EXERCISE 29.3

Q1.
Solution:

We know that the vector equation of the plane passing through a point $\vec{a}$ and normal to $\vec{n}$ is given as:

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ …… (1)

Here,

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{n} = 4\hat{i} + 2\hat{j} - 3\hat{k}$

Now, substitute the values in equation (1), we get

$\left[\vec{r} - \left(2\hat{i} - \hat{j} + \hat{k}\right)\right].\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$

$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left(2\hat{i} - \hat{j} + \hat{k}\right).\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$

$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left[(2)(4) + (-1)(2) + (1)(-3)\right] = 0$

$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left[8 - 2 - 3\right] = 0$

$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - 3 = 0$

Hence, the required equation of plane is:

$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 3$

Q2. i
Solution:
Given:
The vector equation of plane is

$\vec{r}.\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$

let, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$

$(x)(12) + (y)(-3) + (z)(4) + 5 = 0$

12x – 3y + 4z + 5 = 0

Hence, the Cartesian form of the equation of the plane is 12x – 3y + 4z + 5 = 0.

ii.
Solution:

Given:
The equation of plane is:
$\vec{r}.\left(-\hat{i}+\hat{j}+2\hat{k}\right)=9$
let, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$,
Then,
$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-\hat{i}+\hat{j}+2\hat{k}\right)=9$
$(x)(-1)+(y)(1)+(z)(2)=9$
-x + y + 2z = 9
Hence, the Cartesian form of the equation of the plane is -x + y + 2z = 9.

Q3.
Solution:
Given:
For xy-plane,
This plane is passing through the origin and is perpendicular to z-axis.
So,
Put $\vec{a}=0.\hat{i}+0.\hat{j}+0\hat{k}$ and $\vec{n}=\hat{k}$ in the vector equation of plane passing through point $\vec{a}$ and perpendicular to vector $\vec{n}$
$\left(\vec{r}-\vec{a}\right).\vec{n}=0$
$\left(\vec{r}-0.\hat{i}-0.\hat{j}-0\hat{k}\right).\hat{k}=0$
$\vec{r}.\hat{k}=0$ …….. (1)
For xz-plane,
It passes through origin and perpendicular to y-axis,
So,
$\vec{a}=0.\hat{i}+0.\hat{j}+0\hat{k}$ and $\vec{n}=\hat{j}$
Equation of xz-plane is given by
$\left(\vec{r}-\vec{a}\right).\vec{n}=0$
$\left(\vec{r}-0.\hat{i}-0.\hat{j}-0\hat{k}\right).\hat{j}=0$
$\vec{r}.\hat{j}=0$ …… (2)
For yz-plane,
It passes through origin and perpendicular to x-axis,
So,

$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k},\ \vec{n} = \hat{i}$

$(\vec{r} - \vec{a}).\vec{n} = 0$

$(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}).\hat{i} = 0$

$\vec{r}.\hat{i} = 0$ ……… (3)

Hence, equation of xy, yz and xz-plane is given by

$\vec{r}.\hat{k} = 0$

$\vec{r}.\hat{i} = 0$

$\vec{r}.\hat{j} = 0$

Q4.i.
Solution:
Given:
The equation of plane is:
2x – y + 2z = 8
By putting the values, we get

$(x\hat{i} + y\hat{j} + z\hat{k})(2\hat{i} - \hat{j} + 2\hat{k}) = 8$

$\vec{r}.(2\hat{i} - \hat{j} + 2\hat{k}) = 8$

Hence, vector equation of the plane is

$\vec{r}.(2\hat{i} - \hat{j} + 2\hat{k}) = 8$

ii.
Solution:
Given:
The Cartesian equation of plane is:
x + y – z = 5
By putting the values, we get

$(x\hat{i} + y\hat{j} + z\hat{k})(\hat{i} + \hat{j} - \hat{k}) = 5$

$\vec{r}(\hat{i} + \hat{j} - \hat{k}) = 5$

Hence, vector equation of the plane is

$\vec{r}(\hat{i} + \hat{j} - \hat{k}) = 5$

iii.
Solution:
Given:

The Cartesian equation of plane is:
x + y = 3
By putting the values, we get

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(\hat{i}+\hat{j}\right)=3$$

$$\vec{r}.\left(\hat{i}+\hat{j}\right)=3$$

Hence, vector equation of the plane is

$$\vec{r}.\left(\hat{i}+\hat{j}\right)=3$$

Q5.
Solution:
We know that the vector equation of the plane passing through a point $\vec{a}$ and perpendicular to $\vec{n}$ is given as:

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \quad \ldots\ldots (1)$$

The given plane is passing through the point (1, -1, 1) and normal to the line joining A(1, 2, 5) and B(-1, 3, 1).
So,

$$\vec{a}=\hat{i}-\hat{j}+\hat{k} \text{ and } \vec{n}=\overrightarrow{AB}$$

$$= \text{Position vector of } \vec{B} - \text{Position vector of } A$$

$$=\left(-\hat{i}+3\hat{j}+\hat{k}\right)-\left(\hat{i}+2\hat{j}+5\hat{k}\right)$$

$$=-\hat{i}+3\hat{j}+\hat{k}-\hat{i}-2\hat{j}-5\hat{k}$$

$$=-2\hat{i}+\hat{j}-4\hat{k}$$

Now, let us substitute $\vec{a}$ and $\vec{n}$ in equation (1)

$$\left[\vec{r}-\left(\hat{i}-\hat{j}+\hat{k}\right)\right].\left(-2\hat{i}+\hat{j}-4\hat{k}\right)=0$$

$$\vec{r}.\left(-2\hat{i}+\hat{j}-4\hat{k}\right)-\left(\hat{i}-\hat{j}+\hat{k}\right)\left(-2\hat{i}+\hat{j}-4\hat{k}\right)=0$$

$$\vec{r}.\left(-2\hat{i}+\hat{j}-4\hat{k}\right)-\left[(1)(-2)+(-1)(1)+(1)(-4)\right]=0$$

$$\vec{r}.\left(-2\hat{i}+\hat{j}-4\hat{k}\right)-\left[-2-1-4\right]=0$$

$$\vec{r}\left(-2\hat{i}+\hat{j}-4\hat{k}\right)-\left[-7\right]=0$$

$$\vec{r}.\left(-2\hat{i}+\hat{j}-4\hat{k}\right)+7=0$$

$$\vec{r}.\left(-2\hat{i}+\hat{j}-4\hat{k}\right)=-7$$

Now, multiply by (-1) on both sides
So,

$\vec{r}\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$

$(x)(2) + (y)(-1) + (2)(4) = 7$

$2x - y + 4z = 7$

Hence, the vector and cartesian equation of plane is

$\vec{r}.\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7,$

$2x - y + 4z = 7$