

### Exercise 31.4

1.

(i) **Solution:**

Here,

A coin is tossed thrice

So, sample space = {HHH, HHT, HTH, HTT, TTT, TTH, THT, THH}

$$n(S) = 8$$

Let,

A = event for getting a head in the first throw

$$A = \{HHH, HHT, HTH, HTT\}$$

$$n(A) = 4$$

And,

B = event of getting a tail in the last throw

$$B = \{HHT, HTT, TTT, THT\}$$

$$n(B) = 4$$

Now,

$$A \cap B = \{HHT, HTT\}$$

$$n(A \cap B) = 2$$

So,

$$P(A) = n(A)/n(S) = 4/8 = \frac{1}{2}$$

$$P(B) = n(B)/n(S) = 4/8 = \frac{1}{2}$$

$$P(A \cap B) = n(A \cap B)/n(S) = 2/8 = \frac{1}{4}$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Thus,

$$P(A) \cdot P(B) = P(A \cap B)$$

Therefore, A and B are independent events.

(ii) **Solution:**

Here,

A coin is tossed thrice

So, sample space = {HHH, HHT, HTH, HTT, TTT, TTH, THT, THH}

$$n(S) = 8$$

Let,

A = event for getting an odd number of heads

$$A = \{HTT, THT, TTH, HHH\}$$

$$n(A) = 4$$

And,

B = event of getting an odd number of tails

$$B = \{HHT, HTH, TTT, THH\}$$

$$n(B) = 4$$

Now,

$$A \cap B = \{ \} = \emptyset$$

$$n(A \cap B) = 0$$

So,

$$P(A) = n(A) / n(S) = 4/8 = 1/2$$

$$P(B) = n(B) / n(S) = 4/8 = 1/2$$

$$P(A \cap B) = n(A \cap B) / n(S) = 0/8 = 0$$

$$P(A) \cdot P(B) = 1/2 \times 1/2 = 1/4$$

Thus,

$$P(A) \cdot P(B) \neq P(A \cap B)$$

Therefore, A and B are not independent events.

**(iii) Solution:**

Here,

A coin is tossed thrice

So, sample space = {HHH, HHT, HTH, HTT, TTT, TTH, THT, THH}

$$n(S) = 8$$

Let,

A = event for getting two heads

$$A = \{HHT, THH, HTH\}$$

$$n(A) = 3$$

And,

B = event of getting a head in the last throw

$$B = \{HHH, HTH, TTH, THH\}$$

$$n(B) = 4$$

Now,

$$A \cap B = \{THH, HTH\}$$

$$n(A \cap B) = 2$$

So,

$$P(A) = n(A) / n(S) = 3/8$$

$$P(B) = n(B) / n(S) = 4/8 = 1/2$$

$$P(A \cap B) = n(A \cap B) / n(S) = 2/8 = 1/4$$

$$P(A) \cdot P(B) = 3/8 \times 1/2 = 3/16$$

Thus,

$$P(A) \cdot P(B) \neq P(A \cap B)$$

Therefore, A and B are not independent events.

**2. Solution:**

When a pair of dice are thrown, we get 36 outcomes

So, the sample space (s) has 36 elements

Let,

A = Event of occurrence of number 4 on the first die

$$= \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$$

$$n(A) = 6$$

And,

B = Event of occurrence of 5 on the second die  
 $= \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)\}$

$$n(B) = 6$$

Now,

$$A \cap B = \{(4, 5)\}$$

$$n(A \cap B) = 1$$

$$P(A) = n(A)/n(S) = 6/36 = 1/6$$

$$P(B) = n(B)/n(S) = 6/36 = 1/6$$

$$P(A \cap B) = 1/36$$

$$P(A) \cdot P(B) = 1/6 \times 1/6 = 1/36$$

Thus,

$$P(A) \cdot P(B) = P(A \cap B)$$

Therefore, A and B are independent events.

3.

(i) **Solution:**

A card is drawn from a pack of 52 cards.

And, it contains 4 kings, 4 Queens and 4 Jacks

Now, if

A = event of drawing a king or a queen

$$P(A) = (4 + 4)/52$$

$$= 8/52$$

$$= 2/13$$

B = event of drawing a queen or a jack

$$P(B) = (4 + 4)/52$$

$$= 8/52$$

$$= 2/13$$

$A \cap B$  = Event of drawing a queen

$$P(A \cap B) = 4/52$$

$$= 1/13$$

Now,

$$P(A) \cdot P(B) = 2/13 \times 2/13$$

$$= 4/169$$

Thus,

$$P(A) \cdot P(B) \neq P(A \cap B)$$

Therefore, A and B are not independent events.

(ii) **Solution:**

A card is drawn from a pack of 52 cards.

And, it contains 26 black cards and 4 kings, of which 2 kings are black

Now, if

A = event of drawing a black card

$$\begin{aligned}P(A) &= (26 + 2) / 52 \\ &= 28/52 \\ &= 7/13\end{aligned}$$

B = event of drawing a king

$$\begin{aligned}P(B) &= 4 / 52 \\ &= 1/13\end{aligned}$$

$A \cap B$  = Event of drawing a black king

$$\begin{aligned}P(A \cap B) &= 2/52 \\ &= 1/26\end{aligned}$$

Now,

$$\begin{aligned}P(A) \cdot P(B) &= 7/13 \times 1/13 \\ &= 7/169\end{aligned}$$

Thus,

$$P(A) \cdot P(B) = P(A \cap B)$$

Therefore, A and B are independent events.

**(iii) Solution:**

A card is drawn from a pack of 52 cards.

And, it contains 13 spades and 4 aces, of which one card is an ace of spade

Now, if

A = event of drawing a spade

$$\begin{aligned}P(A) &= 13/52 \\ &= 1/4\end{aligned}$$

B = event of drawing an ace

$$\begin{aligned}P(B) &= 4 / 52 \\ &= 1/13\end{aligned}$$

$A \cap B$  = Event of drawing an ace of spade

$$P(A \cap B) = 1/52$$

Now,

$$\begin{aligned}P(A) \cdot P(B) &= 1/4 \times 1/13 \\ &= 1/52\end{aligned}$$

Thus,

$$P(A) \cdot P(B) = P(A \cap B)$$

Therefore, A and B are independent events.