EXERCISE 29.7

Q1.i.

Solution:

Given:

$\vec{r} = (2\hat{i} - \hat{k}) + \lambda\hat{i} + \mu(\hat{i} - 2\hat{j} - \hat{k})$

We know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

Clearly,

$\vec{a} = 2\hat{i} - \hat{k}, \vec{b} = \hat{i}, \vec{c} = \hat{i} - 2\hat{j} - \hat{k}$.

Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,

Hence,

$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & -2 & -1 \end{vmatrix}$$

$\Rightarrow \vec{n} = \hat{i}(0 - 0) - \hat{j}(-1 - 0) + \hat{k}(-2 - 0)$

$\Rightarrow \vec{n} = \hat{j} - 2\hat{k}$

We know that vector equation of a plane in scalar product form is given as

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ ...... (1)

Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,

$\vec{r}.(\hat{j} - 2\hat{k}) = (2\hat{i} - \hat{k}).(\hat{j} - 2\hat{k})$

$\Rightarrow \vec{r}.(\hat{j} - 2\hat{k}) = (2)(0) + (0)(1) + (-1)(-2)$

$\Rightarrow \vec{r}.(\hat{j} - 2\hat{k}) = 2$

Hence, the required equation is $\vec{r}.(\hat{j} - 2\hat{k}) = 2$.

ii.

Solution:

Given:

$\vec{r} = (1 + s - t)\hat{i} + (2 - s)\hat{j} + (3 - 2s + 2t)\hat{k}$

Let us simplify, we get

$\Rightarrow \vec{r} = \hat{i} + s\hat{i} - t\hat{i} + 2\hat{j} - s\hat{j} + 3\hat{k} - 2s\hat{k} + 2t\hat{k}$

$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + s\hat{i} - s\hat{j} - 2s\hat{k} - t\hat{i} + 2t\hat{k}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + (\hat{i} - \hat{j} - 2\hat{k})s + (-\hat{i} + 2\hat{k})t$

Now, we know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

Clearly,

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} - \hat{j} - 2\hat{k}, \vec{c} = -\hat{i} + 2\hat{k}$.

Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,

Hence,

$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix}$$

$$\Rightarrow \vec{n} = \hat{i}(-2 - 0) - \hat{j}(2 - 2) + \hat{k}(0 - 1)$$

$$\Rightarrow \vec{n} = -2\hat{i} - \hat{k}$$

We know that vector equation of a plane in scalar product form is given as

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ …… (1)

Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,

$$\vec{r}.\left(-2\hat{i} - \hat{k}\right) = (\hat{i} + 2\hat{j} + 3\hat{k}).(-2\hat{i} - \hat{k})$$

$$\Rightarrow \vec{r}.\left(-2\hat{i} - \hat{k}\right) = (1)(-2) + (2)(0) + (3)(-1)$$

$$\Rightarrow \vec{r}.\left(-2\hat{i} - \hat{k}\right) = -5$$

$$\Rightarrow \vec{r}.\left(2\hat{i} + \hat{k}\right) = 5$$

Hence, the required equation is $\vec{r}.\left(2\hat{i} + \hat{k}\right) = 5$.

iii.

Solution:

Given,

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$$

We know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

Clearly,

$\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = -\hat{i} + \hat{j} - 2\hat{k}$.

Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,

Hence,

$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$$

$\Rightarrow \vec{n} = \hat{i}(-4 + 1) - \hat{j}(-2 - 1) + \hat{k}(1 + 2)$
$\Rightarrow \vec{n} = -3\hat{i} + 3\hat{j} + 3\hat{k}$

We know that vector equation of a plane in scalar product form is given as
$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ ...... (1)
Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,

$\vec{r}.\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right) = (\hat{i} + \hat{j}).\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right)$
$\Rightarrow \vec{r}.\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right) = (1)(-3) + (1)(3) + (0)(3)$
$\Rightarrow \vec{r}.\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right) = 0$
$\Rightarrow \vec{r}.\left(\hat{i} - \hat{j} - \hat{k}\right) = 0$
Hence, the required equation is $\vec{r}.\left(\hat{i} - \hat{j} - \hat{k}\right) = 0$.

iv.
Solution:
Given,
$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 3\hat{k})$
We know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.
Clearly,
$\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 4\hat{i} - 2\hat{j} + 3\hat{k}$.
Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,
Hence,
$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix}$$
$\Rightarrow \vec{n} = \hat{i}(3 - (-2)) - \hat{j}(3 - 4) + \hat{k}(-2 - 4)$
$\Rightarrow \vec{n} = 5\hat{i} + \hat{j} - 6\hat{k}$

We know that vector equation of a plane in scalar product form is given as
$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ ...... (1)
Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,
$\vec{r}.\left(5\hat{i} + \hat{j} - 6\hat{k}\right) = (\hat{i} - \hat{j}).\left(5\hat{i} + \hat{j} - 6\hat{k}\right)$
$\Rightarrow \vec{r}.\left(5\hat{i} + \hat{j} - 6\hat{k}\right) = (1)(5) + (-1)(1) + (0)(-6)$

$\Rightarrow \vec{r}.\left(5\hat{i} + \hat{j} - 6\hat{k}\right) = 4$

Hence, the required equation is $\vec{r}.\left(5\hat{i} + \hat{j} - 6\hat{k}\right) = 4$.

Q2.i.

Solution:

Given:

$\vec{r} = (\hat{i} - \hat{j}) + \left(-\hat{i} + \hat{j} + 2\hat{k}\right)s + \left(\hat{i} + 2\hat{j} + \hat{k}\right)t$

Now, we know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

Clearly,

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + \hat{j} + 2\hat{k}, \vec{c} = \hat{i} + 2\hat{j} + \hat{k}$.

Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,

Hence,

$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

$\Rightarrow \vec{n} = \hat{i}(1 - 4) - \hat{j}(-1 - 2) + \hat{k}(-2 - 1)$

$\Rightarrow \vec{n} = -3\hat{i} + 3\hat{j} - 3\hat{k}$

We know that vector equation of a plane in scalar product form is given as

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ …… (1)

Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,

$\vec{r}.\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right) = (\hat{i} - \hat{j}).\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right)$

$\Rightarrow \vec{r}.\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right) = (1)(-3) + (-1)(3) + (0)(-3)$

$\Rightarrow \vec{r}.\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right) = -6$

$\Rightarrow \vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 2$

Now, put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

So, we have,

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right) = -6$

$(x)(-3) + (y)(3) + (z)(-3) = -6$

$-3x + 3y - 3z = -6$

Divide by -3, we get

$x - y + z = 2$

Hence, the required equation is $x - y + z = 2$.

ii.

Solution:

Given:

$\vec{r} = (1 + s + t)\hat{i} + (2 - s + t)\hat{j} + (3 - 2s + 2t)\hat{k}$

Let us simplify, we get

$\Rightarrow \vec{r} = \hat{i} + s\hat{i} + t\hat{i} + 2\hat{j} - s\hat{j} + t\hat{j} + 3\hat{k} - 2s\hat{k} + 2t\hat{k}$

$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + s\hat{i} - s\hat{j} - 2s\hat{k} + t\hat{i} + t\hat{j} + 2t\hat{k}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + (\hat{i} - \hat{j} - 2\hat{k})s + (\hat{i} + \hat{j} + 2\hat{k})t$

Now, we know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

Clearly,

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} - \hat{j} - 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k}$.

Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,

Hence,

$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ 1 & 1 & 2 \end{vmatrix}$$

$\Rightarrow \vec{n} = \hat{i}(-2 + 2) - \hat{j}(2 + 2) + \hat{k}(1 + 1)$

$\Rightarrow \vec{n} = -4\hat{j} + 2\hat{k}$

We know that vector equation of a plane in scalar product form is given as

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ …… (1)

Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,

$\vec{r}.(-4\hat{j} + 2\hat{k}) = (\hat{i} + 2\hat{j} + 3\hat{k}).(-4\hat{j} + 2\hat{k})$

$\Rightarrow \vec{r}.(-4\hat{j} + 2\hat{k}) = (1)(0) + (2)(-4) + (3)(2)$

$\Rightarrow \vec{r}.(-4\hat{j} + 2\hat{k}) = -2$

$\Rightarrow \vec{r}.(2\hat{j} - \hat{k}) = 1$

Now, put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

So, we have,

$(x\hat{i} + y\hat{j} + z\hat{k})(-4\hat{j} + 2\hat{k}) = -2$

$(x)(0) + (y)(-4) + (z)(2) = -2$

-4y + 2z = -2

Divide by -2, we get

2y – z = 1

Hence, the required equation of plane is 2y – z = 1.

Q3.i.

Solution:

Given equation of plane is:

$\vec{r} = (\lambda - 2\mu)\hat{i} + (3 - \mu)\hat{j} + (2\lambda + \mu)\hat{k}$

Let us simplify, we get

$\Rightarrow \vec{r} = \lambda\hat{i} - 2\mu\hat{i} + 3\hat{j} - \mu\hat{j} + 2\lambda\hat{k} + \mu\hat{k}$

$\Rightarrow \vec{r} = 3\hat{j} + \lambda\hat{i} + 2\lambda\hat{k} - 2\mu\hat{i} - \mu\hat{j} + \mu\hat{k}$

$\Rightarrow \vec{r} = 3\hat{j} + \lambda(\hat{i} + 2\hat{k}) + (-2\hat{i} - \hat{j} + \hat{k})\mu$

Now,

$\vec{r} = 3\hat{j} + \lambda(\hat{i} + 2\hat{k}) + (-2\hat{i} - \hat{j} + \hat{k})\mu$

We know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

Clearly,

$\vec{a} = 3\hat{j}, \vec{b} = \hat{i} + 2\hat{k}, \vec{c} = -2\hat{i} - \hat{j} + \hat{k}$.

Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,

Hence,

$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$\Rightarrow \vec{n} = \hat{i}(0 - (-2)) - \hat{j}(1 - (-4)) + \hat{k}(-1 - 0)$

$\Rightarrow \vec{n} = 2\hat{i} - 5\hat{j} - \hat{k}$

We know that vector equation of a plane in scalar product form is given as

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ ...... (1)

Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,

$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) = (3\hat{j}).(2\hat{i} - 5\hat{j} - \hat{k})$

$\Rightarrow \vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) = (0)(2) + (3)(-5) + (0)(-1)$

$\Rightarrow \vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) = -15$

Hence, the required equation is $\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) + 15 = 0$.

ii.

Solution:

Given:

$\vec{r} = (2\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(5\hat{i} - 2\hat{j} + 7\hat{k})$

We know that $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents a plane passing through a point having position vector $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

Clearly,

$\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{c} = 5\hat{i} - 2\hat{j} + 7\hat{k}$.

Now, the plane is perpendicular to $\vec{b} \times \vec{c}$,

Hence,

$$\vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$$

$\Rightarrow \vec{n} = \hat{i}(14 + 6) - \hat{j}(7 - 15) + \hat{k}(-2 - 10)$

$\Rightarrow \vec{n} = 20\hat{i} + 8\hat{j} - 12\hat{k}$

We know that vector equation of a plane in scalar product form is given as

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ …… (1)

Substitute the value of $\vec{a}$ and $\vec{n}$ in equation (1) we get,

$\vec{r}.(20\hat{i} + 8\hat{j} - 12\hat{k}) = (2\hat{i} + 2\hat{j} - \hat{k}).(20\hat{i} + 8\hat{j} - 12\hat{k})$

$\Rightarrow \vec{r}.(20\hat{i} + 8\hat{j} - 12\hat{k}) = (2)(20) + (2)(8) + (-1)(-12)$

$\Rightarrow \vec{r}.(20\hat{i} + 8\hat{j} - 12\hat{k}) = 40 + 16 + 12$

$\Rightarrow \vec{r}.(20\hat{i} + 8\hat{j} - 12\hat{k}) = 68$

Divide by 4, we get

$\Rightarrow \vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$

Hence, the required equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$.