EXERCISE 30.1

Q1.
Solution:
Let us consider the data in the tabular form:

Gadget	Foundry	Machine-shop	Profit
A	10	5	Rs 30
B	6	4	Rs 20
Firm’s capacity per week	1000	600	

Now, let x and y be the required weekly production of gadgets A and B.
Given:
Profit on each gadget A is = Rs 30
So, profit on x gadget of type A = 30x

Profit on each gadget B is = Rs 20
So, profit on x gadget of type B = 20y

Let total profit be ‘Z’
So, Z = 30x + 20y

Case I:
Given:
Production of one gadget A requires 10hours per week for foundry.
Production of one gadget B requires 6hours per week for foundry.

So, x units of gadget A requires 10x hours per week and
y unit of gadget B requires 6y hours per week.
Maximum capacity of foundry per week is 1000 hours, so => $10x + 6y \leq 1000$

Case II:
Given:
Production of one gadget A requires 5hours per week of machine-shop.
Production of one unit of gadget B requires 4hours per week of machine-shop.

So, x units of gadget A requires 5x hours per week and
y unit of gadget B requires 4y hours per week.
Maximum capacity of machine-shop per week is 600 hours, so => $5x + 4y \leq 600$

Hence, the required mathematical formulation of linear programming is:
Maximize Z = 30x + 20y
Subject to
$10x + 6y \leq 1000$
$5x + 4y \leq 600$
Where, $x, y \geq 0$

Q2.
Solution:
Let us consider the data in the tabular form:

Product	Machine hours	Labour hours	Profit
A	1	1	Rs 60
B	-	1	Rs 80
Total capacity Minimum supply of product B is 200 units.	400 for A	500	

Now, let x and y be the required production of products A and B.

Given:
Profit on one unit of product A is = Rs 60
So, profit on x unit of product A = 60x

Profit on one unit of product B is = Rs 80
So, profit on x unit of product B = 80y

Let total profit be 'Z'
So, Z = 60x + 80y

First constraint:
Given, Minimum supply of product B is 200
So, $y \geq 200$

Second constraint:
Given:
Production of one unit of product A requires 1hour per week of machine hours.
So, x units of product A requires 1x hour per week and

Total machine hours available for product A is 400hours,
So, $x \le 400$

Third constraint:
Given:
Production of one unit of product A requires 1hour per week of labour hours.
Production of one unit of product B requires 1hour per week of labour hours.
So, x units of product A requires 1x hour per week and
y units of product B requires 1x hour per week.
Total labour hours available is 500hours,
So, $x + y \le 500$

Hence, the required mathematical formulation of linear programming is:
Minimize $Z = 60x + 80y$
Subject to
$x \le 400$
$x + y \le 500$
Where, $x, y \ge 0$

Q3.
Solution:
Let us consider the data in the tabular form:

Product	Machine (M_1)	Machine (M_2)	Profit
A	4	2	3
B	3	2	2
C	5	4	4
Capacity maximum	2000	2500	

Now, let x, y and z units be the required production of products A,B and C.

Given:
Profit on one unit of product A is = Rs 3
So, profit on x unit of product A = 3x

Profit on one unit of product B is = Rs 20
So, profit on x unit of product B = 2y

Profit on one unit of product C is = Rs 4
So, profit on x unit of product C = 4z

Let total profit be 'U'
So, U = 3x + 2y + 4z

First Constraint:
Given:
One unit of product A requires 4minutes on machine, M_1
One unit of product B requires 3minutes on machine, M_1
One unit of product C requires 5minutes on machine, M_1
So,
x unit of product A requires 4x minutes on machine, M_1
y unit of product B requires 3y minutes on machine, M_1
z unit of product C requires 5z minutes on machine, M_1
Total minutes on M_1 = 2000 minutes
i.e., $4x + 3y + 5z \leq 2000$

Second constraint:
Given:
One unit of product A requires 2minutes on machine, M_2
One unit of product B requires 2minutes on machine, M_2
One unit of product C requires 4minutes on machine, M_2
So,
x unit of product A requires 2x minutes on machine, M_2
y unit of product B requires 2y minutes on machine, M_2
z unit of product C requires 4z minutes on machine, M_2
Total minutes on M_2 = 2500 minutes
i.e., $2x + 2y + 4z \leq 2500$

Other constraints:
Given:
Firm must manufacture 100A's, 200B's and 50C's but not more than 150A's.
$100 \leq x \leq 150$
$y \geq 200$
$z \geq 50$

Hence, the required mathematical formulation of linear programming is:
Maximize U = 3x + 2y + 4z
Subject to
$4x + 3y + 5z \leq 2000$

$2x + 2y + 4z \leq 2500$
$100 \leq x \leq 150$
$y \geq 200$
$z \geq 50$
Where, $x, y, z \geq 0$

Q4.
Solution:
Let us consider the data in the tabular form:

Product	M_1	M_2	Profit
A	1	2	2
B	1	1	3
Capacity	6 hours 40minutes = 400minutes	10 hours = 600 minutes	

Now, let x and y be the required production of products A and B.

Given:
Profit on one unit of product A is = Rs 2
So, profit on x unit of product A = 2x

Profit on one unit of product B is = Rs 3
So, profit on x unit of product B = 3y

Let total profit be 'Z'
So, Z = 2x + 3y

First Constraint:
Given:
One unit of product A requires 1minutes on machine, M_1
One unit of product B requires 1minutes on machine, M_1
So,
x unit of product A requires 1x minutes on machine, M_1
y unit of product B requires 1y minutes on machine, M_1
Total minutes on M_1 = 2000 minutes
i.e., $x + y \leq 400$

Second constraint:
Given:
One unit of product A requires 2minutes on machine, M_2

One unit of product B requires 1minutes on machine, M_2
So,
x unit of product A requires 2x minutes on machine, M_2
y unit of product B requires 1y minutes on machine, M_2
Total minutes on M_2 = 2500 minutes
i.e., $2x + y \leq 600$

Hence, the required mathematical formulation of linear programming is:
Maximize Z = 2x + 3y
Subject to $x + y \leq 400$
$2x + y \leq 600$
Where, $x, y \geq 0$

Q5.
Solution:
Let us consider the data in the tabular form:

Plant	A	B	C	Cost
I	50	100	100	2500
II	60	60	200	3500
Monthly demand	2500	3000	7000	

Now, let x and y be the required days of plant I and II to minimize cost.

Given:
Plant I costs per day = Rs 2500
So, cost to run plant x requires per month = Rs 2500x

Plant II costs per day = Rs 3500
So, cost to run plant y requires per month = Rs 3500y

Let total cost per month be 'Z'
So, Z = 2500x + 3500y

First Constraint:
Given:
Production of type A from plant I requires = 50
Production of type A from plant II requires = 60
So,
x unit of production of type A from plant I requires = 50x

y unit of production of type A from plant II requires = 60y
Total demand of type A per month = 2500
i.e., $50x + 60y \geq 2500$

Second Constraint:
Given:
Production of type B from plant I requires = 100
Production of type B from plant II requires = 60
So,
x unit of production of type B from plant I requires = 100x
y unit of production of type B from plant II requires = 60y
Total demand of type B per month = 3000
i.e., $100x + 60y \geq 3000$

Third Constraint:
Given:
Production of type C from plant I requires = 100
Production of type C from plant II requires = 200
So,
x unit of production of type C from plant I requires = 100x
y unit of production of type C from plant II requires = 200y
Total demand of type C per month = 7000
i.e., $100x + 200y \geq 7000$

Hence, the required mathematical formulation of linear programming is:
Minimize $Z = 2500x + 3500y$
Subject to
$50x + 60y \geq 2500$
$100x + 60y \geq 3000$
$100x + 200y \geq 7000$