EXERCISE 29.9

Q1.

Solution:

Given:

Point given by the equation is: $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$

Plane given by the equation is: $\vec{r}.\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9 = 0$

Where, the normal vector is: $\vec{n} = 3\hat{i} - 4\hat{j} + 12\hat{k}$

We know, the distance of $\vec{a}$ from the plane $\vec{r}.\vec{n} - d = 0$ is given by:

$$p = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right|$$

Now substitute the values of $\vec{a}$ and $\vec{n}$, we get

$$\Rightarrow p = \left|\frac{(2\hat{i} - \hat{j} - 4\hat{k}).(3\hat{i} - 4\hat{j} + 12\hat{k}) - 9}{|3\hat{i} - 4\hat{j} + 12\hat{k}|}\right|$$

= 47/13 units

Hence, the distance of the point $2\hat{i} - \hat{j} - 4\hat{k}$ from the plane $\vec{r}.\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9 = 0$ is 47/13 units.

Q2.

Solution:

Given:

Points given by the equation is: $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Plane given by the equation: $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

Where, the normal vector is: $\vec{n} = 5\hat{i} + 2\hat{j} - 7\hat{k}$

We know, the distance of $\vec{a}$ from the plane $\vec{r}.\vec{n} - d = 0$ is given by:

$$p = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right|$$

Distance of $\hat{i} - \hat{j} + 3\hat{k}$ from the plane $= \left|\frac{(\hat{i} - \hat{j} + 3\hat{k}).(5\hat{i} + 2\hat{j} - 7\hat{k}) + 9}{|5\hat{i} + 2\hat{j} - 7\hat{k}|}\right|$

$= \frac{9}{\sqrt{78}}$ units

And,

Distance of $3\hat{i} + 3\hat{j} + 3\hat{k}$ from the plane $= \left|\frac{(3\hat{i}+3\hat{j}+3\hat{k}).(5\hat{i}+2\hat{j}-7\hat{k})+9}{|5\hat{i}+2\hat{j}-7\hat{k}|}\right|$

$= \frac{9}{\sqrt{78}}$ units

Hence, the points $\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + 3\hat{j} + 3\hat{k}$ are equidistant from the plane $\vec{r}.(5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$.

Q3.

Solution:

Given:

Point: A(2, 3, –5)

Plane: π = x + 2y – 2z – 9 = 0

We know, the distance of point (x_1, y_1, z_1) from the plane π: ax + by + cz + d = 0 is given by:

$$p = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

Now, substitute the values A(2, 3, –5), we get

Distance of the plane from A $= \left|\frac{(1)(2)+(2)(3)+(-2)(-5)-9}{\sqrt{(1^2+2^2+(-2)^2}}\right|$

= 3 units

Hence, the distance of the point (2, 3, –5) from the plane is 3 units.

Q4.

Solution:

Given:

The equation of plane is:

x + 2y – 2z + 8 = 0

Since the planes are parallel to x + 2y – 2z + 8 = 0, they must be of the form:

x + 2y – 2z + θ = 0

We know, the distance of point (x_1, y_1, z_1) from the plane π: ax + by + cz + d = 0 is given by:

$$p = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

It is given that, the distance of the planes from (2, 1, 1) is 2 units.

$$\Rightarrow \left|\frac{(1)(2)+(2)(1)+(-2)(1)+\theta}{\sqrt{1^2+2^2+(-2)^2}}\right| = 2$$

$$\left|\frac{2+\theta}{3}\right| = 2$$

$$\frac{2+\theta}{3} = 2 \text{ or } \frac{2+\theta}{3} = -2$$

$\theta = 4$ or -8

Now, substitute the values in $x + 2y - 2z + \theta = 0$, we get

$x + 2y - 2z + 4 = 0$

$x + 2y - 2z - 8 = 0$

Hence, the required planes are:

$x + 2y - 2z + 4 = 0$ and $x + 2y - 2z - 8 = 0$.

Q5.

Solution:

Given:

Points: A(1, 1, 1) and B(–3, 0, 1)

Plane: $\pi = 3x + 4y - 12z + 13 = 0$

We know, the distance of point (x_1, y_1, z_1) from the plane π: $ax + by + cz + d = 0$ is given by:

$$p = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

So, the Distance of (1, 1, 1) from the plane $= \left|\frac{(3)(1)+(4)(1)+(-12)(1)+13}{\sqrt{3^2+4^2+(-12)^2}}\right|$

$= \frac{8}{13}$ units

And,

Distance of (–3, 0, 1) from the plane $= \left|\frac{(3)(-3)+(4)(0)+(-12)(1)+13}{\sqrt{3^2+4^2+(-12)^2}}\right|$

$= \frac{8}{13}$ units

Hence, the points (1, 1, 1) and (–3, 0, 1) are equidistant from the plane $3x + 4y - 12z + 13 = 0$.