## EXERCISE 29.4

**Q1.**

**Solution:**

Given:

Normal vector, $\vec{n} = \hat{i}$

Now,

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{\hat{k}}{|\hat{k}|} = \frac{\hat{k}}{1} = \hat{k}$$

The equation of a plane in normal form is

$$\vec{r}.\hat{n} = d$$

Where, d is the distance of the plane from the origin.

Let us substitute $\hat{n} = \hat{k}$ and d = 3, we get

$$\vec{r}.\hat{k} = 3$$

**Q2.**

**Solution:**

We know that,

The vector equation of a plane in normal form is given as

$\vec{r}.\hat{n} = d$ …… (1)

Here, distance 'd' is unit from origin.

Where, d = 5 unit

$$\vec{n} = \hat{i} - 2\hat{j} - 2\hat{k}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{9}}$$

$$\hat{n} = \frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right)$$

Now let us substitute the value of d and $\vec{n}$ in equation (1)

Hence, the required equation of plane is

$$\vec{r}.\frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$$

**Q3.**

**Solution:**

Given:

The equation of plane is,

$$2x - 3y - 6z = 14$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right) = 14$$

Dividing the equation by $\sqrt{(2)^2 + (-3)^2 + (-6)^2}$

$$\vec{r}.\frac{\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)}{\sqrt{4+9+36}} = \frac{14}{\sqrt{4+9+36}}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = \frac{14}{7}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2 \quad \text{........ (1)}$$

We know that the vector equation of the plane with distance d from origin and normal to unit vector $\vec{n}$ is given as:

$$\vec{r}.\hat{n} = d \quad \text{........ (2)}$$

Now by comparing equation (1) and (2), we get

d = 2 and

$$\hat{n} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

Hence, the distance of plane from origin = 2 units

Direction cosine of normal to plane = 2/7, -3/7, 6/7

**Q4.**

**Solution:**

Given:

The equation of plane is,

$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) + 6 = 0$$

$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = -6$$

Let us multiply by (-1) on both sides, we get

$$\vec{r}.\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = 6$$

$$\vec{r}.\vec{n} = 6 \quad \text{....... (1)}$$

Here, $\vec{n} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$$|\vec{n}| = \sqrt{(-1)^2 + (2)^2 + (-2)^2}$$
$$= \sqrt{1 + 4 + 4}$$
$$= \sqrt{9}$$
$$= 3$$

Divide equation (1) by $|\vec{n}| = 3$ on both sides, we get

$$\vec{r}.\frac{\vec{n}}{|\vec{n}|} = \frac{6}{|\vec{n}|}$$

$$\vec{r}.\frac{1}{3}\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = \frac{6}{3}$$

$$\vec{r}\left(-\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2 \quad \text{……. (2)}$$

We know that the vector equation of the plane with distance d from origin and normal to unit vector $\vec{n}$ is given as:

$$\vec{r}.\hat{n} = d \quad \text{…….. (3)}$$

By comparing equation (2) and (3), we get

d = 2 and

$$\hat{n} = -\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$$

Hence, the distance of plane from origin = 2 units

Direction cosine of normal to plane = -1/3, 2/3, -2/3

**Q5.**

**Solution:**

Given:

The equation of plane is,

2x – 3y + 6z + 14 = 0

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$

$$\vec{r}.\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$

Let us multiply by (-1) on both sides, we get

$$\vec{r}.\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right) = 14 \quad \text{……. (1)}$$

So,

$$\vec{r}.\vec{n} = 14$$

$\vec{n} = -2\hat{i} + 3\hat{j} - 6\hat{k}$

$|\vec{n}| = \sqrt{(-2)^2 + (3)^2 + (-6)^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

$= 7$

Divide equation (1) by $|\vec{n}| = 7$ on both sides, we get

$\vec{r}.\frac{\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right)}{7} = \frac{14}{7}$

$\vec{r}.\left(-\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2$ …… (2)

We know that the vector equation of the plane with distance d from origin and normal to unit vector $\vec{n}$ is given as:

$\vec{r}.\hat{n} = d$ …….. (3)

By comparing equation (2) and (3), we get

d = 2 and

$\hat{n} = -\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$

Hence, the distance of plane from origin = 2 units

Direction cosine of normal to plane = -2/7, 3/7, -6/7