EXERCISE 22.9

Question. 1
Solution:
From the question it is given that, x^2 dy + y (x + y) dx = 0
The given differential equation can be written in standard form as,
$dy/dx = - (y(x + y))/x^2$
So, it is a homogeneous equation,
Let us assume, y = vx and (dy/dx) = v + x (dv/dx)
Then, $v + x (dv/dx) = - (vx (x + vx))/x^2$
$v + x (dv/dx) = - v – v^2$
Transposing,
$x (dv/dx) = - v – v – v^2$
$x (dv/dx) = - 2v – v^2$
Now, taking like variables on same side,
$dv/(2v + v^2) = - dx/x$
Integrating on both side we get,
$\int(1/(2v + v^2)dv = - \int dx/x$
$\int(1/(v^2 + 2v + 1 - 1)dv = - \int dx/x$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$
$\int(1/((v + 1)^2 – 1^2)dv = - \int dx/x$
½ log [(v + 1 - 1)/(v + 1 + 1)] = - log x + log c
$\text{Log } [v/(v + 2)]^{1/2} = - \log (c/x)$
$v/(v + 2) = c^2/x^2$
$((y/x)/((y/x) + 2)) = c^2/x^2$
By simplification we get,
$y/(y + 2x) = c^2/x^2$
$yx^2 = (y + 2x) c^2$

Question. 2
Solution:
From the question it is given that, (dy/dx) = (y - x)/(y + x)
The given differential equation is a homogeneous equation,
Let us assume, y = vx and (dy/dx) = v + x (dv/dx)
v + x (dv/dx) = (vx - x)/(vx + x)
Then, v + x (dv/dx) = (v - 1)/(v + 1)
x (dv/dx) = ((v - 1)/(v + 1)) – v

$x (dv/dx) = (v – 1 – v^2 - v)/(v + 1)$
On dividing we get,
$x (dv/dx) = - (1 + v^2)/(v + 1)$
Now, taking like variables on same side,
$((v + 1)/(v^2 + 1)) = - dx/x$
Integrating on both side we get,
$\int((v + 1)/(v^2 + 1))\, dv = - \int dx/x$
$\int(v/(v^2 + 1))\, dv + \int(1/(v^2 + 1))\, dv = - \int dx/x$
$½ \int(2v/(v^2 + 1))\, dv + \int(1/(v^2 + 1))\, dv = - \int dx/x$
$½ \log [v^2 + 1] + \tan^{-1} v = - \log x + \log c$
Then, $\log [(y^2 + x^2)/x^2] + 2 \tan^{-1} (y/x) = 2 \log (c/x)$
$\log [x^2 + y^2] – 2 \log x + 2 \tan^{-1} (y/x) = 2 \log (c/x)$
$\log (x^2 + y^2) + 2 \tan^{-1} (y/x) = 2 \log c$
$\log (x^2 + y^2) + 2 \tan^{-1} (y/x) = k$

Question. 3
Solution:
From the question it is given that, $(dy/dx) = (y^2 – x^2)/(2xy)$
The given differential equation is a homogeneous equation,
Let us assume, $y = vx$ and $(dy/dx) = v + x (dv/dx)$
$v + x (dv/dx) = (v^2x^2 – x^2)/(2xvx)$
Then, $x (dv/dx) = ((v^2 - 1)/(2v)) – (v/1)$
$x (dv/dx) = (v^2 – 1 – 2v^2)/(2v)$
$x(dv/dx) = (-1 – v^2)/2v$
Now, taking like variables on same side,
$((2v)/(v^2 + 1))\, dv = - dx/x$
Integrating on both side we get,
$\int((2v)/(v^2 + 1))\, dv = - \int dx/x$
$\log (1 + v^2) = - \log x + \log c$
$1 + v^2 = c/x$
Now substitute the value of v,
$1 + y^2/x^2 = c/x$
$x^2 + y^2 = cx$

Question. 4
Solution:
From the question it is given that, $x (dy/dx) = (x + y)/x$

The given differential equation can be written in standard form as,
dy/dx = (x + y)/x
So, it is a homogeneous equation,
Let us assume, y = vx and (dy/dx) = v + x (dv/dx)
dy/dx = v + x (dv/dx)
Then, v + x (dv/dx) = (x + vx)/x
v + x (dv/dx) = (x/x) + (vx/x)
v + x (dv/dx) = 1 + v
Now, taking like variables on same side,
dv = dx/x
Integrating on both side we get,
∫dv = ∫dx/x
v = log x + c
Now substitute the value of v,
y/x = log x + c
y = x log x + cx

Question. 5
Solution:
From the question it is given that, $(x^2 – y^2)dx – 2xy\ dy = 0$
The given differential equation can be written in standard form as,
$dy/dx = (x^2 – y^2)/2xy$
So, it is a homogeneous equation,
Let us assume, y = vx and (dy/dx) = v + x (dv/dx)
$v + x (dv/dx) = (x^2 – v^2x^2)/(2xvx)$
Then, $x (dv/dx) = ((1 – v^2)/(2v)) – (v/1)$
$x (dv/dx) = (1 – v^2 – 2v^2)/(2v)$
$x(dv/dx) = (1 – 3v^2)/2v$
Now, taking like variables on same side,
$((2v)/(1 – 3v^2))\ dv = dx/x$
Integrating on both side we get,
$\int((2v)/(1 – 3v^2))\ dv = - \int dx/x$
$1/-3\int((-6v)/(1 – 3v^2))\ dv = - 3\int dx/x$
$\log (1 - 3v^2) = -3 \log x + \log c$
$1 - 3v^2 = c/x^3$
Now substitute the value of v,
$x^3 (1 – (3y^2/x^2)) = c/$

$x^3 (x^2 - 3y^2)/x^2 = c$

$x(x^2 - 3y^2) = c$