

Exercise 31.3

1. Solution:

Given,

$$P(A) = 7/13, P(B) = 9/13 \text{ and } P(A \cap B) = 4/13$$

We know that,

$$\begin{aligned} P(A/B) &= P(A \cap B) / P(B) \\ &= (4/13) / (9/13) \\ &= 4/9 \end{aligned}$$

2. Solution:

Given,

$$P(A) = 0.6, P(B) = 0.3 \text{ and } P(A \cap B) = 0.2$$

We know that,

$$\begin{aligned} P(A/B) &= P(A \cap B) / P(B) \\ &= 0.2 / 0.3 \\ &= 2/3 \end{aligned}$$

And,

$$\begin{aligned} P(B/A) &= P(A \cap B) / P(A) \\ &= 0.2 / 0.6 \\ &= 1/3 \end{aligned}$$

Therefore, $P(A/B) = 2/3$ and $P(B/A) = 1/3$.

3. Solution:

Given,

$$P(A \cap B) = 0.32 \text{ and } P(B) = 0.5$$

We know that,

$$\begin{aligned} P(A/B) &= P(A \cap B) / P(B) \\ &= 0.32 / 0.5 \\ &= 16/25 \\ &= 0.64 \end{aligned}$$

Therefore, $P(A/B) = 0.64$

4. Solution:

Given,

$$P(A) = 0.4, P(B) = 0.8 \text{ and } P(B/A) = 0.6$$

We know that,

$$\begin{aligned} P(B/A) &= P(A \cap B) / P(A) \\ 0.6 &= P(A \cap B) / 0.4 \\ P(A \cap B) &= 0.6 \times 0.4 \\ &= 0.24 \end{aligned}$$

Now,

$$\begin{aligned}P(A/B) &= P(A \cap B) / P(B) \\ &= 0.24 / 0.8 \\ &= 0.3\end{aligned}$$

And,

$$\begin{aligned}P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.4 + 0.8 - 0.24 \\ &= 0.96\end{aligned}$$

Therefore, $P(A/B) = 0.3$ and $P(A \cup B) = 0.96$

5.

(i) Solution:

Given,

$$P(A) = 1/3, P(B) = 1/4 \text{ and } P(A \cup B) = 5/12$$

We know that,

$$\begin{aligned}P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 5/12 &= 1/3 + 1/4 - P(A \cap B)\end{aligned}$$

$$\begin{aligned}P(A \cap B) &= 1/3 + 1/4 - 5/12 \\ &= (4 + 3 - 5) / 12 \\ &= 2/12 \\ &= 1/6\end{aligned}$$

Now,

$$\begin{aligned}P(A/B) &= P(A \cap B) / P(B) \\ &= (1/6) / (1/4) \\ &= 4/6 \\ &= 2/3\end{aligned}$$

And,

$$\begin{aligned}P(B/A) &= P(A \cap B) / P(A) \\ &= (1/6) / (1/3) \\ &= 3/6 \\ &= 1/2\end{aligned}$$

Therefore, $P(A/B) = 2/3$ and $P(B/A) = 1/2$.

(ii) Solution:

Given,

$$P(A) = 6/11, P(B) = 5/11 \text{ and } P(A \cup B) = 7/11$$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

So,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\begin{aligned}P(A \cap B) &= 6/11 + 5/11 - 7/11 \\ &= 4/11\end{aligned}$$

Now,

$$\begin{aligned}P(A/B) &= P(A \cap B) / P(B) \\ &= (4/11) / (5/11)\end{aligned}$$

$$= 4/5$$

And,

$$P(B/A) = P(A \cap B)/P(A)$$

$$= (4/11) / (6/11)$$

$$= 4/6$$

$$= 2/3$$

Therefore, $P(A/B) = 4/5$ and $P(B/A) = 2/3$.

