Exercise 31.1

1. Solution:

Here,
The sample space S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
Let,
A = Number on the card drawn is even
So, A = {2, 4, 6, 8, 10}
n (A) = 5

And,
B = Number on the card is greater than 3
So, B = {4, 5, 6, 7, 8, 9, 10}
n (B) = 7

Now,
A ∩ B = {4, 6, 8, 10}
n (A ∩ B) = 4

Thus, the required probability is given by
P(A/B) = n (A ∩ B)/ n (B)
= 4/7

2. Solution:

Let b and g represent the boy and the girl child respectively. If a family has two children, the sample space will be
S = {(b, b), (b, g), (g, b), (g, g)}
n (S) = 4

Let A be the event that both children are girls.
A = {(g, g)}
n (A) = 1

(i) Let B the event that the youngest child is a girl
B = {(b, g), (g, g)}
n (B) = 2

So,
A ∩ B = {(g, g)}
n (A ∩ B) = 1
P (A ∩ B) = n (A ∩ B)/ n (S)
= ¼

Now, the conditional probability that both are girls, given that the youngest child is a girl, is given by

P (A/B) = P (A ∩ B)/ P(B)
= ¼ / ½
= ½

Therefore, the required probability is ½.

(ii) Let C the event that at least one child is a girl
C = {(b, g), (g, b), (g, g)}
n (B) = 3

So,
A ∩ C = {(g, g)}
n (A ∩ C) = 1
P (A ∩ C) = n (A ∩ C)/ n (S)
= ¼

Now, the conditional probability that both are girls, given that the youngest child is a girl, is given by
P (A/C) = P (A ∩ C)/ P(C)
= ¼ / ¾
= 1/3

Therefore, the required probability is 1/3.

3. Solution:

Let A be the event of having two different numbers on the dice.
So,
A = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}
n (A) = 30

And,
Let B be the event getting a sum of 4 on the dice
B = {(1, 3), (2, 2), (3, 1)}
n (B) = 3

Now,
A ∩ B = {(1, 3), (3, 1)}
n (A ∩ B) = 2

Hence, the required conditional probability is given by
P(B/A) = n (A ∩ B)/ n (A)

= 2/30
= 1/15

4. Solution:

Let A be the event of a head appearing on the first two tosses
A = {HHT, HHH}
n (A) = 2
And, B be the event of getting a head on the third toss
B = {HHH, HTH, THH, TTH}
n (B) = 4
Now,
A ⋂ B = {HHH}
n (A ⋂ B) = 1

Hence, the required conditional probability is given by
P (B/A) = n (A ⋂ B)/ n (A)
= ½

5. Solution:

Let A be the event of 4 appearing on the third toss, if a die is thrown three times

$$A = \{(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)\\ (2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)\\ (3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)\\ (4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4)\\ (5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4)\\ (6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)\}$$

n (A) = 36

And, let B be the event of 6 and 5 appearing respectively on first two tosses, if the die is tossed three times
B = {(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)}
n (B) = 6

Now,
A ⋂ B = {(6, 5, 4)}
n (A ⋂ B) = 1

Hence, the required probability is given by
P(A/B) = n (A ⋂ B)/ n (B)
= 1/6

6. Solution:

Given,
P(B) = 0.5, P(A ∩ B) = 0.32
We know that,
P(A/B) = P(A ∩ B)/ P(B)
= 0.32/ 0.5
= 32/50
= 16/25
Therefore, P(A/B) = 16/25

7. Solution:

Given,
P(A) = 0.4, P(B) = 0.3 and P(B/A) = 0.5
We know that,
P(B/A) = P(A ∩ B)/ P(A)
0.5 = P(A ∩ B)/ 0.4
P(A ∩ B) = 0.5 x 0.4
= 0.2
Now,
P(A/B) = P(A ∩ B)/ P(B)
= 0.2/ 0.3
Thus,
P(A/B) = 2/3

8. Solution:

Given,
P(A) = 1/3, P (B) = 1/5 and P(A ∩ B) = 11/30
We know that,
P(A ∩ B) = P(A) + P(B) – P(A ∩ B)
11/30 = 1/3 + 1/5 – P(A ∩ B)
P(A ∩ B) = 1/3 + 1/5 - 11/30
= (10 + 6 - 11)/ 30
= 5/30
= 1/6
Now,
P(A/B) = P(A ∩ B)/ P(B)
= (1/6)/ (1/5)
= 5/6
And,
P(B/A) = P(A ∩ B)/ P(A)
= (1/6)/ (1/3)
= 3/6
= ½
Thus, P(A/B) = 5/6 and P(B/A) = ½.