EXERCISE 29.11

Q1.
Solution:

Given:

Equation of line is

$\vec{r} = (2\hat{i} + 3\hat{j} + 9\hat{k}) + k(2\hat{i} + 3\hat{j} + 9\hat{k})$

And the equation of the plane is

$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 5$

As we know that the angle θ between the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and a plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here,

$a_1 = 1$, $b_1 = 1$ and $c_1 = 1$ and

$a_2 = 2$, $b_2 = 3$ and $c_2 = 4$

Hence, the angle between them is given by

$$\sin\theta = \frac{1 \times 2 + 1 \times 3 + 1 \times 4}{\sqrt{(1 + 1 + 1)(4 + 9 + 16)}}$$

$$= \frac{2 + 3 + 4}{\sqrt{3 \times 29}}$$

$$= \frac{9}{\sqrt{87}}$$

Q2.
Solution:

As we know that the angle θ between the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and a plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \ldots\ldots (1)$$

Now, given equation of the line is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{1}$$

So, $a_1 = 1$, $b_1 = -1$ and $c_1 = 1$

Equation of plane is $2x + y - z - 4 = 0$

So, $a_2 = 2$, $b_2 = 1$, $c_2 = -1$ and $d_2 = -4$

Now, let us substitute the values in equation (1), we get

$$\sin\theta = \frac{1\times2 + (-1)\times1 + 1\times(-1)}{\sqrt{1^2+(-1)^2+1^2}\sqrt{2^2+1^2+(-1)^2}}$$

$$= \frac{2-1-1}{\sqrt{1+1+1}\sqrt{4+1+1}}$$

$$= \frac{0}{\sqrt{1+1+1}\sqrt{4+1+1}}$$

$$= 0$$

$\sin\theta = 0$

Hence, the angle between the plane and the line is 0°.

Q3.

Solution:

As we know that the angle θ between the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and a plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad \ldots\ldots (1)$$

Given that the line is passing through A(3, – 4, – 2) and B(12, 2, 0)

The direction ratios of line AB = (12 – 3, 2 – (– 4), 0 – (– 2))

= (12 – 3, 2 + 4, 0 + 2)

= (9, 6, 2)

So,

$a_1 = 9$, $b_1 = 6$ and $c_1 = 2$

Given equation of plane is $3x - y + z = 1$

So,

$a_2 = 3$, $b_2 = -1$ and $c_2 = 1$

Now, substitute the obtained values in equation (1), we get

$$\sin\theta = \frac{3\times9+6\times-1+2\times1}{\sqrt{9^2+6^2+2^2}\sqrt{3^2+1^2+(-1)^2}}$$

$$= \frac{27-6+2}{\sqrt{81+36+4}\sqrt{9+1+1}}$$

$$= \frac{23}{\sqrt{121}\sqrt{11}}$$

$$= \frac{23}{11\sqrt{11}}$$

$$\theta = \sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

Hence, the required angle is $\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$.

Q4.

Solution:

We know that line $\vec{r} = \vec{a} + k\vec{b}$ is parallel to the plane $\vec{r}.\vec{n} = d$ if $\vec{b}.\vec{n} = 0$ …… (1)

Given the equation of the line is

$\vec{r} = \hat{i} + k(2\hat{i} - m\hat{j} - 3\hat{k})$

The equation of the plane is

$\vec{r}.(m\hat{i} + 3\hat{j} + \hat{k}) = 4$

So,

$\vec{b} = (2\hat{i} - m\hat{j} - 3\hat{k})$

$\vec{n} = (m\hat{i} + 3\hat{j} + \hat{k})$

Now substitute the values in equation (1), we get

$(2\hat{i} - m\hat{j} - 3\hat{k}).(m\hat{i} + 3\hat{j} + \hat{k}) = 0$

$2m - 3m - 3 = 0$

$-m = 3$

$m = -3$

Hence, the value of m is -3.

Q5.

Solution:

We know that line $\vec{r} = \vec{a} + k\vec{b}$ and plane $\vec{r}.\vec{n} = d$ is parallel if
$\vec{b}.\vec{n} = 0$ ...... (1)

Given, the equation of the line
$\vec{r} = (2\hat{i} + 5\hat{j} + 7\hat{k}) + k(\hat{i} + 3\hat{j} + 4\hat{k})$ and equation of plane is the
$\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 7$,
So,
$\vec{b} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{n} = \hat{i} + \hat{j} - \hat{k}$
Now,
$\vec{b}.\vec{n} = (\hat{i} + 3\hat{j} + 4\hat{k})(\hat{i} + \hat{j} - \hat{k})$
$= 1 + 3 - 4 = 0$
So, the line and the plane are parallel.

We know that the distance (D) of a plane $\vec{r}.\vec{n} = d$ from a point $\vec{a}$ is given by

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right|$$

$\vec{a} = (2\hat{i} + 5\hat{j} + 7\hat{k})$

$$D = \left|\frac{(2\hat{i} + 5\hat{j} + 7\hat{k})(\hat{i} + \hat{j} - \hat{k}) - 7}{\sqrt{(1)^2 + (1)^2 + (-1)^2}}\right|$$

$$= \left|\frac{(2)(1) + (5)(1) + (7)(-1) - 7}{\sqrt{1+1+1}}\right|$$

$$= \left|\frac{2 + 5 - 7 - 7}{\sqrt{3}}\right|$$

$$= \left|\frac{-7}{\sqrt{3}}\right|$$

So,

$$D = \frac{7}{\sqrt{3}}$$

Hence, the required distance between plane and line is $D = \frac{7}{\sqrt{3}}$ unit.