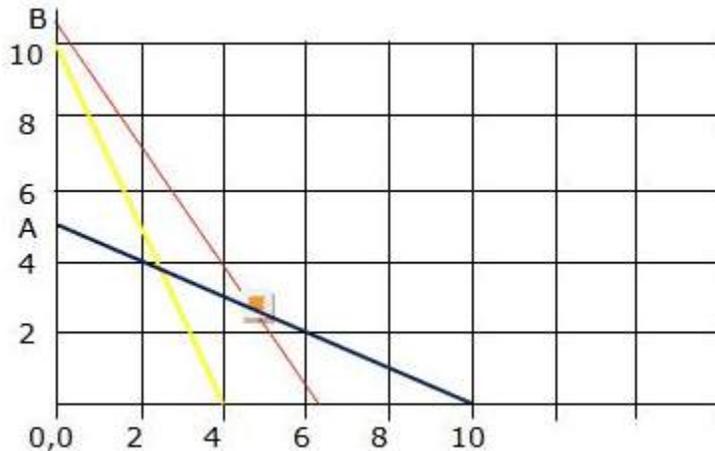


### EXERCISE 30.3

**Q1.**

**Solution:**



Let us consider  $x$  and  $y$  be number of 25gms of food packets,  $F_1$  and  $F_2$   
Minimum cost of diet  $Z = 0.20x + 0.15y$

Therefore the constraints are:

$$0.25x + 0.1y \geq 1; \text{ when } x = 0, y = 10 \text{ and } y = 0, x = 4$$

$$0.75x + 1.5y \geq 7.5; \text{ when } x = 0, y = 5 \text{ and } y = 0, x = 10$$

$$1.6x + 0.8y \geq 10; \text{ when } x = 0, y = 25/2 \text{ and } y = 0, x = 25/4$$

The feasible region is the open region B-E-10

The minimum cost of the diet can be checked by finding the value of  $Z$  at corner points B, E and 10.

| Corner points | Value of $Z = 20x + 15y$ |
|---------------|--------------------------|
| 0, 12.5       | 187.5                    |
| 10, 0         | 200                      |
| 5, 2.5        | 137.5                    |

Since the feasible region is an open region so we plot  $20x + 15y < 137.5$ , to check whether the resulting open half plane has any point common with the feasible region.

Since it has common points  $Z = 20x + 15y$

Hence there is no optimal minimum value subject to the given constraints.

**Q2.**

**Solution:**

Let us consider the required quantity of food A and B be  $x$  and  $y$  units.

Given:

Costs of one unit of food A and B are Rs 4 and Rs 3 per unit.

So, cost of  $x$  unit of food A and  $y$  unit of food B =  $4x$  and  $3y$ .

Let  $Z$  be minimum total cost,

So,  $Z = 4x + 3y$

First Constraint:

One unit of food A and B contain = 200 and 100 units of vitamins

So,  $x$  units of food A and  $y$  units of food B contain =  $200x$  and  $100y$  units of vitamins

Minimum requirement of vitamin = 4000 units

So,

$$200x + 100y \geq 4000$$

$$2x + y \geq 40$$

Second constraint:

One unit of food A and B contain = 1 and 2 units of minerals

So,  $x$  units of food A and  $y$  units of food B contain =  $x$  and  $2y$  units of minerals

Minimum requirement of minerals = 50 units

So,

$$x + 2y \geq 50$$

Third constraint:

One unit of food A and B contain = 40 calories each

So,  $x$  units of food A and  $y$  units of food B contain =  $40x$  and  $40y$  units of calories

Minimum requirement of calories = 1400 units

So,

$$40x + 40y \geq 1400$$

$$2x + 2y \geq 70$$

$$x + y \geq 35$$

Hence, the required mathematical formulation of linear programming is:

Minimize  $Z = 4x + 3y$

Subject to constraints

$$2x + y \geq 40$$

$$x + 2y \geq 50$$

$$x + y \geq 35$$

Where,  $x, y \geq 0$

Now, region  $2x + y \geq 40$ :

The line  $2x + y = 40$  meets axes at  $A_1(20, 0)$ ,  $B_1(0, 40)$  region not containing origin represents  $2x + y \geq 40$  as  $(0, 0)$  does not satisfy  $2x + y \geq 40$ .

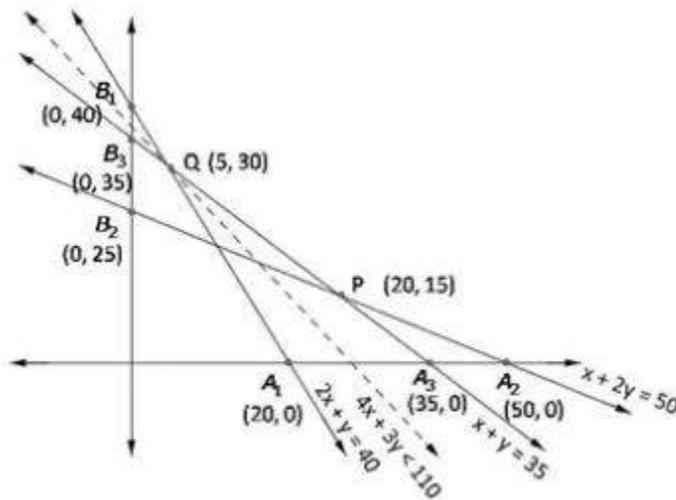
Region  $x + 2y \geq 50$ :

The line  $x + 2y = 50$  meets axes at  $A_2(50, 0)$ ,  $B_2(0, 25)$  region not containing origin represents  $x + 2y \geq 50$  as  $(0, 0)$  does not satisfy  $x + 2y \geq 50$ .

Region  $x + y \geq 35$ :

The line  $x + y = 35$  meets axes at  $A_3(35, 0)$ ,  $B_3(0, 35)$  region not containing origin represents  $x + y \geq 35$  as  $(0, 0)$  does not satisfy  $x + y \geq 35$ .

Region  $x, y \geq 0$ : it represents first quadrant in  $xy$ -plane.



Unbounded shaded region  $A_2 P Q B_1$  represents feasible region with corner points  $A_2(50, 0)$ ,  $P(20,15)$ ,  $Q(5, 30)$ ,  $B_1(0, 40)$

The value of  $Z = 4x + 3y$  at

$$A_2(50, 0) = 4(50) + 3(0) = 2000$$

$$P(20, 15) = 4(20) + 3(15) = 125$$

$$Q(5, 30) = 4(5) + 3(30) = 110$$

$$B_1(0, 40) = 4(0) + 3(40) = 110$$

Hence, smallest value of  $Z = 110$

Open half plane  $4x + 3y < 110$  has no point in common with feasible region

So, smallest value is the minimum value.

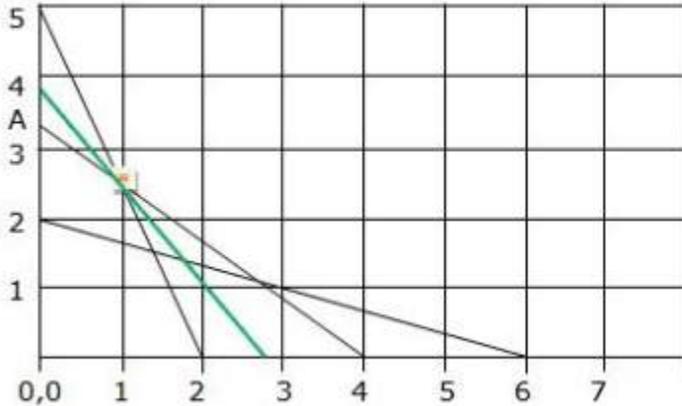
Quantity of food A =  $x = 5$  units

Quantity of food B =  $y = 30$  units

Minimum cost = Rs 110

**Q3.**

**Solution:**



Let us consider  $x$  and  $y$  be units of food, F I and F II

The objective function is to minimize the function  $Z = 0.6x + y$

Where,

$10x + 4y \geq 20$ ; requirement of calcium, line 5-2

$5x + 6y \geq 20$ ; requirement of protein, line A-4

$2x + 6y \geq 12$ ; requirement of calories, line 2-6

The feasible region is the open unbounded region 5-F-E-6

The function  $20x + 15y < 57.5$  needs to be plotted to check if there are any common points. The green line shows that there are no common points.

So,

| Corner points | Value of $Z = 0.6x + y$ |
|---------------|-------------------------|
| 0, 5          | 5                       |
| F(1, 2.5)     | 3.1                     |
| E(2.67, 1.11) | 2.71                    |
| 6, 0          | 3.6                     |

The minimum cost occurs when Food I is 1 unit and Food II is 2.5 units.

Since it is an unbounded region plotting  $Z < 3.1$  gives the green line which has no common points.

So, (1, 2.5) is said to be a minimum point.

**Q4.**

**Solution:**

Let us consider the required quantity of food A and B be  $x$  and  $y$  units.

Given:

Costs of one unit of food A and B are 10paise per unit.

So, cost of  $x$  unit of food A and  $y$  unit of food B =  $10x$  and  $10y$ .

Let  $Z$  be minimum total cost,

So,  $Z = 10x + 10y$

First Constraint:

One unit of food A and B contain = 0.12mg and 0.10mg of Thiamin

So,  $x$  units of food A and  $y$  units of food B contain =  $0.12x$  and  $0.10y$  mg of Thiamin

Minimum requirement of Thiamin = 0.5mg

So,

$$0.12x + 0.10y \geq 0.5$$

$$12x + 10y \geq 50$$

$$6x + 5y \geq 25$$

Second constraint:

One unit of food A and B contain = 100 and 150 calories

So,  $x$  units of food A and  $y$  units of food B contain =  $100x$  and  $150y$  units of calories

Minimum requirement of calories = 600 units

So,

$$100x + 150y \geq 600$$

$$2x + 3y \geq 12$$

Hence, the required mathematical formulation of linear programming is:

Minimize  $Z = 10x + 10y$

Subject to constraints

$$6x + 5y \geq 25$$

$$2x + 3y \geq 12$$

Where,  $x, y \geq 0$

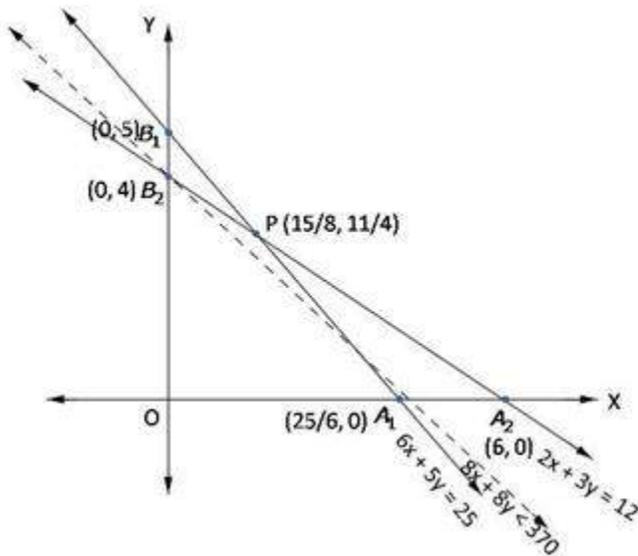
Now, region  $6x + 5y \geq 25$ :

The line  $6x + 5y = 25$  meets axes at  $A_1(25/6, 0)$ ,  $B_1(0, 5)$  region not containing origin represents  $6x + 5y \geq 25$  as  $(0, 0)$  does not satisfy  $6x + 5y \geq 25$ .

Region  $2x + 3y \geq 12$ :

The line  $2x + 3y = 12$  meets axes at  $A_2(6, 0)$ ,  $B_2(0, 4)$  region not containing origin represents  $2x + 3y \geq 12$  as  $(0, 0)$  does not satisfy  $2x + 3y \geq 12$ .

Region  $x, y \geq 0$ : it represents first quadrant in  $xy$ -plane.



Unbounded shaded region  $A_2 P B_1$  represents feasible region with corner points  $A_2(6, 0)$ ,  $P(15/8, 11/4)$ ,  $B_1(0, 5)$

The value of  $Z = 10x + 10y$  at

$$A_2(6, 0) = 10(6) + 10(0) = 60$$

$$P(15/8, 11/4) = 10(15/8) + 10(11/4) = 370/8 = 46 \frac{1}{4}$$

$$B_1(0, 5) = 10(0) + 10(5) = 50$$

Hence, smallest value of  $Z = 46 \frac{1}{4}$

Open half plane  $10x + 10y < 370/8$  has no point in common with feasible region

So, smallest value is the minimum value.

Required quantity of food A =  $x = 15/8$  units

Required quantity of food B =  $y = 11/4$  units

Minimum cost = Rs 46.25

### Q5.

#### Solution:

Let us consider the required quantity of food A and B be  $x$  and  $y$  kg.

Given:

Costs of one kg of food A and B are Rs 5 and Rs 8 kg.

So, cost of  $x$  kg of food A and  $y$  kg of food B =  $5x$  and  $8y$ .

Let  $Z$  be minimum total cost,

$$\text{So, } Z = 5x + 8y$$

First Constraint:

One kg of food A and B contain = 1 and 2 units of vitamin A

So,  $x$  kg of food A and  $y$  kg of food B contain =  $x$  and  $2y$  kg of vitamin A  
Minimum requirement of vitamin A = 6 units

So,  
 $x + 2y \geq 6$

Second constraint:

One kg of food A and B contain = 1 unit of Vitamin B each

So,  $x$  kg of food A and  $y$  kg of food B contain =  $x$  and  $y$  units of vitamin B  
Minimum requirement of vitamin B = 7 units

So,  
 $x + y \geq 7$

Third constraint:

One kg of food A and B contain = 1 and 3 units of vitamin C

So,  $x$  kg of food A and  $y$  kg of food B contain =  $x$  and  $3y$  units of vitamin C  
Minimum requirement of vitamin C = 11 units

So,  
 $x + 3y \geq 11$

Fourth constraint:

One kg of food A and B contain = 2 and 1 units of vitamin D

So,  $x$  kg of food A and  $y$  kg of food B contain =  $2x$  and  $y$  units of vitamin D  
Minimum requirement of vitamin D = 9 units

So,  
 $2x + y \geq 9$

Hence, the required mathematical formulation of linear programming is:

Minimize  $Z = 5x + 8y$

Subject to constraints

$$x + 2y \geq 6$$

$$x + y \geq 7$$

$$x + 3y \geq 11$$

$$2x + y \geq 9$$

Where,  $x, y \geq 0$

Now, region  $x + 2y \geq 6$ :

The line  $x + 2y = 6$  meets axes at  $A_1(6, 0)$ ,  $B_1(0, 3)$  region not containing origin represents  $x + 2y \geq 6$  as  $(0, 0)$  does not satisfy  $x + 2y \geq 6$ .

Region  $x + y \geq 7$ :

The line  $x + y = 7$  meets axes at  $A_2(7, 0)$ ,  $B_2(0, 7)$  region not containing origin represents  $x + y \geq 7$  as  $(0, 0)$  does not satisfy  $x + y \geq 7$ .

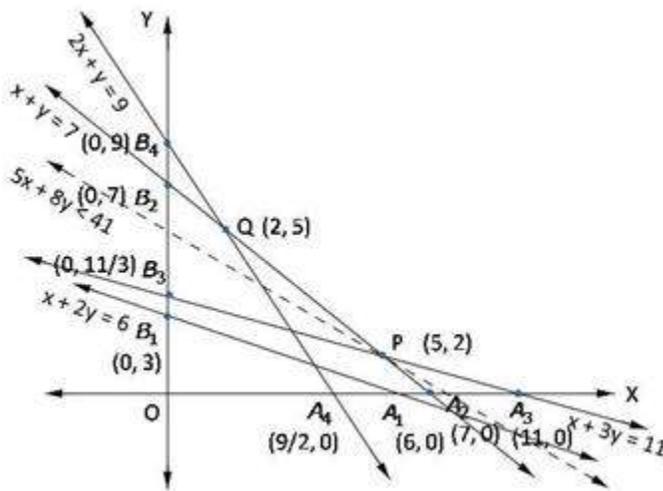
Region  $x + 3y \geq 11$ :

The line  $x + 3y = 11$  meets axes at  $A_3(11, 0)$ ,  $B_3(0, 11/3)$  region not containing origin represents  $x + 3y \geq 11$  as  $(0, 0)$  does not satisfy  $x + 3y \geq 11$ .

Region  $2x + y \geq 9$ :

The line  $2x + y = 9$  meets axes at  $A_4(9/2, 0)$ ,  $B_4(0, 9)$  region not containing origin represents  $2x + y \geq 9$  as  $(0, 0)$  does not satisfy  $2x + y \geq 9$ .

Region  $x, y \geq 0$ : it represents first quadrant in  $xy$ -plane.



Unbounded shaded region  $A_2 P Q B_4$  represents feasible region with corner points  $A_3(11, 0)$ ,  $P(5, 2)$ ,  $Q(2, 5)$ ,  $B_4(0, 9)$

The value of  $Z = 5x + 8y$  at

$$A_3(11, 0) = 5(11) + 8(0) = 55$$

$$P(5, 2) = 5(5) + 8(2) = 41$$

$$Q(2, 5) = 5(2) + 8(5) = 50$$

$$B_4(0, 9) = 5(0) + 8(9) = 72$$

Hence, smallest value of  $Z = 41$

Open half plane  $5x + 8y < 41$  has no point in common with feasible region

So, smallest value is the minimum value.

Last cost of mixture = Rs 41