Exercise 31.6

1. Solution:

Given,
Bag A contains 5 white and 6 black balls
Bag B contains 4 white and 3 black balls
Now, there are two ways of transferring a ball from bag A to bag B
Way – 1
By transferring one white ball from bag A to bag B, then drawing one black ball from bag B.
Way – 2
By transferring one black ball from bag A to bag B, then drawing one black ball from bag B.
Let E_1, E_2 and A be events as below:
E_1 – One white ball drawn from bag A
E_2 – One black ball drawn from bag A
A – One black ball drawn from bag B
So, we have
$P(E_1) = 5/11$
$P(E_2) = 6/11$
Now,
$P(A/E_1) = 3/8$ [Since, E_1 has increased one white ball in bag B]
$P(A/E_2) = 4/8$ [Since, E_2 has increased one black ball in bag B]

By the law of total probability, we get
$P(A) = P(E_1).\ P(A/E_1) + P(E_2).\ P(A/E_2)$
$= (5/11 \times 3/8) + (6/11 \times 4/8)$
$= 15/88 + 24/88$
$= 39/88$
Hence, the required probability is 39/88.

2. Solution:

Given,
Purse (I) contains 2 silver and 4 copper coins
Purse (II) contains 4 silver and 3 copper coins
Now, one coin is drawn from one of the two purse and it is silver
Let E_1, E_2 and A be events as below:
E_1 – selecting purse I
E_2 – selecting purse II
A – drawing a silver coin
So, we have
$P(E_1) = 1/2$
$P(E_2) = 1/2$ [As there are only 2 purses]
Now,
$P(A/E_1)$ = P(drawing a silver from purse I)
$= 2/6$

= 1/3
$P(A/E_2)$ = P(drawing a silver coin from purse II)
= 4/7

By the law of total probability, we get
$P(A) = P(E_1). P(A/E_1) + P(E_2). P(A/E_2)$
= (1/2 x 1/3) + (1/2 x 4/7)
= 1/6 + 4/14
= (7 + 12)/42
= 19/42
Hence, the required probability is 19/42.

3. Solution:

Given,
Bag I contains 4 yellow and 5 red balls
Bag II contains 6 yellow and 3 red balls
Now, there are two ways of transferring a ball from bag I to bag II
Way – 1
By transferring one yellow ball from bag I to bag II, then one yellow ball is drawn from bag II.
Way – 2
By transferring one red ball from bag I to bag II, then one yellow ball is drawn from bag II.
Let E_1, E_2 and A be events as below:
E_1 – One yellow ball drawn from bag I
E_2 – One red ball drawn from bag I
A – One yellow ball drawn from bag II
So, we have
$P(E_1) = 4/9$
$P(E_2) = 5/9$
Now,
$P(A/E_1) = 7/10$ [Since, E_1 has increased one yellow ball in bag II]
$P(A/E_2) = 6/10$ [Since, E_2 has increased one red ball in bag II]

By the law of total probability, we get
$P(A) = P(E_1). P(A/E_1) + P(E_2). P(A/E_2)$
= (4/9 x 7/10) + (5/9 x 6/10)
= (28 + 30)/90
= 58/90
= 29/45
Hence, the required probability is 29/45.

4. Solution:

Given,
Bag I contains 3 white and 2 black balls
Bag II contains 2 white and 4 black balls

One bag is chosen at random, then one ball is drawn and it is white
Let E_1, E_2 and A be events as below:
E_1 – Choosing bag I
E_2 – Choosing bag II
A – Drawing one white ball
So, we have
$P(E_1) = 1/2$
$P(E_2) = 1/2$ [Since, there are only 2 bags]
Now,
$P(A/E_1)$ = P(drawing a white ball from Bag I)
= 3/5
$P(A/E_2)$ = P(drawing a white ball from Bag II)
= 2/6

By the law of total probability, we get
$P(A) = P(E_1). P(A/E_1) + P(E_2). P(A/E_2)$
= (½ x 3/5) + (½ x 2/6)
= 3/10 + 2/12
= (18 + 10)/60
= 28/60
= 7/15
Hence, the required probability is 7/15.

5. Solution:

Given,
Bag I contains 1 white, 2 black and 3 red balls
Bag II contains 2 white, 1 black and 1 red balls
Bag III contains 4 white, 5 black and 3 red balls
One bag is chosen at random, then one red ball and one white ball is drawn
Let E_1, E_2, E_3 and A be events as below:
E_1 – Choosing bag I
E_2 – Choosing bag II
E_3 – Choosing bag III
A – Drawing one red and one white ball
So, we have
$P(E_1) = 1/3$
$P(E_2) = 1/3$
$P(E_3) = 1/3$ [Since, there are only 3 bags]
Now,
$P(A/E_1)$ = P(drawing one red and one white ball from Bag I)
$= ({}^1C_1 \times {}^3C_1)/ {}^6C_2$
= (1 x 3)/ (30/2)
= 1/5
$P(A/E_2)$ = P(drawing one red and one white ball from Bag II)
$= ({}^2C_1 \times {}^1C_1)/ {}^4C_2$

= (2 x 1)/ (12/2)
= 1/3
$P(A/E_3)$ = P(drawing one red and one white ball from Bag III)

= $({}^4C_1 \times {}^3C_1)/ {}^{12}C_2$
= (4 x 3)/ (132/2)
=2/11

By the law of total probability, we get
$P(A) = P(E_1). P(A/E_1) + P(E_2). P(A/E_2) + P(E_3). P(A/E_3)$
= (1/3 x 1/5) + (1/3 x 1/3) + (1/3 x 2/11)
= 1/15 + 1/9 + 2/33
= (33 + 55 + 30)/495
= 118/495
Hence, the required probability is 118/495.