KCET-2016 (Mathematics)

1. The Set A has 4 elements and the set B has 5 elements then the number of injective mappings that can be defined from A to B is
   a. 144
   b. 72
   c. 60
   d. 120

2. Let f : R $\rightarrow$R be defined by f(x) =2x+6 which is a bijective mapping then $f^{-1}(x)$ is given by A
   a. $\frac{x}{2}-3$
   b. 2x+6
   c. x–3
   d. 6x+2

3. Let $*$ be binary operation defined on R by $a * b=\frac{a+b}{4} \forall a, b \in R$ then the operation $*$ is
   a. Commutative and Associative
   b. Commutative but not Associative
   c. Associative but not Commutative
   d. Neither Associative nor Commutative

4. The value of $\sin^{-1}\left(\cos\frac{53\pi}{5}\right)$
   a. $\frac{3\pi}{5}$
   b. $\frac{-3\pi}{5}$
   c. $\frac{\pi}{10}$
   d. $\frac{-\pi}{10}$

5. If $3\tan^{-1} x + \cot^{-1} x = \pi$ then x equal to
   a. 0
   b. 1
   c. –1
   d. 1/2

6. The simplified form of $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ is equal to
   a. 0
   b. $\frac{\pi}{4}$
   c. $\frac{\pi}{2}$
   d. π

7. If x y z are all different and not equal to zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}=0$ then the value of $x^{-1}+y^{-1}+z^{-1}$ is equal to
   a. x y z
   b. $x^{-1}y^{-1}z^{-1}$
   c. – x – y – z
   d. –1

8. If A is any square matrix of order 3×3 then |3A| is equal to

a. 3|A|
b. $\frac{1}{3}|A|$
c. 27|A|
d. 9|A|

9. If $y = e^{\sin^{-1}(t^2-1)}$ & $x = e^{\sec^{-1}\left(\frac{1}{t^2-1}\right)}$ then $\frac{dy}{dx}$ is equal to

a. $\frac{x}{y}$
b. $\frac{-y}{x}$
c. $\frac{y}{x}$
d. $\frac{-x}{y}$

10. If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}, B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$ then A – B is equal to

a. I
b. 0
c. 2I
d. $\frac{1}{2}I$

11. If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is equal to

a. $\frac{\log x}{\log(x-y)}$
b. $\frac{e^x}{x^{x-y}}$
c. $\frac{\log x}{(1+\log x)^2}$.
d. $\frac{1}{y} - \frac{1}{x-y}$

12. If A is a matrix of order m × n and B is a matrix such that AB' and B'A are both defined, the order of the matrix B is

a. m × m
b. n × n
c. n × m
d. m × n

13. The value of $\int \frac{e^x(1+x)dx}{\cos^2(e^x.x)}$ is equal to

a. –cot (e x^x)+c
b. tan (e^x. x)+c
c. tan(e^x)+c
d. cot(e^x)+c

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14. If x, y, z are not equal and $\neq 0, \neq 1$ the value of $\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$ is equal to

a. $\log(x\,y\,z)$
b. $\log(6\,x\,y\,z)$
c. 0
d. $\log(x+y+z)$

15. The function f(x) = [x] where [x] the greatest integer function, is continuous at

a. 1.5
b. 4
c. 1
d. –2

16. The value of $\int \frac{e^x(x^2\tan^{-1}x + \tan^{-1}x + 1)}{x^2+1}\,dx$ is equal to

a. $e^x \tan^{-1} x + c$
b. $\tan^{-1}(e^x) + c$
c. $\tan^{-1}(x^e) + c$
d. $e^{\tan^{-1}x} + c$

17. If $2\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|$ then the angle of between $\vec{a}\,\&\,\vec{b}$ is

a. 30°
b. 0°
c. 90°
d. 60°

18. If $x^m y^n = (x+y)^{(m+n)}$ then $\frac{dy}{dx}$ is equal to

a. $\frac{x+y}{xy}$
b. xy
c. 0
d. $\frac{y}{x}$

19. The general solution of $\cot\theta + \tan\theta = 2$ is

a. $\theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{8}$
b. $\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$
c. $\theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}$
d. $\theta = n\pi + (-1)^n \pi/8$

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20. The value of $\int_{-\pi/4}^{\pi/4} \sin^{103}x.\cos^{101}x\ dx$ is

a. $\left(\frac{\pi}{4}\right)^{103}$

b. $\left(\frac{\pi}{4}\right)^{101}$

c. 2

d. 0

21. The length of latus rectum of the parabola $4y^2+3x + 3y + 1 = 0$ is

a. $\frac{4}{3}$

b. 7

c. 12

d. $\frac{3}{4}$

22. The value of $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}$ dx is equal to

a. 0

b. $\frac{x^3}{3} + c$

c. $\frac{3}{x^3} + c$

d. $\frac{1}{x} + c$

23. The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is

a. 1

b. $-(\log_{10} x)^2$

c. $(\log_x 10)^2$

d. $\frac{x^2}{100}$

24. The slope of the tangent to the curve $x=t^2+ 3t - 8$, $y =2t^2-2t-5$ at the point $(2,-1)$ is

a. $\frac{22}{7}$

b. $\frac{6}{7}$

c. $\frac{7}{6}$

d. $\frac{-6}{7}$

25. The real part of $(1-\cos\theta +i \sin \theta)^{-1}$ is

a. $\frac{1}{2}$

b. $\frac{1}{1+\cos\theta}$

c. $\tan\frac{\theta}{2}$

d. $\cot\frac{\theta}{2}$

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26. $\int_{0}^{\pi/2} \frac{\sin^{1000}x \, dx}{\sin^{1000}x + \cos^{1000}x}$ is equal to

a. 1000
b. 1
c. $\frac{\pi}{2}$
d. $\frac{\pi}{4}$

27. If 1 + sin θ + sin²θ + ………..upto ∞ = $2\sqrt{3}+4$ then θ =

a. $\frac{\pi}{6}$
b. $\frac{\pi}{4}$
c. $\frac{\pi}{3}$
d. $\frac{3\pi}{4}$

28. $\lim_{x\to 0} \frac{xe^x - \sin x}{x}$ is equal to

a. 3
b. 1
c. 0
d. 2

29. If $\tan^{-1}(x^2 + y^2) = \alpha$ then $\frac{dy}{dx}$ is equal to

a. $\frac{-x}{y}$
b. xy
c. $\frac{x}{y}$
d. −xy

30. The simplified form of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

a. 0
b. 1
c. −1
d. i

31. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$

a. Touch each other
b. Cut each other at right angle
c. Cut at an angle $\pi/3$
d. Cut at an angle $\pi/4$

32. The equation of the normal to the curve $y(1 + x^2) = 2 - x$ where the tangent crosses x-axis is

a. $5x - y - 10 = 0$
b. $x - 5y - 10 = 0$
c. $5x + y + 10 = 0$
d. $x + 5y + 10 = 0$

33. The maximum value of $\left(\frac{1}{x}\right)^x$ is.

a. e

b. e^e

c. $e^{\frac{1}{e}}$

d. $\left(\frac{1}{e}\right)^e$

34. The solution for the differential equation $\frac{dy}{y}+\frac{dx}{y}=0$ is.

a. $\frac{1}{y}+\frac{1}{x}=c$

b. log x. log y =c

c. x y = c

d. x + y =c

35. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)\right]^{\frac{3}{4}}=\frac{d^2y}{dx^2}$

a. Order =2
Degree = 3

b. Order =2
Degree = 4

c. Order =2
Degree =3/4

d. Order =2
Degree = not found

36. If $\vec{a}$ *and* $\vec{b}$ are unit vectors then what is the angle between $\vec{a}$ *and* $\vec{b}$ for $\sqrt{3}\,\vec{a}-\vec{b}$ to be unit vector?

a. 30°

b. 45°

c. 60°

d. 90°

37. The sum of 1st n terms of the series

$$\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+........$$

a. $\frac{n+2}{3}$

b. $\frac{n(n+2)}{3}$

c. $\frac{n(n-2)}{3}$

d. $\frac{n(n-2)}{6}$

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38. The 11[th] term in the expansion of $\left(x+\frac{1}{\sqrt{x}}\right)^{14}$ is

a. $\frac{999}{x}$

b. $\frac{1001}{x}$

c. i

d. $\frac{x}{1001}$

39. Suppose $\vec{a}+\vec{b}+\vec{c}=0, |\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, then the angle between $\vec{a}\,\&\,\vec{b}$ is

a. π

b. $\frac{\pi}{2}$

c. $\frac{\pi}{3}$

d. $\frac{\pi}{4}$

40. If a = 3, b = 4, c = 5 each one of $\vec{a}, \vec{b}\,\&\,\vec{c}$ is perpendicular to the sum of the remaining then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is equal to

a. $\frac{5}{\sqrt{2}}$

b. $\frac{2}{\sqrt{5}}$

c. $5\sqrt{2}$

d. $\sqrt{5}$

41. If the straight lines 2x + 3y – 3 = 0 and x + ky + 7 = 0 are perpendicular, then the value of k is

a. $\frac{2}{3}$

b. $\frac{3}{2}$

c. $\frac{-2}{3}$

d. $\frac{-3}{2}$

42. The rate of change of area of a circle with respect to its radius at r =2 cms is

a. 4

b. 2π

c. 2

d. 4π

43. The value of tan $\frac{\pi}{8}$ is equal to

a. $\frac{1}{2}$

b. $\sqrt{2}+1$

c. $\frac{1}{\sqrt{2}+1}$

d. $1-\sqrt{2}$

44. Area lying between the curves $y^2 = 2x$ and $y = x$ is

a. $\frac{2}{3}$ sq.units
b. $\frac{1}{3}$ sq.units
c. $\frac{1}{4}$ sq.units
d. $\frac{3}{4}$ sq.units

45. If $P(A \cap B) = \frac{7}{10}$ and $P(B) = \frac{17}{20}$., where P stands for probability then $P(A|B)$ is equal to

a. $\frac{7}{8}$
b. $\frac{17}{20}$
c. $\frac{14}{17}$
d. $\frac{1}{8}$

46. The coefficient of variation of two distributions are 60 and 70.The standard deviation are 21 and 16 respectively, then their mean is

a. 35
b. 23
c. 28.25
d. 22.85

47. Two cards are drawn at random from a pack of 52 cards. The probability of these two being "Aces" is

a. $\frac{1}{26}$
b. $\frac{1}{221}$
c. $\frac{1}{2}$
d. $\frac{1}{13}$

48. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then x^2 is equal to

a. $1 - y^2$
b. y^2
c. 0
d. $\sqrt{1-y}$

49. The value of $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$ is

a. 10
b. 0
c. 8
d. 3

50. The contra positive of the converse of the statement "If x is a prime number then x is odd" is

a. If x is not a prime number then x is odd.

b. If x is not an odd number then x is not a prime number.

c. If x is a prime number then it is not odd.

d. If x is not a prime number then x is not an odd.

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51. Two dice are thrown simultaneously, the probability of obtaining a total score of 5 is

a. $\frac{1}{18}$
b. $\frac{1}{12}$
c. $\frac{1}{9}$
d. $\frac{1}{6}$

52. If $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ and $A + A^T = I$,

Where I is the unit matrix of 2×2 & A^T is the transpose of A, then the value of θ is equal to.

a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$
c. π
d. $\frac{3\pi}{2}$

53. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then $A^2 - 5A$ is equal to

a. I
b. -I
c. 7I
d. -7I

54. The value of x if $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector is

a. $\pm\frac{1}{\sqrt{3}}$
b. $\pm\sqrt{3}$
c. ± 3
d. $\pm\frac{1}{3}$

55. If x = 2 + 3 cosθ and y = 1 – 3 sin θ represent a circle then the Centre and radius is

a. (2,1), 9
b. (2,1),3
c. $(1,2), \frac{1}{3}$
d. (–2,–1),3

56. The vector equation of the plane which is at a distance of $\frac{3}{\sqrt{14}}$ from the origin and the normal from the origin is $2\hat{i} - 3\hat{j} + \hat{k}$ is

a. $\vec{r}.\left(2\hat{i} - 3\hat{j} + \hat{k}\right) = 3$
b. $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 9$
c. $\vec{r}.(\hat{i} + 2\hat{j}) = 3$
d. $\vec{r}.\left(2\hat{i} + \hat{k}\right) = 3$

57. Find the co-ordinates of the foot of the perpendicular drawn from the origin to the plane 5y + 8 =0:

a. $\left(0, -\frac{18}{5}, 2\right)$
b. $\left(0, \frac{8}{5}, 0\right)$

c. $\left(\frac{8}{25}, 0, 0\right)$
d. $\left(0, -\frac{8}{5}, 0\right)$

58. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of a vector $\vec{a}$ then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to
    a. 2
    b. 3
    c. −1
    d. 0

59. The value of the $\sin 1° + \sin 2° + \ldots\ldots + \sin 359°$ is equal to
    a. 0
    b. 1
    c. −1
    d. 180

60. Integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$ is
    a. x
    b. $\log x$
    c. $\frac{1}{x}$
    d. $-x$

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ANSWER KEYS

1. (d)	2. (a)	3. (b)	4. (d)	5. (b)	6. (b)	7. (d)	8. (c)	9. (b)	10. (d)
11. (c)	12. (d)	13. (b)	14. (c)	15. (a)	16. (a)	17. (d)	18. (d)	19. (b)	20. (d)
21. (d)	22. (b)	23. (b)	24. (b)	25. (a)	26. (d)	27. (c)	28. (c)	29. (a)	30. (a)
31. (b)	32. (a)	33. (c)	34. (c)	35. (d)	36. (a)	37. (b)	38. (b)	39. (c)	40. (c)
41. (c)	42. (d)	43. (c)	44. (a)	45. (c)	46. (a,d)	47. (b)	48. (a)	49. (d)	50. (d)
51. (c)	52. (a)	53. (d)	54. (a)	55. (b)	56. (a)	57. (d)	58. (c)	59. (a)	60. (c)

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Solution

1. (d)

   No. of elements in set A = 4

   No. of elements in set B = 5

   No. of injective mapping = ${}^{5}P_{4} = \frac{5!}{1!} = 120$

2. (a)

   f (x) = 2 x + 6

   y = 2 x + 6

   y – 6 = 2 x

   $\frac{y-6}{2} = x \quad \left\{ put\ y = x\ \&\ x = f^{-1}(x) \right\}$

   $f^{-1}(x) = \frac{x}{2} - 3$

3. (b)

   $a*b = \frac{a+b}{2} (given)$

   Commutative property $\Rightarrow \quad a*b = b*a$

   $$\frac{a+b}{4} = \frac{b+a}{4}$$

   The given operation is commutative.

   Associative property $\Rightarrow \quad a*(b*c) = (a*b)*c$

   $$\Rightarrow \quad \frac{a+\frac{b+c}{4}}{4} = \frac{\frac{a+b}{4}+c}{4}$$

   $$\Rightarrow \quad \frac{4a+b+c}{16} = \frac{a+b+4c}{16}$$

   $$\Rightarrow \quad L.H.S. \neq \text{R.H.S.}$$

   The given operation is not associative.

4. (d)

$$\sin^{-1}\left(\cos\frac{53\pi}{5}\right)$$

$$\Rightarrow \sin^{-1}\left[\cos\left(10\pi+\frac{3\pi}{5}\right)\right]$$

$$\Rightarrow \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$\Rightarrow \sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right]$$

$$\Rightarrow \frac{\pi}{2}-\frac{3\pi}{5}=\frac{-\pi}{10}$$

5. (b)

$$3\tan^{-1}x+\cot^{-1}x=\pi$$

$$\Rightarrow 2\tan^{-1}x+\frac{\pi}{2}=\pi$$

$$\Rightarrow 2\tan^{-1}x=\frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{4}$$

$$\Rightarrow x=\tan\left(\frac{\pi}{4}\right)=1$$

6. (b)

$$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{1-\frac{y}{x}}{1+\frac{y}{x}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right)-\left[\tan^{-1}1-\tan^{-1}\frac{y}{x}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)-\tan^{-1}1$$

$$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right)+\cot^{-1}\left(\frac{x}{y}\right)-\frac{\pi}{4}$$

$$\Rightarrow \frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

7. (d)

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$$

$\Rightarrow \ (1+x)\left[(1+y)(1+z)-1\right]-\left[1+z-1\right]+\left[1-1-y\right]=0$

$\Rightarrow \ (1+x)(1+y)(1+z)-(1+x)-z-y=0$

$\Rightarrow 1+x+y+z+xy+xz+yz+xyz=x+y+z+1$

$\Rightarrow xy+yz+zx=-xyz$

$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=-1$

$\Rightarrow x^{-1}+y^{-1}+z^{-1}=-1$

8. (c)

Use property $|KA| = K^n |A|$ $\{here$ n = order of matrix$\}$

$\Rightarrow |3A| = 3^3 |A| = 27|A|$

9. (b)

$y = e^{\sin^{-1}(t^2-1)}$; $x = e^{\sec^{-1}\left(\frac{1}{t^2-1}\right)}$

$y = e^{\sin^{-1}(t^2-1)}$; $x = e^{\cos^{-1}(t^2-1)}$

$\Rightarrow xy = e^{\sin^{-1}(t^2-1)} . e^{\cos^{-1}(t^2-1)}$

$\Rightarrow xy = e^{\sin^{-1}(t^2-1)+\cos^{-1}(t^2-1)}$

$\Rightarrow xy = e^{\frac{\pi}{2}}$

On differentiating both side with respect to x

$\Rightarrow y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

10. (d)

$$\Rightarrow A-B=\frac{1}{\pi}\begin{bmatrix}\sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x)\end{bmatrix}-\frac{1}{\pi}\begin{bmatrix}-\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x)\end{bmatrix}$$

$$\Rightarrow A-B=\frac{1}{\pi}\begin{bmatrix}\sin^{-1}(\pi x)+\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right)-\tan^{-1}\left(\frac{x}{\pi}\right)\\ \sin^{-1}\left(\frac{x}{\pi}\right)-\sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x)+\tan^{-1}(\pi x)\end{bmatrix}$$

$$\Rightarrow A-B=\frac{1}{\pi}\begin{bmatrix}\frac{\pi}{2} & 0\\ 0 & \frac{\pi}{2}\end{bmatrix}\Rightarrow A-B=\frac{1}{\pi}.\frac{\pi}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow A-B=\frac{I}{2}$$

11. (c)

$x^y=e^{x-y}$

on taking log both sides

$$\Rightarrow y\ \log\ x=x-y\Rightarrow y=\frac{x}{1+\log x} \qquad \text{......(1)}$$

On differentiating both side

$$\Rightarrow \frac{y}{x}+\log x\frac{dy}{dx}=1-\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(\log x+1)=1-\frac{y}{x}$$

Put value of y from eq. (1)

$$\Rightarrow \frac{dy}{dx}(1+\log x)=1-\frac{\frac{x}{(1+\log x)}}{x}$$

$$\Rightarrow \frac{dy}{dx}(1+\log x)=\frac{1+\log x-1}{(1+\log x)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\log x}{(1+\log x)2}$$

12 (d)

Given order of A is = m × n

Let the order of B is = v × u

If AB' is defined = $A_{m\times n} \times B'_{u\times v}$

This is possible when n = u

If B'A is defined = $B'_{u\times v} \times A_{m\times n}$

This is possible when v = m

The order of $B = v \times u = m \times n$

13 (b)

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$$

$\because e^x x = t$

$e^x(1+x)dx = dt$

$$\Rightarrow \int \frac{\text{dt}}{\cos^2 \text{t}}$$

$$\Rightarrow \int \sec^2 t \ dt = \tan t + x \Rightarrow \tan(e^x.x) + c$$

14 (c)

$$\Rightarrow \begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$$

$$R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} \log x & \log y & \log z \\ \log 2 & \log 2 & \log 2 \\ \log 3 & \log 3 & \log 3 \end{vmatrix}$$

$$\Rightarrow \log 2 \ \log 3 \begin{vmatrix} \log x & \log y & \log z \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \ 0 \ \{\text{because two row's are same}\}$$

15 (a)

$f(x)=[x]$

As we know Greatest Integer Functions are discontinuous at all integers

LHL= $\lim\limits_{x\to 1.5^-}[x]=1$

RHL= $\lim\limits_{x\to 1.5^+}[x]=1$

F (1.5) = [1.5] =1

LHL=RHL=f (1.5)

Function f (x) is continuous at x = 1.5

16 (a)

$$\int \frac{e^x\left(x^2\tan^{-1}x+\tan^{-1}x+1\right)}{x^2+1}dx$$

$$\Rightarrow \int e^x\left[\tan^{-1}x+\frac{1}{1+x^2}\right]dx$$

$$\Rightarrow e^x\tan^{-1}x+c$$

17 (d)

$$\Rightarrow 2\vec{a}.\vec{b}=\left|\vec{a}\right|.\left|\vec{b}\right| \ \Rightarrow 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta=\left|\vec{a}\right|\left|\vec{b}\right|$$

$$\Rightarrow \cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$$

18 (d)

$x^m y^n=\left(x+y\right)^{m+n}$

On taking log both sides

$$\Rightarrow \log\left(x^m y^n\right)=\log(x+y)^{m+n} \ \Rightarrow m\log x+n\log y=(m+n)\log(x+y)$$

$$\Rightarrow \frac{m}{x}+\frac{n}{y}\frac{dy}{dx}=\frac{(m+n)}{(x+y)}\left(1+\frac{dy}{dx}\right) \ \Rightarrow \frac{m}{x}-\frac{(m+n)}{(x+y)}=\left[\frac{(m+n)}{(x+y)}-\frac{n}{y}\right]\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\left[m(x+y)-(m+n)x\right]/x}{\left[(m+n)y-n(x+y)\right]/y} \ \Rightarrow \frac{dy}{dx}=\frac{[my-nx]/x}{[my-nx]/y}=\frac{y}{x}$$

19 (b)

$\Rightarrow \cot\theta + \tan\theta = 2$

$\Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = 2$

$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{2\sin\theta\cos\theta} = 1$

$\Rightarrow 1 = \sin 2\theta$

$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$

20 (d)

$I = \int_{-\pi/4}^{\pi/4} \sin^{103} x.\cos^{101} x \, dx$ ……. (1)

$put \ x = -x$

$I = -\int_{-\pi/4}^{\pi/4} \sin^{103} x.\cos^{101} x \, dx$ …….. (2)

We can say that the given function is an add function

Add equation (1) and (2)

$\Rightarrow$ 2I = 0

$\Rightarrow$ I=0

21 (d)

$$\Rightarrow 4y^2+3y=-3x-1$$

$$\Rightarrow 4\left(y^2+\frac{3}{4}y\right)=-3x-1$$

$$\Rightarrow 4\left(y^2+\frac{3}{4}y+\frac{9}{64}-\frac{9}{64}\right)=-3x-1$$

$$\Rightarrow 4\left(\left(y+\frac{3}{8}\right)^2-\frac{9}{64}\right)=-3x-1$$

$$\Rightarrow \left(y+\frac{3}{8}\right)^2=\frac{-3x-1}{4}+\frac{9}{64}$$

$$\Rightarrow \left(y+\frac{3}{8}\right)^2=\frac{-3x}{4}-\frac{7}{64}=\frac{-3}{4}\left[x+\frac{7}{48}\right]$$

$$\Rightarrow LLR=4a=\left|4\times\left(\frac{-3}{4\times 4}\right)\right|=\left|\frac{-3}{4}\right|=\frac{3}{4}$$

22 (b)

$$\Rightarrow \int\frac{e^{6\log x}-e^{5\log x}}{e^{4\log x}-e^{3\log x}}dx$$

$$\Rightarrow \int\frac{x^6-x^5}{x^4-x^3}dx$$

$$\Rightarrow \int\frac{x^5(x-1)}{x^3(x-1)}dx\Rightarrow\int x^2dx\Rightarrow\frac{x^3}{3}+c$$

23. (b)

$let\ u=\log_{10}x$

$$v=\log_x 10=\frac{1}{\log_{10}\mathrm{x}}=\frac{1}{u} \qquad \ldots\ldots(1)$$

$$\Rightarrow\ vu=1$$

On differentiating both side with respect to *v*.

$$\Rightarrow u+v\frac{du}{dv}=0$$

$$\Rightarrow\frac{du}{dv}=\frac{-u}{v} \qquad \left\{from\ \text{eq. }(1)\quad v=\frac{1}{u}\right\}$$

$$\Rightarrow \frac{du}{dv} = -u^2 = -\left(\log_{10} x\right)^2$$

24. (b)

$$given \quad x = t^2 + 3t - 8$$
$$y = 2t^2 - 2t - 5$$

Substitute x = 2 in given eq.

$$\Rightarrow \ 2 = t^2 + 3t - 8$$
$$\Rightarrow \ t = -5, \ 2$$

Now substitute y = -1

$$\Rightarrow \ -1 = 2t^2 - 2t - 5$$
$$\Rightarrow \ t = 2, \ \ -1$$

Common value of t is 2.

$$\frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right|_{t=2} = \left.\frac{4t-2}{2t+3}\right|_{t=2} = \frac{6}{7}$$

25. (a)

$$\Rightarrow \left(1 - \cos\theta + i\sin\theta\right)^{-1}$$

$$\Rightarrow \frac{1}{\left(1-\cos\theta + i\sin\theta\right)} \times \frac{\left(1-\cos\theta - i\sin\theta\right)}{\left(1-\cos\theta - i\sin\theta\right)} \Rightarrow \frac{\left(1-\cos\theta\right) - i\sin\theta}{\left(1-\cos\theta\right)^2 + \sin^2\theta}$$

$$\Rightarrow \frac{\left(1-\cos\theta\right) - i\sin\theta}{4\sin^4\frac{\theta}{2} + 4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}} \Rightarrow \frac{2\sin^2\frac{\theta}{2} - i\left[2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]}{4\sin^2\frac{\theta}{2}\left[\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right]}$$

$$\Rightarrow \frac{\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}} \Rightarrow \operatorname{Re}\left(\frac{1}{2} - \frac{1}{2}i\cot\frac{\theta}{2}\right) \Rightarrow \frac{1}{2}$$

26. (d)

$$I = \int_0^{\pi/2} \frac{\sin^{1000} x dx}{\sin^{1000} x + \cos^{1000} x} \qquad \text{.........} \ (1)$$

Apply king property

$$I = \int_0^{\pi/2} \frac{\cos^{1000} x dx}{\cos^{1000} x + \sin^{1000} x} \qquad \text{.........} \ (2)$$

On adding eq. (1) and eq. (2)

$$2I = \int_0^{\pi/2} 1.dx$$

$$I = \frac{\pi}{4}$$

27. (c)

$$\Rightarrow \frac{1}{1-\sin\theta} = 2\sqrt{3}+4$$

$$\Rightarrow \frac{1}{2\sqrt{3}+4} \times \frac{2\sqrt{3}-4}{2\sqrt{3}-4} = 1-\sin\theta$$

$$\Rightarrow 1-\sin\theta = \frac{2\sqrt{3}-4}{-4}$$

$$\Rightarrow \sin\theta = 1 + \frac{\sqrt{3}-2}{2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

28. (c)

$$\lim_{x\to 0} \frac{xe^x - \sin x}{x}\left(\frac{0}{0}\right)$$

L-Hospital Rule

$$\Rightarrow \lim_{x\to 0} \frac{xe^x + e^x - \cos x}{1} = 0+1-1=0$$

29. (a)

$$\Rightarrow \tan^{-1}\left(x^2+y^2\right) = \alpha$$

$$\Rightarrow x^2+y^2 = \tan\alpha$$

$$\Rightarrow 2x+2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

30. (a)

$$\Rightarrow i^n + i^{n+1} + i^{n+2} + i^{n+3}$$

$$\Rightarrow (i)^n + i(i)^n - (i)^n - i(i)^n$$

$$\Rightarrow 0$$

31. (b)

$x^3 - 3xy^2 + 2 = 0$

$\Rightarrow 3x^2 - 6xy\dfrac{dy}{dx} - 3y^2 = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3\left(x^2 - y^2\right)}{6xy} = m_1$ ....(1)

$\Rightarrow 3yx^2 - y^3 = 2$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-6xy}{3\left(x^2 - y^2\right)} = m_2$ .....(2)

$\because\ m_1 \times m_2 = -1$

$\therefore$ Both curves cut each other at right angle

32. (a)

$\because$ Tangent crosses x-axis.

Put y = 0 $\Rightarrow$ x = 2

$\left.\dfrac{dy}{dx}\right|_{(2,0)} = \dfrac{-1-2xy}{\left(1+x^2\right)}$

$\left.\dfrac{dy}{dx}\right|_{(2,0)} = \dfrac{-1}{5} = m_T$

$m_N = 5$

Eq. of normal at (2, 0) is y – 0 = 5(x – 2) $\Rightarrow$5 x –y =10

33. (c)

$\Rightarrow y = \dfrac{1}{x^x}$

$\Rightarrow \log y = -x \log x$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{-x}{x} + \left(-\log x\right)$

$\dfrac{dy}{dx} = -x^{-x}\left(1 + \log x\right) = 0$

$\Rightarrow 1 + \log x = 0$

$\Rightarrow x = \dfrac{1}{e}\left(\text{Point of maxima}\right)$

$\max\left(y\right) = \left(e\right)^{1/e}$

34. (c)

$$\frac{dy}{y}+\frac{dx}{x}=0$$

On integrating both sides

$\Rightarrow \log y+\log x=\log c$

$\Rightarrow xy=c$

35. (d)

$$1+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)=\left(\frac{d^2y}{dx^2}\right)^{4/3}$$

Order = 2

Degree = not defined $\left(\text{because of } \sin\frac{dy}{dx}\right)$

36. (a)

$$\left|\sqrt{3}\,\vec{a}-\vec{b}\right|=1$$

On squaring both sides

$\Rightarrow\ 3\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\sqrt{3}\,\vec{a}.\vec{b}=1$

$\Rightarrow 4-2\sqrt{3}\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta=1$

$\Rightarrow \cos\alpha=\frac{3}{2\sqrt{3}}=\frac{\sqrt{3}}{2}$

$\Rightarrow \theta=\frac{\pi}{6}$

37. (b)

$$T_n=\frac{1^2+2^2+3^2+.........+n^2}{1+2+3+.........+n}$$

$$T_n=\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}=\frac{2n+1}{3}$$

$$S_n=\sum T_n=\frac{1}{3}\sum(2n+1)$$

$$=\frac{1}{3}\left(\frac{2n(n+1)}{2}+n\right)$$

$$=\frac{1}{3}\left(n^2+2n\right)$$

38. (b)

$$T_{11}={}^{14}C_{10}\,(x)^{14-10}\left(\frac{1}{\sqrt{x}}\right)^{10}$$

$$={}^{14}C_{10}\;x^4.\frac{1}{x^5}$$

$$=\frac{1001}{x}$$

39. (c)

$$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0$$

$$\Rightarrow \left|\vec{a}+\vec{b}\right|^2=\left|-\vec{c}\right|^2$$

$$\Rightarrow \left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta=\left|\vec{c}\right|^2$$

$$\Rightarrow 9+25+2\times15\cos\theta=49$$

$$\Rightarrow 30\cos\theta=15$$

$$\Rightarrow \cos\theta={}^1/_2$$

$$\Rightarrow \theta={}^{\pi}/_3$$

40. (c)

Given

$$\vec{a}.\left(\vec{b}+\vec{c}\right)=0$$

$$\vec{b}.\left(\vec{a}+\vec{c}\right)=0$$

$$\vec{c}.\left(\vec{a}+\vec{b}\right)=0$$

$$\overline{\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0}$$

$$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)$$

$$=9+16+25+0$$

$$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right|=5\sqrt{2}$$

41. (c)

$$m_1=\frac{-2}{3}$$

$$m_2=\frac{-1}{K}$$

$\Rightarrow m_1 m_2 = -1$

$\Rightarrow \dfrac{2}{3K} = -1 \Rightarrow K = \dfrac{-2}{3}$

42. (d)

A = πr^2

$$\Rightarrow \left.\frac{dA}{dr}\right|_{r=2} = 2\pi r = 2\pi \times 2 = 4\pi$$

43. (c)

$\Rightarrow \tan\dfrac{\pi}{8}$

$$\Rightarrow \frac{\sin 2\left(\pi/8\right)}{1+\cos 2\left(\pi/8\right)}$$

$$\Rightarrow \frac{\sin \pi/4}{1+\cos \pi/4} \Rightarrow \frac{1}{1+\sqrt{2}}$$

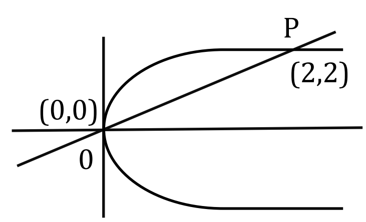
44. (a)

Point of contact of both curves is

$\Rightarrow x^2 = 2x$

$\Rightarrow \quad x = 0, \quad x = 2$

$\quad y = 0, \quad y = 2$



$$\Rightarrow \int_0^2 \left(\sqrt{2}\sqrt{x} - x\right) dx$$

$$\Rightarrow \sqrt{2}\left[\frac{2x^{3/2}}{3}\right]_0^2 - \left[\frac{x^2}{2}\right]_0^2$$

$\Rightarrow 8/3 - 2 = 2/3$ sq. units

45. (c)

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)} = \frac{7/10}{17/20}$$

$= \frac{14}{17}$

46. (a,d)

Coefficient of variation $Cv_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$\Rightarrow \quad 60 = \frac{21}{\bar{x}_1} \times 100 \quad ......(1)$

$\Rightarrow \quad Cv_2 = \frac{\sigma_2}{\bar{x}_2} \times 100 \quad \Rightarrow \quad 70 = \frac{16}{\bar{x}_2} \times 100 \quad ......(2)$

From Eq. (1) & (2)

$\bar{x}_1 = 35$ & $\bar{x}_2 = 22.85$

47. (b)

Probability of two cards being aces is = $\frac{{}^{4}C_2}{{}^{52}C_2} = \frac{1}{221}$

48. (a)

$\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}x = \frac{\pi}{2} - \sin^{-1}y$

$\Rightarrow \quad \sin^{-1}x = \cos^{-1}y$

$\Rightarrow \quad \sin^{-1}x = \sin^{-1}\sqrt{1-y^2}$

$\Rightarrow \quad x^2 = 1 - y^2$

49. (d)

$I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx \quad .... (1)$

Apply king

$I = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}} dx \quad .... (2)$

On adding eq. (1) and eq. (2)

$$\Rightarrow 2I = \int_2^8 1.dx = 6 \Rightarrow I = 3$$

50. (d)

The converse of the given statement $p \to q$ is $q \to p$

The contra positive of the given statement is $q \to p$ is $\sqcup$ p→$\sqcup$ q

Hence, the contra positive of the converse of the given statement is " If x is not a prime number, then x is not odd"

51. (c)

Total ways of obtaining score of 5 is {(1, 4), (2, 3), (3, 2), (4, 1)}

Required probability is $= \frac{4}{6^2} = \frac{4}{36} = \frac{1}{9}$

52. (a)

$A + A^T = I$

$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} + \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos 2\theta & 0 \\ 0 & 2\cos 2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2\cos 2\theta = 1 \quad \Rightarrow \quad \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{3} \quad \Rightarrow \quad \theta = \frac{\pi}{6}$$

53. (d)

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (\lambda - 2)(\lambda - 3) + 1 = 0$$

$$\Rightarrow \quad \lambda^2 - 5\lambda + 6 + 1 = 0$$

$\Rightarrow$ Put $\lambda = A$

$\Rightarrow \quad A^2 - 5A + 7I = 0$

$\Rightarrow \quad A^2 - 5A = -7I$

54. (a)

$|x(\hat{i} + \hat{j} + \hat{k})| = 1$

$\Rightarrow \quad \sqrt{x^2 + x^2 + x^2} = 1$

$\Rightarrow \quad 3x^2 = 1$

$\Rightarrow \quad x = \pm \dfrac{1}{\sqrt{3}}$

55. (b)

$x = 2 + 3\cos\theta \quad \Rightarrow \quad \left(\dfrac{x-2}{3}\right) = \cos\theta \qquad \text{........ (i)}$

$y = 1 - 3\sin\theta \quad \Rightarrow \quad \left(\dfrac{y-1}{-3}\right) = \sin\theta \qquad \text{........ (ii)}$

$\Rightarrow$ Equation $(1)^2$ + Equation $(2)^2$

$\Rightarrow \quad \text{Sin}^2\theta + \cos^2\theta = \left(\dfrac{x-2}{3}\right)^2 + \left(\dfrac{y-1}{-3}\right)^2$

$\Rightarrow \quad (x-2)^2 + (y-1)^2 = 9$

Centre $\equiv (2, 1)$; radius = 3

56. (a)

Vector equation of a plane is $\vec{r} \cdot \hat{n} = d$

$\Rightarrow \quad \vec{r} \cdot \dfrac{(2\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{14}} = \dfrac{3}{\sqrt{14}}$

$\Rightarrow \quad \vec{r} \cdot (2\hat{i} - 3j + \hat{k}) = 3$

57. (d)

Let foot of perpendicular drawn from origin is (x_1, y_1, z_1)

$\Rightarrow \quad \dfrac{x_1 - 0}{0} = \dfrac{y_1 - 0}{5} = \dfrac{z_1 - 0}{0} = \dfrac{-(0+0+0+8)}{0^2 + 5^2 + 0^2}$

$\Rightarrow \quad \dfrac{x_1}{0} = \dfrac{y_1}{5} = \dfrac{z_1}{0} = \dfrac{-8}{25}$

$\Rightarrow \quad x_1 = 0,\ y_1 = \frac{-8}{5}, z_1 = 0$

Co-ordinate is = $\left(0, \frac{-8}{5}, 0\right)$

58. (c)

As we know that

$\Rightarrow \quad co^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\beta}{2} = 1$

$\Rightarrow \quad \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

59. (a)

$\sin 1° + \sin 2° + \ldots\ldots + \sin 359°$

$\Rightarrow \quad (\sin 1° + \sin 359°) + (\sin 2° + \sin 358°) + \ldots.. + \sin 180°$

$\Rightarrow \quad (\sin 1° - \sin 1°) + (\sin 2° - \sin 2°) + \ldots\ldots + \sin 180°$

$\Rightarrow \quad 0 + 0 + \ldots\ldots + 0 = 0$

60. (c)

$x\frac{dy}{dx} - y = x^4 - 3x$

$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$

I. F = $e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$