

Calculus Formulas

<p>l'Hopital's Rule If $\frac{f(a)}{g(a)} = \frac{0}{0}$ or $= \frac{\infty}{\infty}$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$</p>	<p>Properties of Log and Ln 1. $\ln 1 = 0$ 2. $\ln e^a = a$ 3. $e^{\ln x} = x$ 4. $\ln x^n = n \ln x$ 5. $\ln(ab) = \ln a + \ln b$ 6. $\ln(\frac{a}{b}) = \ln a - \ln b$</p>	<p>The Fundamental Theorem of Calculus $\int_a^b f(x) dx = F(b) - F(a)$ where $F'(x) = f(x)$</p>	<p>2nd Fundamental Theorem of Calculus $\frac{d}{dx} \int_a^{g(x)} f(x) dx = f(g(x)) \cdot g'(x)$</p>
<p>Average Rate of Change (slope of the secant line) If the points $(a, f(a))$ and $(b, f(b))$ are on the graph of $f(x)$ the average rate of change of $f(x)$ on the interval $[a,b]$ is</p> $\frac{f(b)-f(a)}{b-a}$	<ul style="list-style-type: none"> • $\frac{d}{dx} x^n = nx^{n-1}$ • $\frac{d}{dx}(fg) = fg' + gf'$ • $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$ • $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ • $\frac{d}{dx}(\sin x) = \cos x$ • $\frac{d}{dx}(\cos x) = -\sin x$ • $\frac{d}{dx}(\tan x) = \sec^2 x$ • $\frac{d}{dx}(\cot x) = -\csc^2 x$ • $\frac{d}{dx}(\sec x) = \sec x \tan x$ • $\frac{d}{dx}(\csc x) = -\csc x \cot x$ • $\frac{d}{dx}(e^x) = e^x$ • $\frac{d}{dx}(a^x) = a^x \ln a$ • $\frac{d}{dx} \ln x = \frac{1}{x}$ • $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ • $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$ 	<p>Average Value If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$ if $f(a) = f(b)$, then $f'(c) = 0$.</p>	<p>If given that $\frac{dy}{dx} = f(x, y)$ and that the solution passes through (x_0, y_0), then</p> $x_{\text{new}} = x_{\text{old}} + \Delta x$ $y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx}_{(x_{\text{old}}, y_{\text{old}})} \cdot \Delta x$
<p>Mean Value & Rolle's Theorem If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$ if $f(a) = f(b)$, then $f'(c) = 0$.</p>	<ul style="list-style-type: none"> • $\int 1 dx = x + C$ • $\int a dx = ax + C$ • $\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$ • $\int \sin x dx = -\cos x + C$ • $\int \cos x dx = \sin x + C$ • $\int \sec^2 x dx = \tan x + C$ • $\int \csc^2 x dx = -\cot x + C$ • $\int \sec x (\tan x) dx = \sec x + C$ • $\int \csc x (\cot x) dx = -\csc x + C$ • $\int \frac{1}{x} dx = \ln x + C$ • $\int e^x dx = e^x + C$ • $\int a^x dx = \frac{a^x}{\ln a} + C; a > 0, a \neq 1$ • $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$ • $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$ • $\int \frac{1}{ x \sqrt{x^2-1}} dx = \sec^{-1} x + C$ • $\int \sin^n(x) dx = \frac{-1}{n} \sin^{n-1}(x) \cos(x) + \frac{n-1}{n} \int \sin^{n-2}(x) dx$ • $\int \cos^n(x) dx = \frac{1}{n} \cos^{n-1}(x) \sin(x) + \frac{n-1}{n} \int \cos^{n-2}(x) dx$ • $\int \tan^n(x) dx = \frac{1}{n-1} \tan^{n-1}(x) + \int \tan^{n-2}(x) dx$ 	<p>Differentiation Rules <u>Chain Rule</u> $\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$ </p> <p><u>Product Rule</u> $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ </p> <p><u>Quotient Rule</u> $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ </p> <p>Logistics Curves $P(t) = \frac{L}{1 + Ce^{-(Lk)t}},$ where L is carrying capacity Maximum growth rate occurs when $P = \frac{1}{2}L$ $\frac{dP}{dt} = kP(L-P) \text{ or}$ $\frac{dP}{dt} = (Lk)P(1 - \frac{P}{L})$ </p>	<p>Curve sketching and analysis $y = f(x)$ must be continuous at each: critical point: $\frac{dy}{dx} = 0$ or undefined. local minimum: $\frac{dy}{dx}$ goes $(-,0,+)$ or $(-,und,+)$ or $\frac{d^2y}{dx^2} > 0$ local maximum: $\frac{dy}{dx}$ goes $(+,0,-)$ or $(+,und,-)$ or $\frac{d^2y}{dx^2} < 0$ Absolute Max/Min.: Compare local extreme values to values at endpoints. pt of inflection: concavity changes. $\frac{d^2y}{dx^2}$ goes $(+,0,-), (-,0,+)$, $(+,und,-)$, or $(-,und,+)$</p>