EXERCISE 29.2

Q1.
Solution:
Given:
Intercepts on the coordinate axes are 2, -3 and 4.
We know that,
The equation of a plane whose intercepts on the coordinate axes are a, b, c is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Where, a = 2, b = -3, c = 4
So now let us substitute the above values, we get
Equation of the required plane is

$$\frac{x}{2} + \frac{y}{-3} + \frac{z}{4} = 1$$

$$\frac{6x - 4y + 3z}{12} = 1$$

6x – 4y + 3z = 12
Hence, the equation of plane is 6x – 4y + 3z = 12.

Q2.i
Solution:
Let us reduce the given equation 4x + 3y – 6z – 12 = 0 in intercept form:
4x + 3y – 6z – 12 = 0
4x + 3y – 6z = 12
Divide by 12, we get

$$\frac{4x}{12} + \frac{3y}{12} - \frac{6z}{12} = \frac{12}{12}$$

$$\frac{x}{3} + \frac{y}{4} - \frac{z}{2} = 1$$

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{(-2)} = 1 \quad \ldots\ldots. (1)$$

We know that this is of the form,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \ldots\ldots. (2)$$

Now, compare equation 1 and 2, we get
a = 3, b = 4, c = -2
Therefore intercepts on the coordinate axes are 3, 4, and -2.

ii.

Solution:

Let us reduce the given equation 2x + 3y – z = 6 in intercept form:

Divide the given equation by 6, we get

$$\frac{2x}{6}+\frac{3y}{6}-\frac{z}{6}=\frac{6}{6}$$

$$\frac{x}{3}+\frac{y}{2}-\frac{z}{6}=1$$

$$\frac{x}{3}+\frac{y}{2}+\frac{z}{(-6)}=1$$

……… (1)

We know that this is of the form,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

……. (2)

Now, compare equation 1 and 2, we get

a = 3, b = 2, c = -6

Therefore intercepts on the coordinate axes are 3, 2, -6.

iii.

Solution:

Given:

The equation of plane is 2x – y + z = 5

Divide the given equation by 5, we get

$$\frac{2x}{5}-\frac{y}{5}+\frac{z}{5}=\frac{5}{5}$$

$$\frac{x}{\left(\frac{5}{2}\right)}+\frac{y}{(-5)}+\frac{z}{5}=1$$

……. (1)

We know that this is of the form,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

……. (2)

Now, compare equation 1 and 2, we get

a = 5/2, b = -5, c = 5

Therefore intercepts on the coordinate axes are 5/2, -5, 5.

Q3.

Solution:

Given:

The plane meets axes in A, B and C.

Let A = (a, 0, 0), B = (0, b, 0) and C = (0, 0, c)
Given that the centroid of the triangle = (α, β, γ)
By using the formula,

$$\text{Centroid} = \frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}$$

$$(\alpha, \beta, \gamma) = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$(\alpha, \beta, \gamma) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

So,
a/3 = α
=> a = 3α …… (i)

b/3 = β
=> b = 3β ……. (2)

c/3 = γ
=> c = 3γ ……. (3)
If a, b, c is intercepts by plane on coordinate axes,
Then equation of plane is given by:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Now substitute the values of a, b, c from equation (1), (2), (3), we get

$$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$$

Multiply by 3 on both sides, we get

$$\frac{3x}{3\alpha} + \frac{3y}{3\beta} + \frac{3z}{3\gamma} = 3$$

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

Hence, the equation of plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.

Q4.
Solution:
Given:
The intercepts on the coordinate axes are equal.
We know that the equation of the plane whose intercepts on the coordinate axes is given as:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

It is given that, a = b = c
So let a = b = c = p
Then,

$$\frac{x}{p}+\frac{y}{p}+\frac{z}{p}=1$$

$$\frac{x+y+z}{p}=1$$

x + y + z = p …… (1)

It is also given that plane is passing through the point (2, 4, 6)
By using equation (1),
x + y + z = p
2 + 4 + 6 = p
p = 12
Now, substitute the value of p back in equation (1), we get
x + y + z = 12
Hence, the required equation of the plane is x + y + z = 12.

Q5.
Solution:
Given:
Plane meets the coordinate axes at A, B and C with centroid of triangle ABC is (1, -2, 3).
We know that the equation of the plane whose intercepts on the coordinate axes is given as:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad \text{……. (1)}$$

By using the formula,
Centroid of a triangle is given by:

$$\text{Centroid}=\frac{x_1+x_2+x_3}{3},\ \frac{y_1+y_2+y_3}{3},\ \frac{z_1+z_2+z_3}{3}$$

$$(1,-2,3)=\left(\frac{a+0+0}{3},\ \frac{0+b+0}{3},\ \frac{0+0+c}{3}\right)$$

$$(1,-2,3)=\left(\frac{a}{3},\ \frac{b}{3},\ \frac{c}{3}\right)$$

Now let us compare LHS and RHS,
a/3 = 1
=> a = 3

b/3 = -2
=> b = -6

c/3 = 3
=> c = 9
Now, substitute the values of a, b, c in equation (1), we get

$$\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$$

6x – 3y + 2z = 18
Hence, the required equation of the plane is 6x – 3y + 2z = 18.