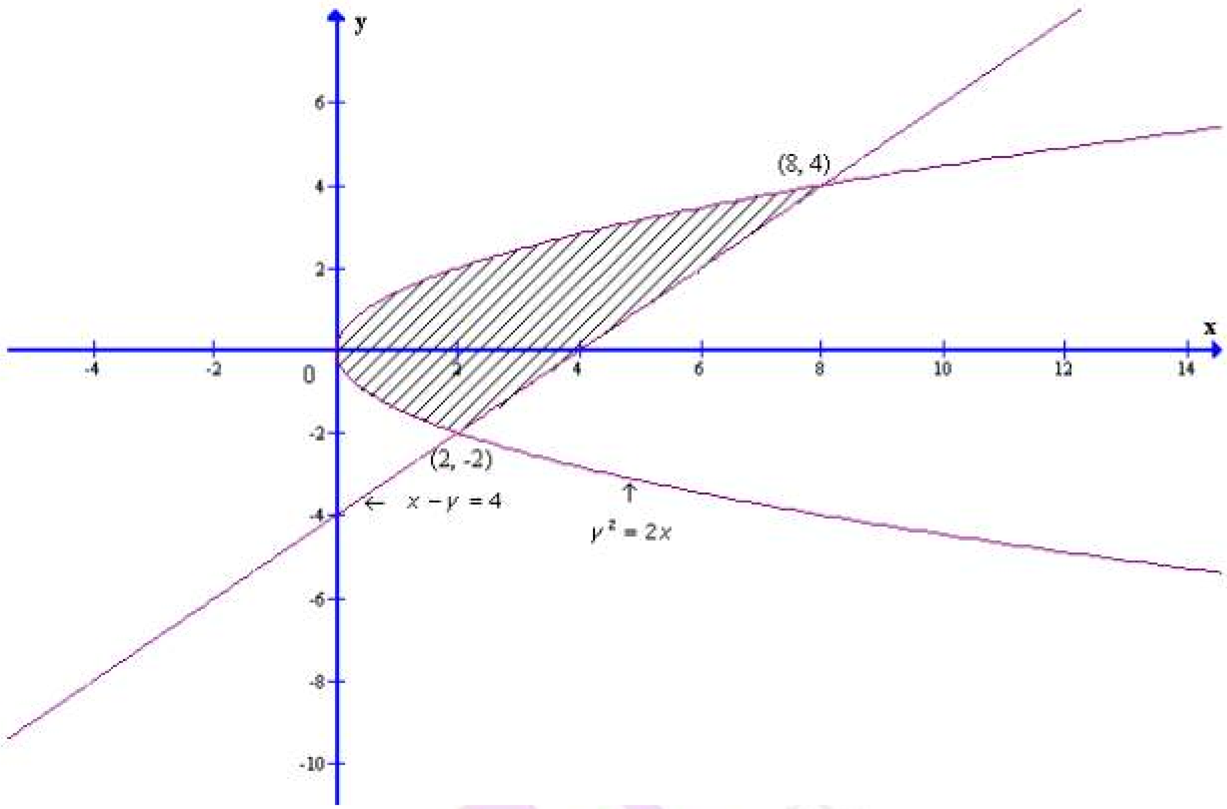
$$= \int_{-2}^{4} \left[\left(\frac{y+4}{2}\right) - \left(\frac{y^2}{4}\right)\right] dy$$

On intigrating we get,

$$= \left[\frac{y^2}{4} + 2y - \frac{y^3}{12}\right]_{-2}^{4}$$

Applying limits, we get,
$= [(4^2/4) + 2(4) – (4^3/12)] - [((-2^2)/4) + 2(-2) – ((-2)^3/12)]$
$= [4 + 8 - (64/12)] – [1 – 4 + (8/12)]$
$= [12 – (16/3)] – [-3 + (2/3)]$
$= 12 – (16/3) + 3 – (2/3)$
$= 15 – 18/3$
$= 15 – 6$
= 9 square units
Therefore, the required area is 9 square units.

Question. 4
Solution:
From the question it is given that, parabola $y^2 = 2x$ and the line x - y = 4,
As shown in the figure,
$y^2 = 2x$ … [equation (i)]
x = y + 4 … [equation (ii)]



Now, we have to find the points of intersection,
So,
$y^2 = 2(y + 4)$
$y^2 = 2y + 8$
Transposing we get,
$y^2 - 2y - 8 = 0$
$y^2 - 4y + 2y - 8 = 0$
$y(y - 4) + 2(y - 4) = 0$
$(y - 4)(y + 2) = 0$
$y = 4, -2$

$$A = \int_{-2}^{4} x_{\text{line}}\, dy - \int_{-2}^{4} x_{\text{parabola}}\, dy$$

Now substitute the values,

$$= \int_{-2}^{4} (y + 4) dy - \int_{-2}^{4} \frac{y^2}{2} dy$$

On intigrating we get,

$$= \left|4y + \frac{y^2}{2} - \frac{1}{6}y^3\right|_{-2}^{4}$$

Applying limits, we get,

$= 4(4 – (-2)) + ½ (4^2 – (-2)^2) – (1/6) (4^3 + 2^3)$

$= 4(4 + 2) + ½ (16 - 4) – (1/6) (64 + 8)$

$= 4(6) + ½ (12) – 1/6 (72)$

$= 24 + 6 – 12$

$= 30 – 12$

= 18 square units

Therefore, the required area is 18 square units.