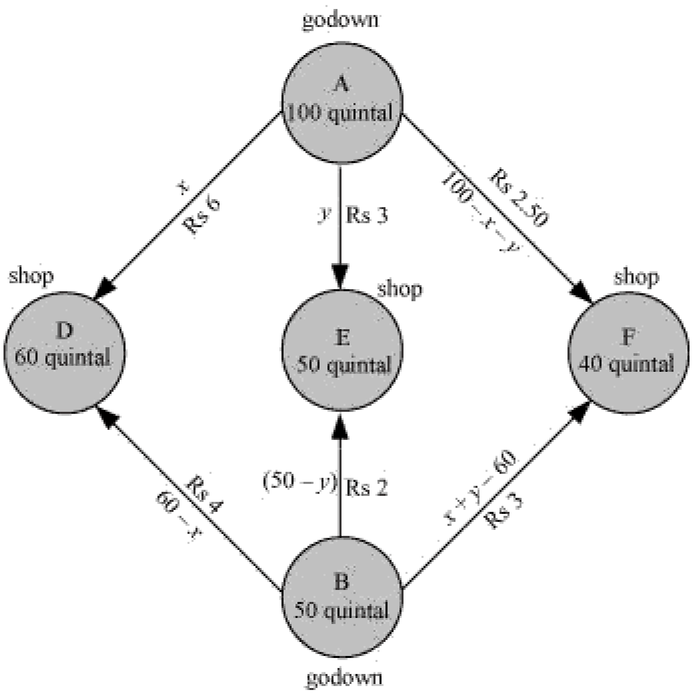
EXERCISE 30.5

Q1.

Solution:

Let us consider godown A supply x and y quintals of grain to the shops D and E.
Then, (100 – x – y) will be supplied to shop F.
The requirement at shop D is 60 quintals since, x quintals are transported from godown A.
Therefore, the remaining (60 − x) quintals will be transported from godown B.

Similarly, (50 − y) quintals and 40 − (100 − x − y) i.e. (x + y − 60) quintals will be transported from godown B to shop E and F respectively.
Here is the diagrammatic representation of the given problem:



$x \geq 0$, $y \geq 0$ and $100 - x - y \geq 0$
$\Rightarrow x \geq 0$, $y \geq 0$, and $x + y \leq 100$

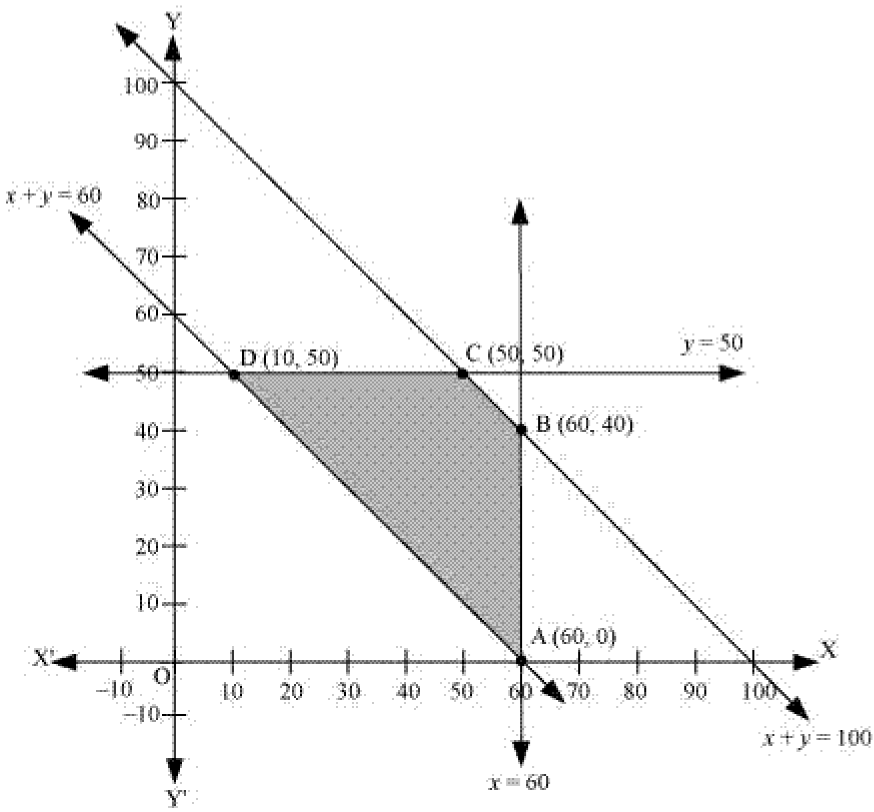
$60 - x \geq 0$, $50 - y \geq 0$, and $x + y - 60 \geq 0$
$\Rightarrow x \leq 60$, $y \leq 50$, and $x + y \geq 60$

Total transportation cost Z is given by,
$Z = 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$
$= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180$

= 2.5x + 1.5y + 410

Hence, the required mathematical formulation of linear programming is:
Minimize Z = 2.5x + 1.5y + 410
subject to the constraints,
$x + y \leq 100$
$x \leq 60$
$y \leq 50$
$x + y \geq 60$
$x, y \geq 0$

The feasible region obtained by the system of constraints is:



The corner points are A(60, 0), B(60, 40), C(50, 50), and D(10, 50).

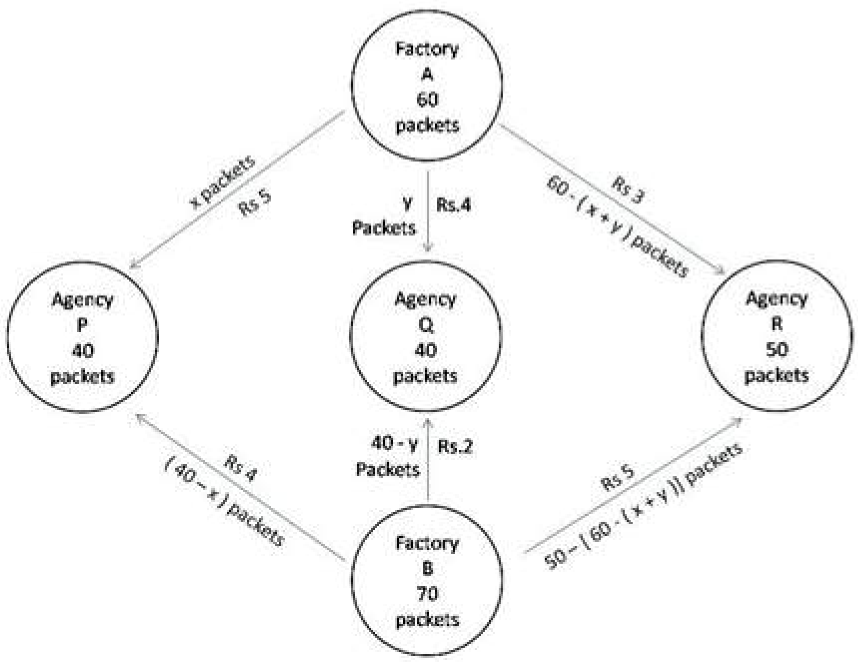
The values of Z at these corner points are as follows.

Corner point	Z = 2.5x + 1.5y + 410
A (60, 0)	560
B (60, 40)	620
C (50, 50)	610

D (10, 50)	510 -> minimum

Hence, the minimum value of Z is 510 at D (10, 50).
Thus, the amount of grain transported from A to D, E, and F is 10 quintals, 50 quintals, and 40 quintals respectively.
From B to D, E, and F is 50 quintals, 0 quintals, and 0 quintals respectively.
The minimum cost is Rs 510.

Q2.
Solution:
The diagrammatic representation of the given problem:



Let x and y packets be transported from factory A to the agencies P and Q respectively. Then, [60 − (x + y)] packets be transported to the agency R.
First constraint:
$x, y \geq 0$ and
Second constraint:
$60 - (x + y) \geq 0$
$(x + y) \leq 60$

The requirement at agency P is 40 packets. Since, x packets are transported from factory A,
Therefore, the remaining (40 − x) packets are transported from factory B.

Similarly, $(40 - y)$ packets are transported by B to Q and $50 - [60 - (x + y)]$ i.e. $(x + y - 10)$ packets will be transported from factory B to agency R respectively.

Number of packets cannot be negative.
Therefore,
Third constraint:
$40 - x \geq 0$
$\Rightarrow x \leq 40$

Fourth constraint:
$40 - y \geq 0$
$\Rightarrow y \leq 40$

Fifth constraint:
$x + y - 10 \geq 0$
$\Rightarrow x + y \geq 10$

So, costs of transportation of each packet from factory A to agency P, Q, R are Rs 5, 4, 3.
Costs of transportation of each packet from factory B to agency P, Q, R are Rs 4, 2, 5.
Let total cost of transportation be Z.
$Z = 5x + 4y + 3[60 - x + y] + 4(40 - x) + 2(40 - y) + 5(x + y - 10]$
$= 3x + 4y + 10$

Hence, the required mathematical formulation of linear programming is:
Minimize $Z = 3x + 4y + 370$
subject to constraints,
$x + y \leq 60$
$x \leq 40$
$y \leq 40$
$x + y \geq 10$
where, $x, y \geq 0$

Let us convert inequations into equations as follows:
$x + y = 60$, $x = 40$, $y = 40$, $x + y = 10$, $x = 0$ and $y = 0$

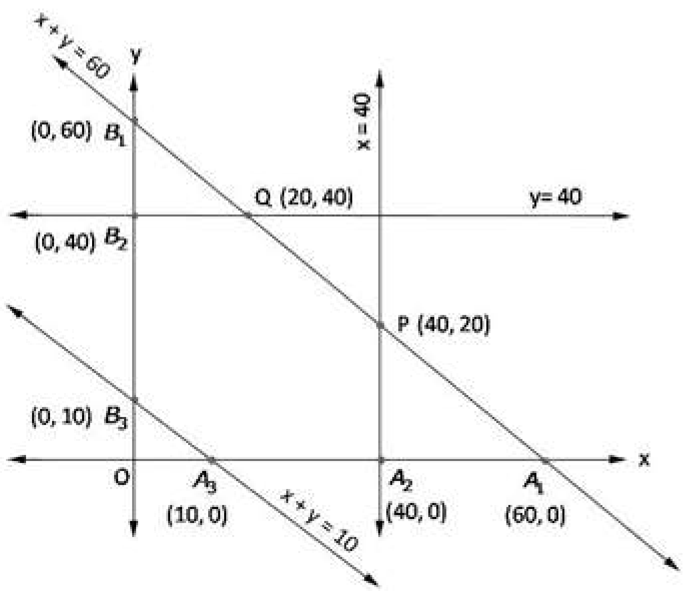
Region represented by $x + y \leq 60$:
The line $x + y = 60$ meets the coordinate axes at $A_1(60, 0)$ and $B_1(0, 60)$ respectively.
Region containing origin represents $x + y \leq 60$ as $(0, 0)$ satisfies $x + y \leq 60$.

Region represented by $x \leq 40$:
The line x = 40 is parallel to y-axis, meets x-axis at $A_2(40, 0)$. Region containing origin represents $x \leq 40$ as (0, 0) satisfies $x \leq 40$.

Region represented by $y \leq 40$:
The line y = 40 is parallel to x-axis, meets y-axis at $B_2(0, 40)$. Region containing origin represents $y \leq 40$ as (0, 0) satisfies $y \leq 40$.

Region represented by $x + y \geq 10$:
The line x + y = 10 meets the coordinate axes at $A_2(10, 0)$ and $B_3(0, 10)$ respectively. Region not containing origin represents $x + y \geq 10$ as (0, 0) does not satisfy $x + y \geq 10$.

Shaded region A_3 A_2 P Q B_2 B_3 represents feasible region.
Point P(40, 20) is obtained by solving x = 40 and x + y = 60
Point Q(20, 40) is obtained by solving y = 40 and x + y = 60



The value of Z = 3x + 4y + 370 at
$A_3(10, 0)$ = 3(10) + 4(0) + 370 = 400
$A_2(40, 0)$ = 3(40) + 4(0) + 370 = 490
P(40, 20) = 3(40) + 4(20) + 370 = 570
Q(20, 40) = 3(20) + 4(40) + 370 = 590
$B_2(0, 40)$ = 3(0) + 4(40) + 370 = 530
$B_3(0, 10)$ = 3(0) + 4(10) + 370 = 410

Hence, minimum value of Z = 400 at x = 10, y = 0
So, from A -> P = 10 packets

From A -> Q = 0 packets
From A -> R = 50 packets
From B -> P = 30 packets
From B -> Q = 40 packets
From B -> R = 0 packets
Therefore minimum cost = Rs 400