Exercise 31.5

1. Solution:

Let the given 2 bags be considered as Bag 1 and Bag 2
Bag 1 contains 6 black and 3 white balls
Bag 2 contains 5 black and 4 white balls
Now,
One ball is drawn from each bag
P(one black from bag 1) = 6/9
$P(B_1) = 2/3$
P(one black from bag 2) = 5/9
$P(B_2) = 5/9$
P(one white from bag 1) = 3/9
$P(W_1) = 1/3$
And,
P(one white from bag 2) = 4/9
$P(W_2) = 4/9$
So,
P(two balls of same colour) $= P[(W_1 \cap W_2) \cup (B_1 \cap B_2)]$
$= P(W_1 \cap W_2) + P(B_1 \cap B_2)$
$= P(W_1).\ P(W_2) + P(B_1).\ P(B_2)$
$= (1/3 \times 4/9) + (2/3 \times 5/9)$
$= 4/27 + 10/27$
$= 14/27$

Thus, the required probability is 14/27.

2. Solution:

Let the given two bags be considered as Bag 1 and Bag 2
Bag 1 contains 3 red and 5 black balls
Bag 2 contains 6 red and 4 black balls
Now,
One ball is drawn from each bag
P(one red from bag 1) = 3/8
$P(R_1) = 3/8$
P(one red from bag 2) = 6/10
$P(R_2) = 3/5$
P(one black from bag 1) = 5/8
$P(B_1) = 5/8$
And,
P(one black from bag 2) = 4/10
$P(W_2) = 2/5$
So,
When one ball is drawn from each bag

P(one ball is red and the other is black)
= $P[(R_1 \cap B_2) \cup (B_1 \cap R_2)]$
= $P(R_1 \cap B_2) + P(B_1 \cap R_2)$
= $P(R_1). P(B_2) + P(B_1). P(R_2)$
= (3/8 x 2/5) + (5/8 x 3/5)
= 6/40 + 15/40
= 21/40

Thus, the required probability is 21/40.

3.
Solution:
Given,
A box contains 10 black and 8 red balls. Total number of balls is 18.
So, P(B) = 10/18 and P(R) = 8/18
Two balls are drawn with replacement

(i)
P(both the balls are red) = $P(R_1 \cap R_2)$
= $P(R_1). P(R_2)$
= (8/18) x (8/18)
= 16/81
Thus, the required probability is 16/81.

(ii)
P(first ball is black and second is red) = $P(B \cap R)$
= P(B). P(R)
= (10/18) x (8/18)
= 20/81
Thus, the required probability is 20/81.

(iii)
P(one of them is red and the other is black) is given by
= $P[(B \cap R) \cup (R \cap B)]$
= $P(B \cap R) + P(R \cap B)$
= P(B). P(R) + P(R). P(B)
= (10/18 x 8/18) + (8/18 x 10/18)
= (20 + 20)/ 81
= 40/81
Thus, the required probability is 40/81.

4. Solution:

Here, two cards are drawn without replacement
In a pack of cards there are total of 4 aces
Let,

A = Event of getting an ace
Now,
P(exactly one ace out of 2 cards) is given by

$$= P\left(\left(A \cap \overline{A}\right) \cup \left(\overline{A} \cap A\right)\right)$$

$$= P(A)P\left(\frac{\overline{A}}{A}\right) + P\left(\overline{A}\right)P\left(\frac{A}{\overline{A}}\right)$$

$$= \frac{4}{52}\cdot\frac{48}{51} + \frac{48}{52}\cdot\frac{4}{51}$$

$$= \frac{96}{663}$$

$$= \frac{32}{221}$$

Therefore, the required probability is 32/221.

5. Solution:

Given,
A speaks truth in 75% cases
B speaks truth in 80% cases
So,

$$P(A) = \frac{75}{100} \quad \Rightarrow P\left(\overline{A}\right) = \frac{25}{100}$$

$$P(B) = \frac{80}{100} \quad \Rightarrow P\left(\overline{B}\right) = \frac{20}{100}$$

Now,

$P(A \text{ and } B \text{ contradict each other})$

$$= P\left[\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right)\right]$$

$$= P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right)$$

$$= P(A)P\left(\overline{B}\right) + P\left(\overline{A}\right)P(B)$$

$$= \frac{75}{100}\cdot\frac{20}{100} + \frac{25}{100}\cdot\frac{80}{100}$$

$$= \frac{1500}{1000} + \frac{2000}{10000}$$

$$= \frac{3500}{10000}$$

$$= 35\%$$

Therefore, the required probability is 35%.

6. Solution:

Given,
Probability of selection of Kamal (K) = P(K) = 1/3

Probability of selection of Monika (M) = P(M) = 1/5

(i)

P(Both of them are selected) is given by

= P(K ∩ M)

= P(K). P(M)

= 1/3 x 1/5

= 1/15

Thus, the required probability is 1/15.

(ii)

P(none of them will be selected) is given by

$= P\left(\overline{K} \cap \overline{M}\right)$

$= P\left(\overline{K}\right) P\left(\overline{M}\right)$

$= \left[1 - P(K)\right]\left[1 - P(M)\right]$

$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)$

$= \frac{2}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

Thus, the required probability is 8/15.

(iii)

P(At least one of them is selected) = 1 – P(none of them will be selected)
= 1 - 8/15 [From the answer of (ii)]
= 7/15

Thus, the required probability is 7/15.

(iv)

P(only one of them will be selected) is given by

$= P\left[\left(K \cap \overline{M}\right) \cup \left(\overline{K} \cap M\right)\right]$

$= P\left(K \cap \overline{M}\right) + P\left(\overline{K} \cap M\right)$

$= P(K) P\left(\overline{M}\right) + P\left(\overline{K}\right) P(M)$

$= \frac{1}{3}\left[1 - P(M)\right] + \left[1 - P(K)\right]\frac{1}{5}$

$= \frac{1}{3}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{3}\right] \cdot \frac{1}{5}$

= (1/3 x 4/5) + (2/3 x 1/5)

= 4/15 + 2/15

= 6/15

= 2/5

Thus, the required probability is 2/5.

7. Solution:

Given,
A bag containing 3 white, 4 red and 5 black balls
And, two balls are drawn without replacement
So,
P(one ball is white and the other is black)
= $P[(W \cap B) \cup (B \cap W)]$
= $P(W \cap B) + P(B \cap W)$
= P(W).P(B/W) + P(B). P(W/B)
= (3/12 x 5/11) + (5/12 x 3/11)
= 15/132 + 15/132
= 30/132
= 5/22
Thus, the required probability is 5/22.