Exercise 31.7

1. Solution:

Given,
Urn I contains 1 white, 2 black and 3 red balls
Urn II contains 2 white, 1 black and 1 red balls
Urn III contains 4 white, 5 black and 4 red balls
Let E_1, E_2, E_3 and A be the events as defined:
E_1 = Selecting urn I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = Drawing 1 white ball and 1 red ball
Now.
$P(E_1) = P(E_2) = P(E_3) = 1/3$ [As there are only 3 urns]
And,
$P(A/E_1)$ = P(Drawing 1 red ball and 1 white ball from urn I)
$= ({}^1C_1 \times {}^3C_1)/ {}^6C_2$
$= (1 \times 3)/ (6 \times 5/2)$
$= 1/5$
$P(A/E_2)$ = P(Drawing 1 red ball and 1 white ball from urn II)
$= ({}^2C_1 \times {}^1C_1)/ {}^4C_2$
$= (2 \times 1)/ (4 \times 3/2)$
$= 1/3$
$P(A/E_3)$ = P(Drawing 1 red ball and 1 white ball from urn III)
$= ({}^4C_1 \times {}^3C_1)/ {}^{12}C_2$
$= (4 \times 3)/ (12 \times 11)/2$
$= 2/11$
We have to find,
P(both balls came from urn I) = $P(E_1/A)$
P(both balls came from urn II) = $P(E_2/A)$
P(both balls came from urn III) = $P(E_3/A)$
So, by Baye's theorem we get

$$p\left(\frac{E_1}{A}\right)=\frac{p(E_1)p\left(\frac{A}{E_1}\right)}{p(E_1)p\left(\frac{A}{E_1}\right)+p(E_2)p\left(\frac{A}{E_2}\right)+p(E_3)p\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$=\frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$=\frac{\frac{1}{5}}{\frac{36+55+30}{165}}$$

$$=\frac{1}{5}\times\frac{165}{118}$$

$$=\frac{33}{118}$$

$$p\left(\frac{E_2}{A}\right)=\frac{p(E_2)p\left(\frac{A}{E_2}\right)}{p(E_1)p\left(\frac{A}{E_1}\right)+p(E_2)p\left(\frac{A}{E_2}\right)+p(E_3)p\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{3}\times\frac{1}{3}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$=\frac{\frac{1}{3}}{\frac{33+55+30}{165}}$$

$$=\frac{1}{3}\times\frac{165}{118}$$

$$=\frac{55}{118}$$

And,

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{11}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{2}{11}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{2}{11}\times\frac{165}{118}$$

$$= \frac{30}{118}$$

Therefore, the required probabilities are 33/118, 55/118 and 30/118.

2. Solution:

Given,
Bag A contains 2 white and 3 red balls
Bag B contains 4 white and 5 red balls
Let E_1, E_2 and A be events as below:
E_1 – Choosing bag A
E_2 – Choosing bag B
A – Drawing one red ball
So, we have
$P(E_1) = P(E_2) = 1/2$ [Since, there are only 2 bags]
Now,
$P(A/E_1)$ = P(drawing a red ball from Bag A)
= 3/5
$P(A/E_2)$ = P(drawing a red ball from Bag B)
= 5/9
So, by Baye's theorem we get

$$p\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{3}{5}+\frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{27+25}{45}}$$

$$= \frac{5}{9}\times\frac{45}{52} = \frac{25}{52}$$

Hence, the required probability is 25/52.

3. Solution:

Given,
Urn I contains 2 white and 3 black balls
Urn II contains 3 white and 2 black balls
Urn III contains 4 white and 1 black balls
Let E_1, E_2, E_3 and A be the events as defined:
E_1 = Selecting urn I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = Drawing 1 white ball
Now.
$P(E_1) = P(E_2) = P(E_3) = 1/3$ [As there are only 3 urns]
And,
$P(A/E_1)$ = P(Drawing one white ball from urn I)
= 2/5
$P(A/E_2)$ = P(Drawing one white ball from urn II)
= 3/5
$P(A/E_3)$ = P(Drawing one white ball from urn III)
= 4/5
According to the question, we need to find
P(Drawn one white ball is from urn 1) = $P(E_1/A)$
So, By Baye's theorem we get

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{4}{5}}$$

$$= \frac{\frac{2}{10}}{\frac{2+3+4}{10}}$$

$$= \frac{2}{9}$$

Therefore, the required probability is 2/9.

4. Solution:

Given,
Urn I contains 7 white and 3 black balls
Urn II contains 4 white and 6 black balls
Urn III contains 2 white and 8 black balls
Let E_1, E_2, E_3 and A be the events as defined:
E_1 = Selecting urn I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = Drawing 2 white balls without replacement
Also given,
$P(E_1) = 0.20$
$P(E_2) = 0.60$
$P(E_3) = 0.20$
Now,
$P(A/E_1)$ = P(Drawing two white balls from urn I)
$= {}^7C_2/{}^{10}C_2$
$= (7x6)/2 \div (10x9)/2$
$= 7/15$
$P(A/E_2)$ = P(Drawing two white balls from urn II)
$= {}^4C_2/{}^{10}C_2$
$= (4x3)/2 \div (10x9)/2$
$= 12/90$
$= 2/15$
$P(A/E_3)$ = P(Drawing two white balls from urn III)
$= {}^2C_2/{}^{10}C_2$
$= 1 \div (10x9)/2$
$= 1/45$
According to the question, we need to find

P(Drawn two white balls are from urn III) = $P(E_3/A)$
So, By Baye's theorem we get

$$P\left(\frac{E_3}{A}\right)=\frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$=\frac{0.2\times\frac{1}{45}}{0.2\times\frac{7}{15}+0.6\times\frac{2}{15}+0.2\times\frac{1}{45}}$$

$$=\frac{\frac{2}{450}}{\frac{14}{150}+\frac{12}{150}+\frac{2}{450}}$$

$$=\frac{\frac{2}{450}}{\frac{42+36+2}{450}}$$

$$=\frac{2}{80}$$

$$=\frac{1}{40}$$

Thus, the required probability is 1/40.

5. Solution:

Let's consider the following events,
E_1 – Getting 1 or 2 in a throw of die
E_2 – Getting 3, 4, 5 or 6 in a throw of die
A – Getting exactly one tail
So clearly, we have
$P(E_1) = 2/6 = 1/3$
$P(E_2) = 4/6 = 2/3$
$P(A/E_1) = 3/8$
$P(A/E_2) = ½$
Now, the required probability is given by

$$= P(E_2/A)$$

$$=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$=\frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}$$

$$=\frac{8}{11}$$

6. Solution:

Let's consider the following events,
E_1 – First group wins
E_2 – Second group wins
A – New product is introduced
It's given that,
$P(E_1) = 0.6$
$P(E_2) = 0.4$
$P(A/E_1) = 0.7$
$P(A/E_2) = 0.3$
So, the required probability $P(E_2/A)$ is given by

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{0.4\times0.3}{0.6\times0.7+0.4\times0.3} = \frac{12}{54} = \frac{2}{9}$$

Hence,
$P(E_2/A) = 2/9$