

EXERCISE 29.8

Q1.

Solution:

Given:

Equation of plane is $2x - 3y + z = 0$ (1)

We know that equation of a plane is parallel to given plane (1) is given as

$2x - 3y + z + k = 0$ (2)

Given that, plane (2) is passing through the point $(1, -1, 2)$. So it must satisfy plane (2),

$$2(1) - 3(-1) + (2) + k = 0$$

$$2 + 3 + 2 + k = 0$$

$$7 + k = 0$$

$$k = -7$$

Now, substitute the value of k in equation (2), we get

$$2x - 3y + z - 7 = 0$$

Hence, the equation of the required plane is, $2x - 3y + z = 7$.

Q2.

Solution:

Given:

The equation of the plane is:

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 2 = 0 \text{ (1)}$$

We know that the equation of a plane is parallel to given plane (1) is given as

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + k = 0 \text{ (2)}$$

Given that, plane (2) is passing through the point $3\hat{i} + 4\hat{j} - \hat{k}$. So it must satisfy the equation (2),

$$3\hat{i} + 4\hat{j} - \hat{k} \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + k = 0$$

$$(3)(2) + (4)(-3) + (-1)(5) + k = 0$$

$$6 - 12 - 5 + k = 0$$

$$k = 11$$

Now, substitute the value of k in equation (2), we get

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 11 = 0$$

Hence, the required equation of plane is,

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 11 = 0$$

Q3.

Solution:

We know that, equation of a plane passing through the line of intersection of two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$(a_1x + b_1y + c_1z + d_1) + k(a_2x + b_2y + c_2z + d_2) = 0$$

Given, equation of plane is,

$$2x - 7y + 4z - 3 = 0 \text{ and}$$

$$3x - 5y + 4z + 11 = 0$$

So equation of plane passing through the line of intersection of given two planes is

$$(2x - 7y + 4z - 3) + k(3x - 5y + 4z + 11) = 0$$

$$2x - 7y + 4z - 3 + 3kx - 5ky + 4kz + 11k = 0$$

$$x(2 + 3k) + y(-7 - 5k) + z(4 + 4k) - 3 + 11k = 0 \dots\dots(1)$$

It is given that, plane (1) is passing through the point $(-2, 1, 3)$. So it must satisfy the equation (1),

$$(-2)(2 + 3k) + (1)(-7 - 5k) + (3)(4 + 4k) - 3 + 11k = 0$$

$$-2 + 12k = 0$$

$$12k = 2$$

$$k = 2/12$$

$$= 1/6$$

Now substitute the value of k in equation (1), we get

$$x(2 + 3k) + y(-7 - 5k) + z(4 + 4k) - 3 + 11k = 0$$

$$x\left(2 + \frac{3}{6}\right) + y\left(-7 - \frac{5}{6}\right) + z\left(4 + \frac{4}{6}\right) - 3 + \frac{11}{6} = 0$$

$$x\left(\frac{12+3}{6}\right) + y\left(\frac{-42-5}{6}\right) + z\left(\frac{24+4}{6}\right) - \frac{18+11}{6} = 0$$

$$\frac{15}{6}x - \frac{47}{6}y + \frac{28}{6}z - \frac{7}{6} = 0$$

Multiply by 6, we get

$$15x - 47y + 28z - 7 = 0$$

Hence, the required equation of plane is $15x - 47y + 28z - 7 = 0$.

Q4.

Solution:

We know that, the equation of a plane passing through the line of intersection of two planes

$\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is given by

$$\vec{r} \cdot (\vec{n}_1 + k\vec{n}_2) = d_1 + kd_2$$

So the equation of the plane passing through the line of intersection of given two planes

$$\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 0 \text{ and } \vec{r} \cdot (\hat{j} + 2\hat{k}) = 0 \text{ is given by}$$

$$\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k} + k(\hat{j} + 2\hat{k})) = 0 \dots\dots (1)$$

It is given that, plane (1) is passing through the point $2\hat{i} + \hat{j} - \hat{k}$. So it must satisfy equation (1).

$$(2\hat{i} + \hat{j} - \hat{k})(\hat{i} + 3\hat{j} - \hat{k}) + k(2\hat{i} + \hat{j} - \hat{k})(\hat{j} + 2\hat{k}) = 0$$

$$(2)(1) + (1)(3) + (-1)(-1) + k[(2)(0) + (1)(1) + (-1)(2)] = 0$$

$$(2 + 3 + 1) + k(1 - 2) = 0$$

$$6 - k = 0$$

$$k = 6$$

Now substitute the value of k in equation (1), we get

$$\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k} + k(\hat{j} + 2\hat{k})) = 0$$

$$\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k} + 6(\hat{j} + 2\hat{k})) = 0$$

$$\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k} + 6\hat{j} + 12\hat{k}) = 0$$

$$\vec{r} \cdot (\hat{i} + 9\hat{j} + 11\hat{k}) = 0$$

Hence, the equation of the required plane is

$$\vec{r} \cdot (\hat{i} + 9\hat{j} + 11\hat{k}) = 0$$

Q5.

Solution:

We know that, equation of a plane passing through the line of intersection of two planes

$$a_1x + b_1y + c_1z + d_1 = 0 \text{ and } a_2x + b_2y + c_2z + d_2 = 0 \text{ is given by}$$

$$(a_1x + b_1y + c_1z + d_1) + k(a_2x + b_2y + c_2z + d_2) = 0$$

So equation of plane passing through the line of intersection of given two planes

$$2x - y = 0 \text{ and } 3z - y = 0 \text{ is}$$

$$(2x - y) + k(3z - y) = 0$$

$$2x - y + 3kz - ky = 0$$

$$x(2) + y(-1 - k) + z(3k) = 0 \dots\dots (1)$$

We know that, two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \dots\dots (2)$$

It is given, plane (1) is perpendicular to plane
 $4x + 5y - 3z = 8 \dots\dots (3)$

By using equation (1) and (3) in equation (2), we get

$$(2) (4) + (-1 - k) (5) + (3k) (-3) = 0$$

$$8 - 5 - 5k - 9k = 0$$

$$3 - 14k = 0$$

$$-14k = -3$$

$$k = 3/14$$

Now, substitute the value of k in equation (1), we get

$$x(2) + y(-1 - k) + z(3k) = 0$$

$$2x + y\left(-1 - \frac{3}{14}\right) + z \cdot 3\left(\frac{3}{14}\right) = 0$$

$$2x + y\left(\frac{-14 - 3}{14}\right) + \frac{9z}{14} = 0$$

$$2x + y\left(-\frac{17}{14}\right) + \frac{9z}{14} = 0$$

Multiply by 14, we get

$$28x - 17y + 9z = 0$$

Hence, the required equation of plane is, $28x - 17y + 9z = 0$.