## EXERCISE 30.4

**Q1.**
**Solution:**
Let us consider a young man drive x km at a speed of 25km/hr and y km at a speed of 40km/hr.
Let Z be the total distance travelled by young man.
So, $Z = x + y$

First constraint:
When speed is 25km/hr, young man spends = Rs 2 per km
When speed is 40km/hr, young man spends = Rs 5 per km
So,
Expenses on x km and y km = Rs 2x and Rs 5y
But young man has only Rs 100
So, $2x + 5y \leq 100$

Second constraint:
Time taken to travel x km = Distance/speed
= x/25 hr

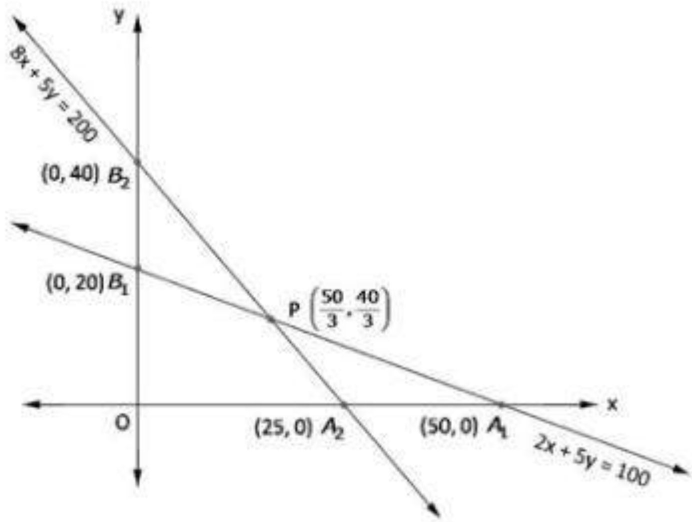
Time taken to travel y km = Distance/speed
= y/40 hr
It is given that, 1hr to travel,
So, $x/25 + y/40 \leq 1$
$40x + 25y \leq 1000$
$8x + 5y \leq 200$

Hence, the required mathematical formulation of linear programming is:
Minimize $Z = x + y$
Subject to constraints
$2x + 5y \leq 100$
$8x + 5y \leq 200$
Where, $x, y \geq 0$

Now, region $2x + 5y \leq 100$:
The line $2x + 5y = 100$ meets axes at $A_1(50, 0)$, $B_1(0, 20)$ region containing origin represents $2x + 5y \leq 100$ as (0, 0) satisfies $2x + 5y \leq 100$.

Region $8x + 5y \leq 200$:
The line $8x + 5y = 200$ meets axes at $A_2(25, 0)$, $B_2(0, 40)$ region containing origin represents $8x + 5y \leq 200$ as (0, 0) satisfies $8x + 5y \leq 200$.

Region $x, y \geq 0$: it represents first quadrant in xy-plane.




Shaded region O $A_2$ P $B_1$ represents feasible region.
Point P(50/3, 40/3) is obtained by solving $8x + 5y = 200$, $2x + y = 100$

The value of $Z = x + y$ at
O(0, 0) = 0 + 0 = 0
$A_2$(25, 0) = 25 + 0 = 25
P(50/3, 40/3) = 50/3 + 40/3 = 30
$B_1$(0, 20) = 0 + 20 = 20

Hence, maximum value of Z = 30 at x = 50/3, y = 40/3
Distance travelled at speed of 25km/hr = 50/3 km
And at speed of 40 km/hr = 40/3 km
So maximum distance = 30 km

**Q2.**
**Solution:**
Let us consider the required quantity of items be A and B.
Given:
Profits on one item A and B = Rs 6 and Rs 4
So, profits on x items of type A and y items on type B = 6x and 4y
Let the total profit be Z,

Z = 6x + 4y

First constraint:
Machine I works 1 hour and 2hours on item A and B
So, x no. of item A and y no. of item B = x hour and 2y hours on machine I
Machine I works at most = 12 hours
$x + 2y \geq 12$

Second constraint:
Machine II works 2 hours and 1hour on item A and B
So, x no. of item A and y no. of item B = 2x hours and y hours on machine II
Machine II works maximum of = 12 hours
$2x + y \geq 12$

Third constraint:
Machine III works 1 hour and 5/4hour on item A and B
So, x no. of item A and y no. of item B = x hours and 5/4y hours on machine III
Machine III works at least = 5 hours
$x + 5/4y \geq 5$
$4x + 5y \geq 20$

Hence, the required mathematical formulation of linear programming is:
Minimize Z = 6x + 4y
Subject to constraints
$x + 2y \geq 12$
$2x + y \geq 12$
$4x + 5y \geq 20$
Where, $x, y \geq 0$

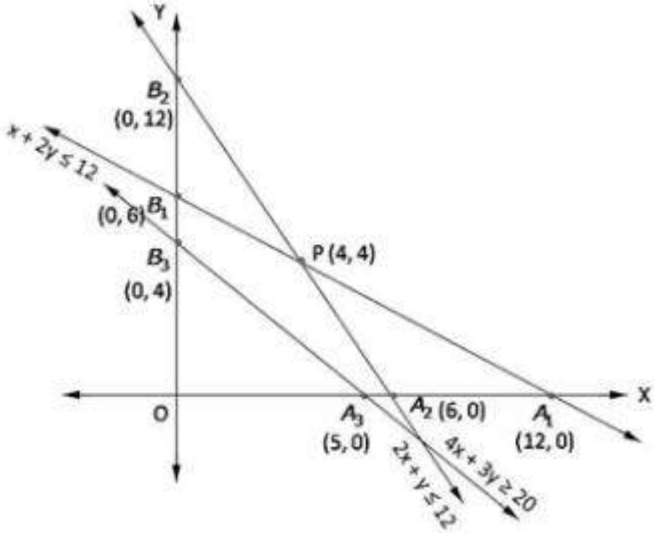
Now, region $x + 2y \geq 12$:
The line x + 2y = 12 meets axes at $A_1(12, 0)$, $B_1(0, 6)$ region containing origin represents $x + 2y \geq 12$ as (0, 0) satisfies $x + 2y \geq 12$.

Region $2x + y \geq 12$:
The line 2x + y = 12 meets axes at $A_1(6, 0)$, $B_1(0, 12)$ region containing origin represents $2x + y \geq 12$ as (0, 0) satisfies $2x + y \geq 12$.

Region $4x + 5y \geq 20$:
The line 4x + 5y = 20 meets axes at $A_3(5, 0)$, $B_3(0, 4)$ region not containing origin

represents $4x + 5y \geq 20$ as (0, 0) does not satisfy $4x + 5y \geq 20$.

Region $x, y \geq 0$: it represents first quadrant in xy-plane.




Shaded region $A_2$ $A_3$ P $B_3$ $B_1$ represents feasible region.

The value of $Z = 6x + 4y$ at
$A_2(6, 0) = 6(6) + 4(0) = 36$
$A_3(5, 0) = 6(5) + 4(0) = 30$
$B_3(0, 4) = 6(0) + 4(4) = 16$
$B_2(0, 6) = 6(0) + 4(6) = 24$
$P(4,4) = 6(4) + 4(4) = 40$

Hence, maximum value of Z = 40 at x = 4, y = 4
Required number of product A =4, product B = 4
So maximum profit = Rs 40

**Q3.**
**Solution:**
Let us consider tailor A and B work for x and y days.
Given:
Tailor A and B earn = Rs 15 and Rs 20
So, tailor A and B earn is x and y days = Rs 15x and 20y
Let the total maximum profit be Z which gives minimum labour cost,
$Z = 15x + 20y$

First constraint:

Tailor A and B stitches = 6 and 10 shirts in a day
So, tailor A and B can stitch = 6x and 10y shirts in a day
But, tries to produce atleast = 60 shirts
So, $6x + 10y \geq 60$
$3x + 5y \geq 30$

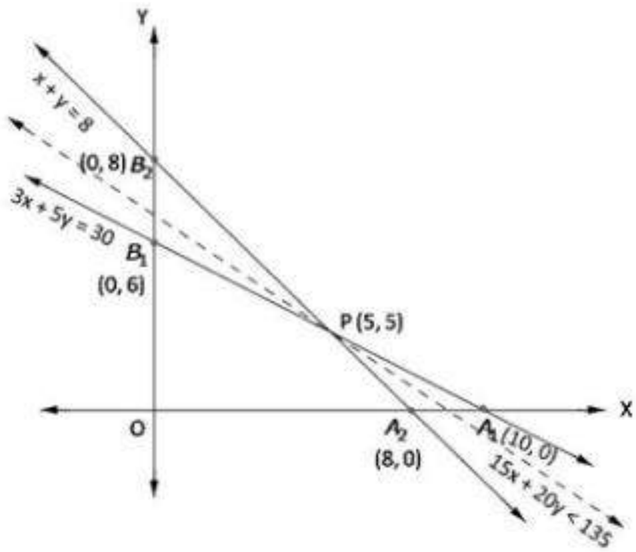
Second constraint:
Tailor A and B stitches = 4 pants each in a day
So, tailor A and B can stitch = 4x and 4y pants in a day
But, tries to produce atleast = 32 pants
So, $4x + 4y \geq 32$
$x + y \geq 8$

Hence, the required mathematical formulation of linear programming is:
Minimize $Z = 15x + 20y$
Subject to constraints
$3x + 5y \geq 30$
$x + y \geq 8$
Where, $x, y \geq 0$

Now, region $3x + 5y \geq 30$:
The line $3x + 5y = 30$ meets axes at $A_1(10, 0)$, $B_1(0, 6)$ region not containing origin represents $3x + 5y \geq 30$ as (0, 0) does not satisfy $3x + 5y \geq 30$.

Region $x + y \geq 8$:
The line $x + y = 8$ meets axes at $A_2(8, 0)$, $B_2(0, 8)$ region not containing origin represents $x + y \geq 8$ as (0, 0) does not satisfy $x + y \geq 8$.

Region $x, y \geq 0$: it represents first quadrant in xy-plane.




Unbounded shaded region $A_1$ P $B_2$ represents feasible region with corner points $A_1$(10, 0), P(5, 3), $B_2$(0, 8)

The value of Z = 15x + 20y at
$A_1$(10, 0) = 15(10) + 20(0) = 150
P(5, 3) = 15(5) + 20(3) = 135
$B_2$(0, 8) = 15(0) + 20(8) = 160

Hence, smallest value of Z = 135
Open half plane 15x + 20y < 135 has no point in common with feasible region
So, smallest value is the minimum value.
Z = 135, at x = 5, y = 3
Therefore tailor A should work for 5 days and B should work for 3 days.

**Q4.**
**Solution:**
Let us consider the factory manufactures x screws of type A and y screws of type B per day.
So, x ≥ 0 and y ≥ 0

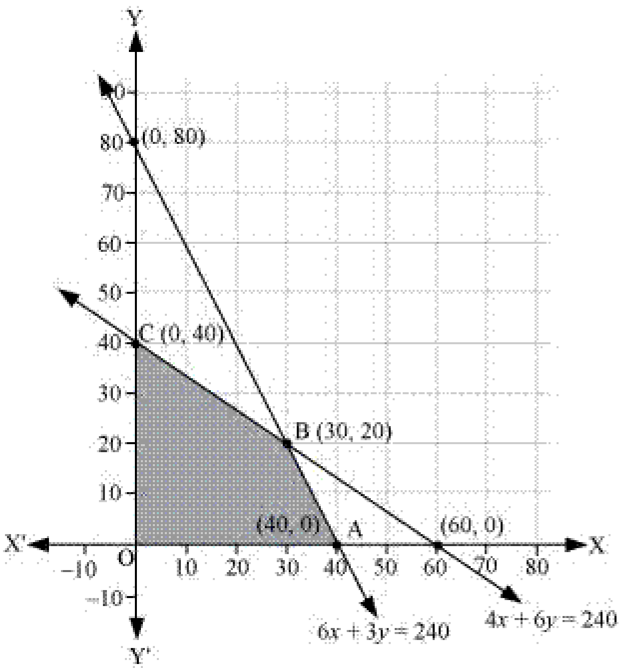
| | Screw A | Screw B | Availability |
|---|---|---|---|
| Automatic Machine (min) | 4 | 6 | 4 x 60 = 120 |
| Hand operated Machine (min) | 6 | 3 | 4 x 60 = 120 |

The profit on package of screws A and screws B is Rs 7 and Rs 10.

So the constraints are:
$4x + 6y \leq 240$
$6x + 3y \leq 240$
Total profit, $Z = 7x + 10y$

Hence, the required mathematical formulation of linear programming is:
Minimize $Z = 7x + 10y$
Subject to constraints
$4x + 6y \leq 240$
$6x + 3y \leq 240$
Where, $x, y \geq 0$

The feasible region obtained by the constraints is:




The corner points are A(40, 0), B(30, 20) and C(0, 40)
So the value of Z at these corner points is:

| Corner point | $Z = 7x + 10y$ | |
|---|---|---|
| A(40, 0) | 280 | |
| B(30, 20) | 410 | maximum |
| C(0, 40) | 400 | |

Hence, the maximum value of Z = 410 at B(30, 20).
So, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

**Q5.**

**Solution:**

Let us consider the required number of belt A and belt B be x and y.
Given:
Profits on belt A and B = Rs 2 and Rs 1.50 per belt
So, profits on x belts of type A and y belts on type B = 2x and 1.5y
Let the total profit be Z,
Z = 2x + 1.5y

First constraint:
Each belt of type A requires twice as much as belt B.
Each belt of type B requires = 1 hour to make, so A requires = 2 hours
So for x and y belts of type A and B requires = 2x and y hours to make
Total time available is equal to production of 1000 belts B = 1000 hours
$2x + y \leq 1000$

Second constraint:
It is given that, supply of leather for = 800 belts per day for both A and B
So, $x + y \leq 800$

Third constraint:
It is given that, buckles available for A = 400 and
Buckles available for B = 700
So, $x \leq 400$
$y \leq 700$

Hence, the required mathematical formulation of linear programming is:
Maximize Z = 2x + 1.5y
Subject to constraints
$2x + y \leq 1000$
$x + y \leq 800$
$x \leq 400$
$y \leq 700$
Where, $x, y \geq 0$

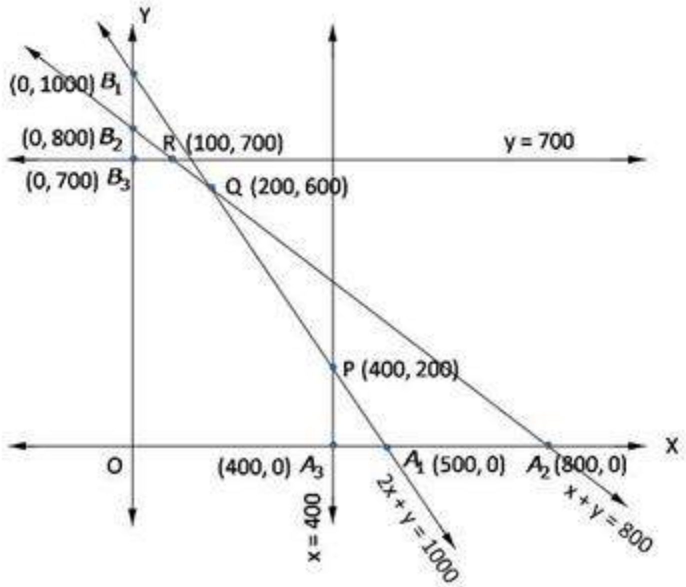
Now, region $2x + y \leq 1000$:
The line 2x + y = 1000 meets axes at $A_1(500, 0)$, $B_1(0, 1000)$ region containing origin represents $2x + y \leq 1000$ as (0, 0) satisfies $2x + y \leq 1000$.

Region $x + y \leq 800$:
The line $x + y = 800$ meets axes at $A_2(800, 0)$, $B_2(0, 800)$ region containing origin represents $x + y \leq 800$ as (0, 0) satisfies $x + y \leq 800$.

Region $x \leq 400$:
The line $x \leq 400$ is parallel to y-axis and meets x-axis at $A_3(400, 0)$ region containing origin represents $x \leq 400$ as (0, 0) satisfies $x \leq 400$.

Region $y \leq 400$:
The line $y \leq 700$ is parallel to x-axis and meets y-axis at $B_3(0, 700)$ region containing origin represents $y \leq 700$ as (0, 0) satisfies $y \leq 700$.

Region $x, y \geq 0$: it represents first quadrant in xy-plane.




Shaded region O $A_3$ P Q R $B_3$ represents feasible region.
Point P is obtained by intersection of $x + y = 800$, $2x + y = 1000$,
R is not point of intersection of $y = 700$, $x + y = 800$.

The value of $Z = 2x + 1.5y$ at
$O(0, 0) = 2(0) + 1.5(0) = 0$
$A_3(400, 0) = 2(400) + 1.5(0) = 800$
$P(400, 200) = 2(400) + 1.5(200) = 1100$
$Q(200, 600) = 2(200) + 1.5(600) = 1300$
$R(100, 700) = 2(100) + 1.5(700) = 1250$
$B_3(0, 700) = 2(0) + 1.5(700) = 1050$

Hence, maximum value of Z = 1300 at x = 200, y = 600
Required no. of belt A = 200,
Required no. of belt B = 600,
So maximum profit = Rs 1300