EXERCISE 29.1

Q1. i.

Solution:

Given:

The three points are:

(2, 1, 0), (3, -2, -2) and (3, 1, 7)

By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Now let us substitute the values,

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = (x-2)(-21-0)-(y-1)(7+2)+z(0+3)=0$$

= -21x + 42 – 9y + 9 + 3z = 0

= -21x – 9y + 3z + 51 = 0

Let us divide by -3, we get

Hence, the equation of plane is 7x + 3y – z – 17 = 0.

ii.

Solution:

Given:

The three points are:

(-5, 0, -6), (-3, 10, -9) and (-2, 6, -6)

By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Now let us substitute the values,

$$\begin{vmatrix} x+5 & y-0 & z+6 \\ -3+5 & 10-0 & -9+6 \\ -2+5 & 6-0 & -6+6 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+5 & y & z+6 \\ 2 & 10 & -3 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

Let us simplify, we get

$$(x+5)(0+18) - y(0+9) + (z+6)(12-30) = 0$$
$$(x+5)(18) - y(9) + (z+6)(-18) = 0$$
$$18x + 90 - 9y - 18z - 108 = 0$$

Divide by 9, we get

Hence, the equation of plane is 2x - y – 2z – 2 = 0.

iii.

Solution:

Given:

The three points are:

(1, 1, 1), (1, -1, 2) and (-2, -2, -2)

By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Now let us substitute the values,

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 2-1 \\ -2-1 & -2-1 & 2-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 1 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

Let us simplify, we get

$$(x-1)(-2+3) - (y-1)(0+3) + (z-1)(0-6) = 0$$
$$(x-1)(1) - (y-1)(3) + (z-1)(-6) = 0$$
$$x - 1 - 3y + 3 - 6z + 6 = 0$$

x – 3y – 6z + 8 = 0

Hence, the equation of plane is x – 3y – 6z + 8 = 0.

iv.

Solution:

Given:

The three points are:

(2, 3, 4), (-3, 5, 1) and (4, -1, 2)

By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Now let us substitute the values,

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -3-2 & 5-3 & 1-4 \\ 4-2 & -1-3 & 2-4 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -5 & 2 & -3 \\ 2 & -4 & -2 \end{vmatrix} = 0$$

Let us simplify, we get

$$(x-2)(-4-12)-(y-3)(10+6)+(z-4)(20-4)=0$$

$$(x-2)(-16)-(y-3)(16)+(z-4)(16)=0$$

$$-16x+32-16y+48+16z-64=0$$

-16x – 16y + 16z + 16 = 0

Divide by -16, we get

Hence, the equation of plane is x + y – z – 1 = 0.

v.

Solution:

Given:

The three points are:

(0, -1, 0), (3, 3, 0) and (1, 1, 1)

By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Now let us substitute the values,

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 3-0 & 3+1 & 0-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 3 & 4 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

Let us simplify, we get

$x(4-0)-(y+1)(3-0)+z(6-4)=0$

$4x-(y+1)(3)+z(2)=0$

$4x-3y-3+2z=0$

4x – 3y + 2z – 3 = 0

Hence, the equation of plane is 4x – 3y + 2z – 3 = 0.

Q2.

Solution:

We have to prove that points (0, -1, -1), (4, 5, 1), (3, 9, 4) and (-4, 4, 4) are coplanar.

Now let us find the equation of plane passing through three point's i.e.

(0, -1, -1), (4, 5, 1), (3, 9, 4)

By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

Now let us substitute the values,

$$\begin{vmatrix} x-0 & y+1 & z+1 \\ 4-0 & 5+1 & 1+1 \\ 3-0 & 9+1 & 4+1 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix}=0$$

Let us simplify, we get

$x(30-20)-(y+1)(20-6)+(z+1)(40-18)=0$

$10x-(y+1)(14)+(z+1)(22)=0$

$10x-14y-14+22z+22=0$

10x – 14y + 22z + 8 = 0

Divide by 2, we get

5x – 7y + 11z + 4 = 0 ……. (1)

By using the fourth point (-4, 4, 4),

Substitute the values as x = -4, y =4, z = 4 in equation (1), we get

$5(-4)-7(4)+11(4)+4=0$

$-20-28+44+4=0$

$-48+48=0$

0 = 0

LHS = RHS

Since, fourth point satisfies the equation of plane passing through three points. So, all the points are coplanar.
Hence, the equation of common plane is 5x – 7y + 11z + 4 = 0.

Q3.i
Solution:
Given:
Four points are:
(0, -1, 0), (2, 1, -1), (1, 1, 1) and (3, 3, 0)
Now let us find the equation of plane passing through three point's i.e.
(0, -1, 0), (2, 1, -1), (1, 1, 1)
By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Now let us substitute the values,

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 2-0 & 1+1 & -1-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

Let us simplify, we get

$$x(2+2)-(y+1)(2+1)+z(4-2)=0$$
$$x(4)-(y+1)(3)+z(2)=0$$
$$4x-3y-3+2z=0$$

4x – 3y – 3 + 2z = 0
4x – 3y + 2z – 3 = 0 ……. (1)

By using the fourth point (3, 3, 0),
Substitute the values as x = 3, y =3, z = 0 in equation (1), we get

$$4x-3y+2z-3=0$$
$$4(3)-3(3)+2(0)-3=0$$
$$12-9+0-3=0$$
$$12-12=0$$

0 = 0
LHS = RHS
Since, fourth point satisfies the equation of plane passing through three points. So, all the

points are coplanar.
Hence, the equation of common plane is 4x – 3y + 2z – 3 = 0.

ii.
Solution:
Given:
Four points are:
(0, 4, 3), (-1, -5, -3), (-2, -2, 1) and (1, 1, -1)
Now let us find the equation of plane passing through three point's i.e.
(0, 4, 3), (-1, -5, -3), (-2, -2, 1)
By using the formula, equation of plane passing through three points is given as:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Now let us substitute the values,

$$\begin{vmatrix} x - 0 & y - 4 & z - 3 \\ -1 - 0 & -5 - 4 & -3 - 3 \\ -2 - 0 & -2 - 4 & 1 - 3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y - 4 & z - 3 \\ -1 & -9 & -6 \\ -2 & -6 & -2 \end{vmatrix} = 0$$

Let us simplify, we get

$$x(18 - 36) - (y - 4)(2 - 12) + (z - 3)(6 - 18) = 0$$
$$x(-18) - (y - 4)(-10) + (z - 3)(-12) = 0$$
$$-18x + 10y - 40 - 12z + 36 = 0$$

-18x + 10y – 12z – 4 = 0 ………. (1)

By using the fourth point (1, 1, -1),
Substitute the values as x = 1, y =1, z = -1 in equation (1), we get

$$-18(1) + 10(1) - 12(-1) - 4 = 0$$
$$-18 + 10 + 12 - 4 = 0$$
$$-22 + 22 = 0$$

0 = 0
LHS = RHS
Since, fourth point satisfies the equation of plane passing through three points. So, all the points are coplanar.
Hence, the equation of common plane is -18x + 10y – 12z – 4 = 0.