

Class: XII Session: 2020-21

Subject: Mathematics

Marking Scheme (Theory)

Sr.No.	Objective type Question Section I	Marks
1	<p>Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$ $\Rightarrow (x_1)^3 = (x_2)^3$ $\Rightarrow x_1 = x_2$, Hence $f(x)$ is one – one</p> <p align="center">OR</p> <p>2^6 reflexive relations</p>	1
2	(1,2)	1
3	<p>Since \sqrt{a} is not defined for $a \in (-\infty, 0)$ $\therefore \sqrt{a} = b$ is not a function.</p> <p align="center">OR</p> <p>$A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \emptyset$</p>	1
4	3x5	1
5	$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ <p align="center">OR</p> $ \text{adj } A = (-4)^{3-1} = 16$	1
6	0	1
7	$e^x(1 - \cot x) + C$ <p align="center">OR</p> <p>$\because f(x)$ is an odd function</p> $\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \, dx = 0$	1
8	$A = 2 \int_0^1 x^2 \, dx = \frac{2}{3} [x^3]_0^1$ $= \frac{2}{3} \text{sq unit}$	1

9	0 OR 3	1 1
10	\hat{j}	1
11	$\frac{1}{2} 2\hat{i} \times (-3\hat{j}) = \frac{1}{2} -6\hat{k} = 3 \text{ sq units}$	1
12	$\begin{aligned} \hat{a} + \hat{b} ^2 &= 1 \\ \Rightarrow \hat{a}^2 + \hat{b}^2 + 2 \hat{a} \cdot \hat{b} &= 1 \\ \Rightarrow 2 \hat{a} \cdot \hat{b} &= 1 - 1 - 1 \\ \Rightarrow \hat{a} \cdot \hat{b} &= \frac{-1}{2} \Rightarrow \hat{a} \hat{b} \cos \theta = \frac{-1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3} \\ \Rightarrow \theta &= \frac{2\pi}{3} \end{aligned}$	1
13	1,0,0	1
14	(0,0,0)	1
15	$1 - \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$	1
16	$\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$	1
	Section II	
17(i)	(b)	1
17(ii)	(a)	1
17(iii)	(c)	1
17(iv)	(a)	1
17(v)	(d)	1
18(i)	(b)	1
18(ii)	(c)	1
18(iii)	(b)	1
18(iv)	(d)	1
18(v)	(d)	1
	Section III	
19	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ $\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$	$\frac{1}{2}$

	$\tan^{-1} \left[\cot \left(\frac{\pi}{4} - \frac{x}{2} \right) \right] = \tan^{-1} \left[\tan \frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2} \right) \right]$	1
	$\tan^{-1} \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right] = \frac{\pi}{4} + \frac{x}{2}$	$\frac{1}{2}$
20	$A^2 = 2A$ $\Rightarrow AA = 2A $ $\Rightarrow A A = 8 A \quad (\because AB = A B \text{ and } 2A = 2^3 A)$ $\Rightarrow A (A - 8) = 0$ $\Rightarrow A = 0 \text{ or } 8$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR	
	$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ $5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ $\Rightarrow A^{-1}(A^2 - 5A + 7I) = A^{-1}0$ $\Rightarrow A - 5I + 7A^{-1} = 0$ $\Rightarrow 7A^{-1} = 5I - A$ $\Rightarrow A^{-1} = \frac{1}{7} \left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \right)$ $\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$	1
21	$\lim_{x \rightarrow 0} \frac{1 - \cos kx}{x \sin x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \left(\frac{kx}{2} \right)}{x \sin x}$ $= \lim_{x \rightarrow 0} \frac{\frac{2 \sin^2 \left(\frac{kx}{2} \right)}{x^2}}{\frac{x \sin x}{x^2}}$ $= \frac{\lim_{x \rightarrow 0} \frac{2 \sin^2 \left(\frac{kx}{2} \right)}{\left(\frac{kx}{2} \right)^2} \times \left(\frac{k}{2} \right)^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{2 \times 1 \times \frac{k^2}{4}}{1}$	$1 \frac{1}{2}$

	$\because f(x) \text{ is continuous at } x = 0$ $\therefore \lim_{x \rightarrow 0} f(x) = f(0)$ $\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$	$\frac{1}{2}$
22	$y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$ $\because \text{normal is perpendicular to } 3x - 4y = 7, \therefore \text{tangent is parallel to it}$ $1 - \frac{1}{x^2} = \frac{3}{4} \Rightarrow x^2 = 4 \Rightarrow x = 2 \ (\because x > 0)$ $\text{when } x = 2, y = 2 + \frac{1}{2} = \frac{5}{2}$ $\therefore \text{Equation of Normal : } y - \frac{5}{2} = -\frac{4}{3}(x - 2) \Rightarrow 8x + 6y = 31$	1 1
23	$I = \int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$ Put, $1 - \tan x = y$ So that, $-\sec^2 x dx = dy$ $= \int \frac{-1 dy}{y^2} = - \int y^{-2} dy$ $= + \frac{1}{y} + c = \frac{1}{1 - \tan x} + c$	1 1
	OR	
	$I = \int_0^1 x (1 - x)^n dx$ $I = \int_0^1 (1 - x)[1 - (1 - x)]^n dx$ $I = \int_0^1 (1 - x) x^n dx = \int_0^1 (x^n - x^{n+1}) dx$ $I = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1$ $I = \left[\left(\frac{1}{n+1} - \frac{1}{n+2} \right) - 0 \right] = \frac{1}{(n+1)(n+2)}$	$\frac{1}{2}$ $\frac{1}{2}$
24	$\text{Area} = 2 \int_0^2 \sqrt{8x} dx$ $= 2 \times 2\sqrt{2} \int_0^2 x^{\frac{1}{2}} dx$	1

	$= 4\sqrt{2} \left[\frac{2}{3}x^{\frac{3}{2}} \right]_0^2$ $= \frac{8}{3}\sqrt{2} \left[2^{\frac{3}{2}} - 0 \right] = \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$ $= \frac{32}{3} \text{ sq units}$	$\frac{1}{2}$ $\frac{1}{2}$		
25	$\frac{dy}{dx} = x^3 \operatorname{cosec} y ; \quad y(0) = 0$ $\int \frac{dy}{\operatorname{cosec} y} = \int x^3 dx$ $\int \sin y dy = \int x^3 dx$ $-\cos y = \frac{x^4}{4} + c$ $-1 = c \quad (\because y = 0, \text{when } x = 0)$ $\cos y = 1 - \frac{x^4}{4}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$		
26	<p>Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$</p> $\vec{d} = 4\hat{i} + 5\hat{k}$ $\therefore \vec{a} + \vec{b} = \vec{d} \Rightarrow \vec{b} = \vec{d} - \vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = -5\hat{i} - 1\hat{j} + 4\hat{k}$ <p>Area of parallelogram = $\vec{a} \times \vec{b} = \sqrt{25 + 1 + 16} = \sqrt{42}$ sq units</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$		
27	<p>Let the normal vector to the plane be \vec{n}</p> <p>Equation of the plane passing through $(1,0,0)$, i.e., \hat{i} is $(\vec{r} - \hat{i}) \cdot \vec{n} = 0 \dots \dots \dots (1)$</p> <p>$\because$ plane (1) contains the line $\vec{r} = \vec{o} + \lambda\hat{j}$</p> <p>$\therefore \hat{i} \cdot \vec{n} = 0$ and $\hat{j} \cdot \vec{n} = 0 \Rightarrow \vec{n} = \hat{k}$</p> <p>Hence equation of the plane is $(\vec{r} - \hat{i}) \cdot \hat{k} = 0$ i.e., $\vec{r} \cdot \hat{k} = 0$</p>	1 1		
28	<p>Let x denote the number of milk chocolates drawn</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px; text-align: center;">X</td> <td style="padding: 5px; text-align: center;">$P(x)$</td> </tr> </table>	X	$P(x)$	
X	$P(x)$			

		<table border="1"> <tr> <td>0</td><td>$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$</td></tr> <tr> <td>1</td><td>$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$</td></tr> <tr> <td>2</td><td>$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$</td></tr> </table>	0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$	1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$	2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$	
0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$								
1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$								
2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$								

Most likely outcome is getting one chocolate of each type

$1\frac{1}{2}$

$\frac{1}{2}$

OR

$$P(\bar{E} | \bar{F}) = P\left(\frac{\bar{E} \cap \bar{F}}{P(\bar{F})}\right) = \frac{(E \cup F)}{P(\bar{F})} = \frac{1 - P(E \cap F)}{1 - P(F)} \quad \dots \quad (1)$$

1

$$\text{Now } P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= 0.8 + 0.7 - 0.6 = 0.9$$

$\frac{1}{2}$

Substituting value of $P(E \cup F)$ in (1)

$$P(\bar{E} | \bar{F}) = \frac{1 - 0.9}{1 - 0.7} = \frac{0.1}{0.3} = \frac{1}{3}$$

$\frac{1}{2}$

Section IV

29

(i) Reflexive :

Since, $a+a=2a$ which is even $\therefore (a,a) \in R \forall a \in Z$

Hence R is reflexive

$\frac{1}{2}$

(ii) Symmetric:

If $(a,b) \in R$, then $a+b = 2\lambda \Rightarrow b+a = 2\lambda$

$\Rightarrow (b,a) \in R$, Hence R is symmetric

1

(iii) Transitive:

If $(a,b) \in R$ and $(b,c) \in R$

then $a+b = 2\lambda \dots (1)$ and $b+c = 2\mu \dots (2)$

Adding (1) and (2) we get

$$a+2b+c=2(\lambda + \mu)$$

$$\Rightarrow a+c=2(\lambda + \mu - b)$$

$$\Rightarrow a+c=2k, \text{ where } \lambda + \mu - b = k \Rightarrow (a,c) \in R$$

Hence R is transitive

$$[0] = \{-4, -2, 0, 2, 4, \dots\}$$

$\frac{1}{2}$

30

Let $u = e^{x \sin^2 x}$ and $v = (\sin x)^x$

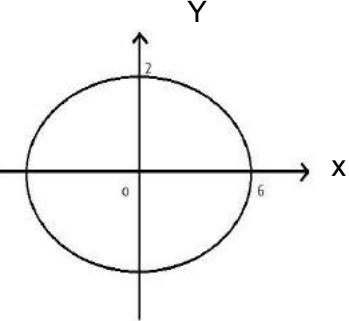
$\frac{1}{2}$

	<p>so that $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ --- (1)</p> <p>Now, $u = e^{x \sin^2 x}$, Differentiating both sides w.r.t. x, we get</p> $\Rightarrow \frac{du}{dx} = e^{x \sin^2 x} [x(\sin 2x) + \sin^2 x] \quad \text{----- (2)}$ <p>Also, $v = (\sin x)^x$</p> $\Rightarrow \log v = x \log(\sin x)$ <p>Differentiating both sides w.r.t. x, we get</p> $\frac{1}{v} \frac{dv}{dx} = x \cot x + \log(\sin x)$ $\frac{dv}{dx} = (\sin x)^x [x \cot x + \log(\sin x)] \quad \text{----- (3)}$ <p>Substituting from - (2), - (3) in - (1) we get</p> $\frac{dy}{dx} = e^{x \sin^2 x} [x \sin 2x + \sin^2 x] + (\sin x)^x [x \cot x + \log(\sin x)]$	1
31	<p>RHD = $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h}$</p> $= \lim_{h \rightarrow 0} \frac{(1-1)}{h} = 0$ <p>LHD = $\lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} = \lim_{h \rightarrow 0} \frac{0-1}{-h}$</p> $= \lim_{h \rightarrow 0} \frac{1}{h} = \infty$ <p>Since, RHD \neq LHD Therefore $f(x)$ is not differentiable at $x = 1$</p>	1 1 1 1
	OR	
	$y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \dots (1)$ $x = a \sec \theta \Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \dots (2)$	

	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \operatorname{cosec} \theta$ <p>Differentiating both sides w.r.t. x, we get</p> $\begin{aligned}\frac{d^2y}{dx^2} &= \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{d\theta}{dx} \\ &= \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{1}{a \sec \theta \tan \theta} \quad [\text{using (2)}] \\ &= \frac{-b}{a \cdot a} \cot^3 \theta\end{aligned}$ $\left. \frac{d^2y}{dx^2} \right _{\theta=\frac{\pi}{6}} = \frac{-b}{a} \left[\cot \frac{\pi}{6} \right]^3 = \frac{-b}{a} (\sqrt{3})^3 = -\frac{3\sqrt{3}b}{a \cdot a}$	$1\frac{1}{2}$
32	$f(x) = \tan x - 4x$ $f'(x) = \sec^2 x - 4$ <p>a) For $f(x)$ to be strictly increasing</p> $\begin{aligned}f'(x) &> 0 \\ \Rightarrow \sec^2 x - 4 &> 0 \\ \Rightarrow \sec^2 x &> 4 \\ \Rightarrow \cos^2 x < \frac{1}{4} &\Rightarrow \cos^2 x < \left(\frac{1}{2}\right)^2 \\ \Rightarrow -\frac{1}{2} < \cos x < \frac{1}{2} &\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}\end{aligned}$ <p>b) For $f(x)$ to be strictly decreasing</p> $\begin{aligned}f'(x) &< 0 \\ \Rightarrow \sec^2 x - 4 &< 0 \\ \Rightarrow \sec^2 x &< 4 \\ \Rightarrow \cos^2 x > \frac{1}{4} &\\ \Rightarrow \cos^2 x &> \left(\frac{1}{2}\right)^2 \\ \Rightarrow \cos x > \frac{1}{2} &\left[\because x \in \left(0, \frac{\pi}{2}\right) \right] \\ \Rightarrow 0 < x < \frac{\pi}{3} &\end{aligned}$	$\frac{1}{2}$

33	<p>Put $x^2 = y$ to make partial fractions</p> $\frac{x^2 + 1}{(x^2 + 2)(x^2 + 3)} = \frac{y + 1}{(y + 2)(y + 3)} = \frac{A}{y + 2} + \frac{B}{y + 3}$ $\Rightarrow y + 1 = A(y + 3) + B(y + 2) \dots \dots \dots (1)$ <p>Comparing coefficients of y and constant terms on both sides of (1) we get</p> $A+B = 1 \text{ and } 3A + 2B = 1$ <p>Solving, we get $A = -1$, $B = 2$</p> $\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx = \int \frac{-1}{x^2+2} dx + 2 \int \frac{1}{x^2+3} dx$ $= -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
34	<p>Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$</p> <p>We get $x^2 + 3x^2 = 4$</p> $\Rightarrow x^2 = 1 \Rightarrow x = 1$ <p>Required Area</p> $= \sqrt{3} \int_0^1 x dx + \int_1^2 \sqrt{2^2 - x^2} dx$ $= \frac{\sqrt{3}}{2} [x^2]_0^1 + \left[\frac{x}{2} \sqrt{2^2 - x^2} + 2 \sin^{-1} \left(\frac{x}{2} \right) \right]_1^2$ $= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6} \right]$ $\frac{2\pi}{3} \text{ sq units}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

OR

	<p>Required Area = $\frac{4}{3} \int_0^6 \sqrt{6^2 - x^2} dx$</p>  $= \frac{4}{3} \left[\frac{x}{2} \sqrt{6^2 - x^2} + 18 \sin^{-1} \left(\frac{x}{6} \right) \right]_0^6$ $= \frac{4}{3} \left[18 \times \frac{\pi}{2} - 0 \right] = 12\pi \text{ sq units}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
35	<p>The given differential equation can be written as</p> $\frac{dy}{dx} = \frac{y + 2x^2}{x} \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 2x$ <p>Here $P = -\frac{1}{x}$, $Q = 2x$</p> $IF = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$ <p>The solutions is :</p> $y \times \frac{1}{x} = \int \left(2x \times \frac{1}{x} \right) dx$ $\Rightarrow \frac{y}{x} = 2x + c$ $\Rightarrow y = 2x^2 + cx$	$\frac{1}{2}$ 1
36	$ A = 1(-1 - 2) - 2(-2 - 0) = -3 + 4 = 1$ <p>A is nonsingular, therefore A^{-1} exists</p> $Adj A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $\Rightarrow A^{-1} = \frac{1}{ A } (Adj A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	$\frac{1}{2}$ $1\frac{1}{2}$

The given equations can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$\frac{1}{2}$

Which is of the form $A'X = B$

$$\Rightarrow X = (A')^{-1}B = (A^{-1})'B$$

1

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow x = 0, \quad y = -5, \quad z = -3$$

$1\frac{1}{2}$

OR

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$1\frac{1}{2}$

$$\Rightarrow AB = 6I$$

$$\Rightarrow A\left(\frac{1}{6}B\right) = I \Rightarrow A^{-1} = \frac{1}{6}(B)$$

1

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = D, \text{ where } D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = A^{-1}D$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

1

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$x = 2, \quad y = -1, \quad z = 4$$

$1\frac{1}{2}$

	$a_2 = 5\hat{i} - 2\hat{j}$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 4\hat{j} + 4\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{i}(12 - 4) - \hat{j}(6 - 6) + \hat{k}(2 - 6)$	1 1
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	$\vec{b}_1 \times \vec{b}_2 = 8\hat{i} + 0\hat{j} - 4\hat{k} = 8\hat{i} - 4\hat{k}$ $\therefore (\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 16 - 16 = 0$	1
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\therefore The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection

	$3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ $\Rightarrow 3 + \lambda = 5 + 3\mu \quad \dots \dots \quad (1)$ $2 + 2\lambda = -2 + 2\mu \quad \dots \dots \quad (2)$ $-4 + 2\lambda = 6\mu \quad \dots \dots \quad (3)$	1
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Solving (1) ad (2) we get, $\mu = -2$ and $\lambda = -4$

Substituting in equation of line we get

$$\vec{r} = 5\hat{i} - 2\hat{j} + (-2)(3\hat{i} + 2\hat{j} - 6\hat{k}) = -\hat{i} - 6\hat{j} - 12\hat{k}$$

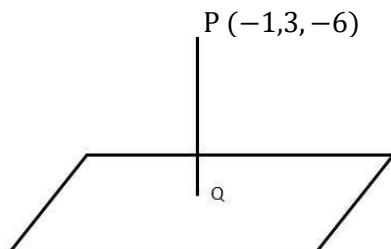
Point of intersection is $(-1, -6, -12)$

1

OR

Let P be the given point and Q be the foot of the perpendicular.

	Equation of PQ $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2} = \lambda$	$1\frac{1}{2}$
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Let coordinates of Q be $(2\lambda - 1, \lambda + 3, -2\lambda - 6)$

Since Q lies in the plane $2x + y - 2z + 5 = 0$

	$\therefore 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$ $\Rightarrow 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$	$\frac{1}{2}$
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$$\Rightarrow 9\lambda + 18 = 0 \quad \Rightarrow \lambda = -2$$

\therefore coordinates of Q are $(-5, 1, -2)$

$$\begin{aligned} \text{Length of the perpendicular} &= \sqrt{(-5+1)^2 + (1-3)^2 + (-2+6)^2} \\ &= 6 \text{ units} \end{aligned}$$

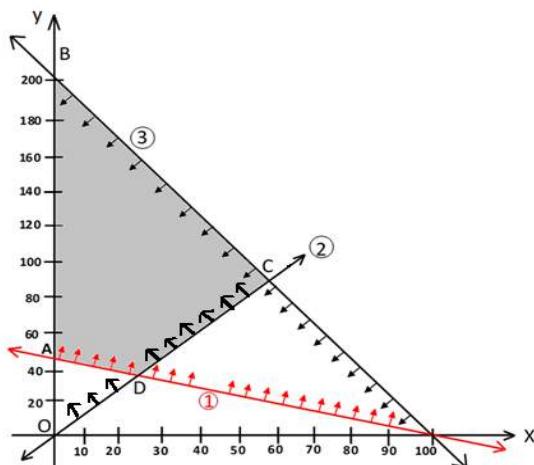
1

1

1

38 $\text{Max } Z = 3x + y$

Subject to $x + 2y \geq 100$ ----- (1)
 $2x - y \leq 0$ ----- (2)
 $2x + y \leq 200$ ----- (3)
 $x \geq 0, y \geq 0$



3

Corner Points	$Z = 3x + y$
A (0, 50)	50
B (0, 200)	200
C (50, 100)	250
D (20, 40)	100

1

$\text{Max } z = 250 \text{ at } x = 50, y = 100$

1

OR

(i)

Corner points	$Z = 3x - 4y$
O(0,0)	0
A(0,8)	-32
B(4,10)	-28
C(6,8)	-14
D(6,5)	-2
E(4,0)	12

$\text{Max } Z = 12 \text{ at } E(4,0)$

$\text{Min } Z = -32 \text{ at } A(0,8)$

$1\frac{1}{2}$

- (ii) Since maximum value of Z occurs at B(4,10) and C(6, 8)
 $\therefore 4p + 10q = 6p + 8q$
 $\Rightarrow 2q = 2p$
 $\Rightarrow p = q$
Number of optimal solution are infinite

1

2

$\frac{1}{2}$