EXERCISE 22.2

Question. 1
Solution:
From the question it is given that,
$y^2 = (x - c)^3$ … [equation (i)]
Now, differentiate the equation (i) with respect x,
$2y\ dy/dx = 3(x - c)^2$
By cross multiplication we get,
$(x - c)^2 = (2y/3)\ (dy/dx)$
Transferring square to Right Hand Side (RHS),
$(x - c) = \{(2y/3)\ (dy/dx)\}^{1/2}$
Then, substitute the value of (x - c) in equation (i),
$y^2 = [\{(2y/3)\ (dy/dx)\}^{1/2}]^3$
$y^2 = \{(2y/3)\ (dy/dx)\}^{3/2}$
Squaring on both the side we get,
$(y^2)^2 = [\{(2y/3)\ (dy/dx)\}^{3/2}]^2$
$y^4 = [\{(2y/3)\ (dy/dx)\}^3$
$y^4 = (2y/3)^3\ (dy/dx)^3$
$y^4= (8y^3/27)\ (dy/dx)^3$
By cross multiplication we get,
$27y^4/y^3 = 8\ (dy/dx)^3$
$27y = 8\ (dy/dx)^3$

Question. 2
Solution:
From the question it is given that,
$y = e^{mx}$ … [equation (i)]
Now, differentiate the equation (i) with respect x,
$dy/dx = me^{mx}$
We know that, from equation (i) $y = e^{mx}$
So, applying log on both side we get,
$\log y = mx$
$m = \log y/x$
Now, substitute the value of m and e^{mx} is equation (i) we get,
$dy/dx = (\log y/x)y$
By cross multiplication we get,

$x(dy/dx) = y \log y$

Question. 3(i)

Solution:

From the question it is given that,

$y^2 = 4ax$ … [equation (i)]

Now, differentiate the equation (i) with respect x,

$2y\,(dy/dx) = 4a$ … [equation (ii)]

From equation (i), $a = y^2/4x$

Then, substitute the value of a in equation (ii),

$2y\,(dy/dx) = 4(y^2/4x)$

$2y\,(dy/dx) = y^2/x$

By cross multiplication we get,

$2x\,(dy/dx) = y^2/y$

$2x\,(dy/dx) = y$

Question. 3(ii)

Solution:

From the question it is given that,

$y = cx + 2c^2 + c^3$ … [equation (i)]

Now, differentiate the equation (i) with respect x,

$dy/dx = c$

Then, substitute the value of c in equation (i),

$y = (dy/dx)\,x + 2(dy/dx)^2 + (dy/dx)^3$

Question. 3(iii)

Solution:

From the question it is given that,

$xy = a^2$ … [equation (i)]

Now, differentiate the equation (i) with respect x,

$x\,dy/dx + y(1) = 0$

$x\,dy/dx + y = 0$

Question. 3(iv)

Solution:

From the question it is given that,

$y = ax^2 + bx + c$ … [equation (i)]

Now, differentiate the equation (i) with respect x,
$dy/dx = 2ax + b$
Then, the above equation is again differentiating with respect to x we get,
$d^2y/dx^2 = 2a$
The above equation is again differentiating with respect to x we get
$d^3y/dx^3 = 0$

Question. 4
Solution:
From the question it is given that,
$y = Ae^{2x} + Be^{-2x}$ … [equation (i)]
Now, differentiate the equation (i) with respect x,
$dy/dx = 2Ae^{2x} - 2Be^{-2x}$
Then, the above equation is again differentiating with respect to x we get,
$d^2y/dx^2 = 4Ae^{2x} + 4Be^{-2x}$
Taking common terms outside,
$d^2y/dx^2 = 4 (Ae^{2x} + Be^{-2x})$
From equation (i) we have $y = Ae^{2x} + Be^{-2x}$
Therefore, $d^2y/dx^2 = 4y$

Question. 5
Solution:
From the question it is given that,
$x = A \cos nt + B \sin nt$ … [equation (i)]
Now, differentiate the equation (i) with respect t,
$dx/dt = - An \sin nt + Bn \cos nt$
Then, the above equation is again differentiating with respect to t we get,
$d^2x/dt^2 = - An^2 \cos nt – Bn^2 \sin nt$
Taking common terms outside,
$d^2x/dt^2 = - n^2 (A \cos nt + B \sin nt)$
From equation (i) we have $x = A \cos nt + B \sin nt$
$d^2x/dt^2 = - n^2x$
By transposing we get,
$d^2x/dt^2 + n^2x = 0$