

EXERCISE 22.6

Question. 1

Solution:

From the question it is given that,

$$(dy/dx) + ((1 + y^2)/y) = 0$$

Transposing we get,

$$dy/dx = - (1 + y^2)/y$$

By cross multiplication,

$$(y/(1 + y^2)) dy = - dx$$

Integrating on both side, we get,

$$\int (y/(1 + y^2)) dy = \int -dx$$

$$\int (2y/(1 + y^2)) dy = -2 \int dx$$

$$\log (1 + y^2) = -2x + c_1$$

$$\text{Therefore, } \frac{1}{2} \log [1 + y^2] + x = c$$

Question. 2

Solution:

From the question it is given that,

$$(dy/dx) = ((1 + y^2)/y^3)$$

By cross multiplication,

$$(y^3/(1 + y^2)) dy = dx$$

Integrating on both side, we get,

$$\int (y - (y/(1 + y^2))) dy = \int dx$$

$$\int y dy - \int (y/(1 + y^2)) dy = \int dx$$

$$\int y dy - \frac{1}{2} \int (2y/(1 + y^2)) dy = \int dx$$

$$(y^2/2) - \frac{1}{2} \log [y^2 + 1] = x + c$$

Question. 3

Solution:

From the question it is given that,

$$(dy/dx) = \sin^2 y$$

By cross multiplication,

$$dy/\sin^2 y = dx$$

We know that, $(1/\sin x) = \operatorname{cosec} x$

$$\operatorname{cosec}^2 y dy = dx$$

Integrating on both side, we get,

$$\int \operatorname{cosec}^2 y \, dy = \int dx + c$$
$$-\cot y = x + c$$

Question. 4**Solution:**

From the question it is given that,

$$(dy/dx) = (1 - \cos 2y)/(1 + \cos 2y)$$

We know that, $1 - \cos 2y = 2 \sin^2 y$ and $1 + \cos 2y = 2 \cos^2 y$

$$\text{So, } dy/dx = (2 \sin^2 y)/(2 \cos^2 y)$$

Also we know that, $\sin \theta / \cos \theta = \tan \theta$

By cross multiplication,

$$dy/\tan^2 y = dx$$

Integrating on both side, we get,

$$\int \cot^2 y \, dy = \int dx$$

$$\int (\operatorname{cosec}^2 y - 1) \, dy = \int dx$$

$$-\cot y - y + c = x$$

$$c = x + y + \cot y$$