EXERCISE 29.12

Q1.

Solution:

(a) Direction ratio of given line are (5 – 3, 1 – 4, 6 – 1) = (2, – 3, 5).

Hence, equation of line is $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$

x = 2r +5, y = -3r + 1, z = 5r +6

For any point on the yz –plane: x = 0

So,

x = 2r + 5 = 0

$r = -\frac{5}{2}$,

$y = -3(-\frac{5}{2}) + 1$

$= \frac{17}{2}$

$z = 5(-\frac{5}{2}) + 6$

$= -\frac{13}{2}$

Hence, the coordinates of this point are $(0, \frac{17}{2}, -\frac{13}{2})$.

(b): Direction ratio of given line are (5 – 3, 1 – 4, 6 – 1) = (2, – 3, 5

Hence, equation of line is

$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$

For any point on zx –plane: y = 0

y = – 3r + 1 = 0

r = 1/3

$x = 2(\frac{1}{3}) + 5$

$= \frac{17}{3}$

$z = 5(\frac{1}{3}) + 6$

$= \frac{23}{3}$

Hence, the coordinates of this point are $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$.

Q2.
Solution:

Let the coordinates of the points A and B be (3, – 4, – 5) and (2, – 3, 1) respectively.

The equation of the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = r$, Where r is constant.

So, the equation of AB is

$$\frac{x-3}{2-3} = \frac{y-(-4)}{(-3)-(-4)} = \frac{z-(-5)}{1-(-5)} = r$$

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = r$$

Any point on the line AB is the form:
– r + 3, r – 4, 6r – 5

Let p be the point of intersection of the line AB and the plane 2x + y + z = 7
Thus, we have,
2(– r + 3) + r – 4 + 6r – 5 = 7
– 2r + 6 + r – 4 + 6r – 5 = 7
5r = 10
r = 2

Now substitute the value of r in – r + 3, r – 4, 6r – 5, the coordinates of P are:
(– 2 + 3, 2 – 4, 12 – 5) = (1, – 2, 7).

Q3.
Solution:

The equation of the given line is
$\vec{r}.(2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ …… (1)

The equation of the given plane is
$\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$ …… (2)

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we get
$[2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})].(\hat{i} - \hat{j} + \hat{k}) = 5$
$(3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$
$\lambda = 0$

Substituting the value of equation (1), we obtain the equation of the line as
$\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k})$
So, the position vector of the point of intersection of the line and the plane is
$\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k})$

This shows that the point of intersection of the given plane and line is given by the coordinates, (2, – 1, 2). The point is (– 1, – 5, – 10).

The distance d between the points, (2, – 1, 2) and (– 1, – 5, – 10) is
$d = \sqrt{9 + 16 + 144}$
$d = \sqrt{169}$
$d = 13$

Q4.
Solution:
To find the point of intersection of the line
$\vec{r} = (2\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane
$\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$

Let us substitute $\vec{r}$ of line in the plane.
$[2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})].(\hat{i} - 2\hat{j} + \hat{k}) = 0$
$2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0$
$3\lambda = 12$
$\lambda = 4$
Now, substitute the value of λ, we get
$\vec{r} = (2\hat{i} - 4\hat{j} + 2\hat{k}) + 4(3\hat{i} + 4\hat{j} + 2\hat{k}) = 14\hat{i} + 12\hat{j} + 10\hat{k}$
Hence, the distance of the point $2\hat{i} + 12\hat{j} + 5\hat{k}$ from $14\hat{i} + 12\hat{j} + 10\hat{k}$ is
$\sqrt{(14 - 2)^2 + (12 - 12)^2 + (10 - 5)^2} = \sqrt{169} = 13$

Q5.

Solution:

Equation of line through the point A(2, – 1, 2) and B (5, 3, 4) is

$$\frac{x-2}{5-2} = \frac{y+1}{3+1} = \frac{z-2}{4-2} = r$$

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r$$

$x = 3r + 2, y = 4r - 1, z = 2r + 2$

Now, let us substitute these values in the equation of plane, we get

$(3r + 2) - (4r - 1) + (2r + 2) = 5$

$r = 0$

$x = 2, y = -1, z = 2$

Distance of (2, – 1, 2) from (– 1, – 5, – 10) is

$= \sqrt{(2-(-1))^2 + (-1-(-5))^2 + (2-(-10))^2}$

$= \sqrt{169}$

$= 13$

Hence, the distance is 13 units.