



|                    |   |               |
|--------------------|---|---------------|
| 9                  | 0<br><b>OR</b><br>3   | 1<br><br>1    |
| 10                 | $f$   | 1             |
| 11                 | $\frac{1}{2} 2\hat{i} \times (-3\hat{j})  = \frac{1}{2} -6\hat{k}  = 3 \text{ sq units}$  | 1             |
| 12                 | $ \hat{a} + \hat{b} ^2 = 1$<br>$\Rightarrow \hat{a}^2 + \hat{b}^2 + 2\hat{a} \cdot \hat{b} = 1$<br>$\Rightarrow 2\hat{a} \cdot \hat{b} = 1 - 1 - 1$<br>$\Rightarrow \hat{a} \cdot \hat{b} = \frac{-1}{2} \Rightarrow  \hat{a}  \hat{b}  \cos \theta = \frac{-1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3}$<br>$\Rightarrow \theta = \frac{2\pi}{3}$ | 1             |
| 13                 | 1,0,0   | 1             |
| 14                 | (0,0,0)   | 1             |
| 15                 | $1 - \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$  | 1             |
| 16                 | $\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$  | 1             |
| <b>Section II</b>  |   |               |
| 17(i)              | (b)   | 1             |
| 17(ii)             | (a)   | 1             |
| 17(iii)            | (c)   | 1             |
| 17(iv)             | (a)   | 1             |
| 17(v)              | (d)   | 1             |
| 18(i)              | (b)   | 1             |
| 18(ii)             | (c)   | 1             |
| 18(iii)            | (b)   | 1             |
| 18(iv)             | (d)   | 1             |
| 18(v)              | (d)   | 1             |
| <b>Section III</b> |   |               |
| 19                 | $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$<br>$\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$                            | $\frac{1}{2}$ |



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|    | $\therefore f(x) \text{ is continuous at } x = 0$ $\therefore \lim_{x \rightarrow 0} f(x) = f(0)$ $\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$  | $\frac{1}{2}$   |
| 22 | $y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$ <p><math>\therefore</math> normal is perpendicular to <math>3x - 4y = 7</math>, <math>\therefore</math> tangent is parallel to it</p> $1 - \frac{1}{x^2} = \frac{3}{4} \Rightarrow x^2 = 4 \Rightarrow x = 2 \quad (\because x > 0)$ <p>when <math>x = 2</math>, <math>y = 2 + \frac{1}{2} = \frac{5}{2}</math></p> <p><math>\therefore</math> Equation of Normal : <math>y - \frac{5}{2} = -\frac{4}{3}(x - 2) \Rightarrow 8x + 6y = 31</math></p>  | 1<br><br>1  |
| 23 | $I = \int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$ <p>Put, <math>1 - \tan x = y</math></p> <p>So that, <math>-\sec^2 x dx = dy</math></p> $= \int \frac{-1 dy}{y^2} = - \int y^{-2} dy$ $= + \frac{1}{y} + c = \frac{1}{1 - \tan x} + c$ <p style="text-align: center;"><b>OR</b></p> $I = \int_0^1 x (1 - x)^n dx$ $I = \int_0^1 (1 - x) [1 - (1 - x)]^n dx$ $I = \int_0^1 (1 - x) x^n dx = \int_0^1 (x^n - x^{n+1}) dx$ $I = \left[ \frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1$ $I = \left[ \left( \frac{1}{n+1} - \frac{1}{n+2} \right) - 0 \right] = \frac{1}{(n+1)(n+2)}$ | 1<br><br>1<br><br>$\frac{1}{2}$<br><br>1<br><br>$\frac{1}{2}$ |
| 24 | $\text{Area} = 2 \int_0^2 \sqrt{8x} dx$ $= 2 \times 2\sqrt{2} \int_0^2 x^{\frac{1}{2}} dx$   | 1   |



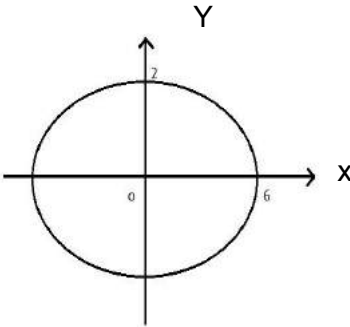


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|    | <p>so that <math>y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}</math>----(1)</p> <p>Now, <math>u = e^{x \sin^2 x}</math> , Differentiating both sides w.r.t. x, we get</p> $\Rightarrow \frac{du}{dx} = e^{x \sin^2 x} [x(\sin 2x) + \sin^2 x] \quad \text{---- ( 2)}$ <p>Also , <math>v = (\sin x)^x</math></p> $\Rightarrow \log v = x \log (\sin x)$ <p>Differentiating both sides w.r.t. x, we get</p> $\frac{1}{v} \frac{dv}{dx} = x \cot x + \log (\sin x)$ $\frac{dv}{dx} = (\sin x)^x [x \cot x + \log(\sin x)] \quad \text{----- (3)}$ <p>Substituting from – (2), – (3) in – (1) we get</p> $\frac{dy}{dx} = e^{x \sin^2 x} [x \sin 2x + \sin^2 x] + (\sin x)^x [x \cot x + \log(\sin x)]$ | <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> |
| 31 | $\text{RHD} = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h}$ $= \lim_{h \rightarrow 0} \frac{(1-1)}{h} = 0$ $\text{LHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} = \lim_{h \rightarrow 0} \frac{0-1}{-h}$ $= \lim_{h \rightarrow 0} \frac{1}{h} = \infty$ <p>Since, RHD <math>\neq</math> LHD<br/>Therefore <math>f(x)</math> is not differentiable at <math>x = 1</math></p> <p style="text-align: center;"><b>OR</b></p> $y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \dots (1)$ $x = a \sec \theta \Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \dots (2)$                | <p>1</p> <p>1</p> <p>1</p>                        |

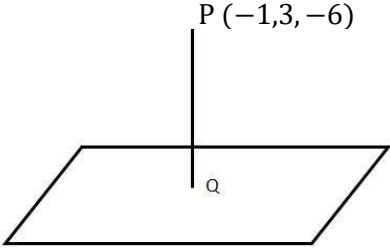






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|    | <p>Required Area = <math>\frac{4}{3} \int_0^6 \sqrt{6^2 - x^2} dx</math></p>  <p> <math display="block">= \frac{4}{3} \left[ \frac{x}{2} \sqrt{6^2 - x^2} + 18 \sin^{-1} \left( \frac{x}{6} \right) \right]_0^6</math> <math display="block">= \frac{4}{3} \left[ 18 \times \frac{\pi}{2} - 0 \right] = 12\pi \text{ sq units}</math> </p>                                  | <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p> |
| 35 | <p>The given differential equation can be written as</p> $\frac{dy}{dx} = \frac{y + 2x^2}{x} \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 2x$ <p>Here <math>P = -\frac{1}{x}</math>, <math>Q = 2x</math></p> $\text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$ <p>The solutions is :</p> $y \times \frac{1}{x} = \int \left( 2x \times \frac{1}{x} \right) dx$ $\Rightarrow \frac{y}{x} = 2x + c$ $\Rightarrow y = 2x^2 + cx$ | <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> |
| 36 | <p><math> A  = 1(-1 - 2) - 2(-2 - 0) = -3 + 4 = 1</math></p> <p>A is nonsingular, therefore <math>A^{-1}</math> exists</p> $\text{Adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $\Rightarrow A^{-1} = \frac{1}{ A } (\text{Adj } A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$  | <p><math>\frac{1}{2}</math></p> <p><math>1\frac{1}{2}</math></p>                  |

|    |  |   |
|----|--|---|
|    | <p>The given equations can be written as:</p> $\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$ <p>Which is of the form <math>A'X = B</math></p> $\Rightarrow X = (A')^{-1}B = (A^{-1})'B$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\Rightarrow x = 0, \quad y = -5, \quad z = -3$ <p style="text-align: center;"><b>OR</b></p> $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $\Rightarrow AB = 6I$ $\Rightarrow A\left(\frac{1}{6}B\right) = I \Rightarrow A^{-1} = \frac{1}{6}(B)$ <p>The given equations can be written as</p> $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $AX = D, \text{ where } D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $\Rightarrow X = A^{-1}D$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $x = 2, \quad y = -1, \quad z = 4$ | <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>1\frac{1}{2}</math></p> <p style="text-align: center;"><math>1\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>1\frac{1}{2}</math></p> |
| 37 | We have $a_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$ $b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$   |   |

|  |   |   |
|--|---|---|
|  | $a_2 = 5i - 2j$ $b_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 4\hat{j} + 4\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{i}(12 - 4) - \hat{j}(6 - 6) + \hat{k}(2 - 6)$ $\vec{b}_1 \times \vec{b}_2 = 8\hat{i} + 0\hat{j} - 4\hat{k} = 8\hat{i} - 4\hat{k}$ $\therefore (\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 16 - 16 = 0$ <p><math>\therefore</math> The lines are intersecting and the shortest distance between the lines is 0.</p> <p>Now for point of intersection</p> $3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ $\Rightarrow 3 + \lambda = 5 + 3\mu \quad \text{--- -- (1)}$ $2 + 2\lambda = -2 + 2\mu \quad \text{--- -- (2)}$ $-4 + 2\lambda = 6\mu \quad \text{--- -- (3)}$ <p>Solving (1) and (2) we get, <math>\mu = -2</math> and <math>\lambda = -4</math></p> <p>Substituting in equation of line we get</p> $\vec{r} = 5\hat{i} - 2\hat{j} + (-2)(3\hat{i} + 2\hat{j} - 6\hat{k}) = -\hat{i} - 6\hat{j} - 12\hat{k}$ <p>Point of intersection is <math>(-1, -6, -12)</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Let P be the given point and Q be the foot of the perpendicular.</p> <p>Equation of PQ <math>\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2} = \lambda</math></p> <div style="text-align: center;">  </div> <p>Let coordinates of Q be <math>(2\lambda - 1, \lambda + 3, -2\lambda - 6)</math></p> <p>Since Q lies in the plane <math>2x + y - 2z + 5 = 0</math></p> $\therefore 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$ $\Rightarrow 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$ | <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p><math>1\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> |
|--|---|---|

$$\Rightarrow 9\lambda + 18 = 0 \quad \Rightarrow \lambda = -2$$

$\therefore$  coordinates of Q are (-5, 1, -2)

$$\begin{aligned} \text{Length of the perpendicular} &= \sqrt{(-5 + 1)^2 + (1 - 3)^2 + (-2 + 6)^2} \\ &= 6 \text{ units} \end{aligned}$$

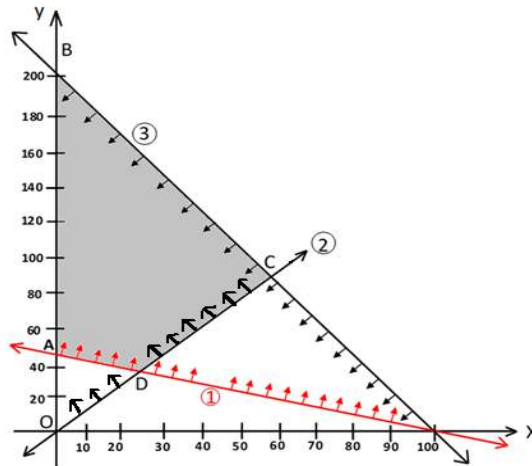
1  
1  
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38

$$\text{Max } Z = 3x + y$$

Subject to

$$\begin{aligned} x + 2y &\geq 100 && \text{----} && (1) \\ 2x - y &\leq 0 && \text{----} && (2) \\ 2x + y &\leq 200 && \text{----} && (3) \\ x &\geq 0, && y &\geq 0 \end{aligned}$$



3

| Corner Points | $Z = 3x + y$ |
|---------------|--------------|
| A (0, 50)     | 50           |
| B (0, 200)    | 200          |
| C (50, 100)   | 250          |
| D (20, 40)    | 100          |

1

$$\text{Max } z = 250 \text{ at } x = 50, \quad y = 100$$

1

OR

(i)

| Corner points | $Z = 3x - 4y$ |
|---------------|---------------|
| O(0,0)        | 0             |
| A(0,8)        | -32           |
| B(4,10)       | -28           |
| C(6,8)        | -14           |
| D(6,5)        | -2            |
| E(4,0)        | 12            |

*Max*  $Z = 12$  at  $E(4,0)$

*Min*  $Z = -32$  at  $A(0,8)$

(ii) Since maximum value of  $Z$  occurs at  $B(4,10)$  and  $C(6, 8)$

$$\therefore 4p + 10q = 6p + 8q$$

$$\Rightarrow 2q = 2p$$

$$\Rightarrow p = q$$

Number of optimal solution are infinite

$1\frac{1}{2}$

1

2

$\frac{1}{2}$