KCET-2018 (Mathematics)

1. If $|x + 5| \geq 10$ then
   a. $x \in (-15, 5]$
   b. $x \in (-5, 5]$
   c. $x \in (-\infty, -15] \cup [5, \infty)$
   d. $x \in [-\infty, -15] \cup [5, \infty)$

2. Everybody in a room shakes hands with everybody else. The total number of handshakes is 45. The total number of persons in the room is
   a. 9
   b. 10
   c. 5
   d. 15

3. The constant term in the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$ is
   a. ${}^{16}C_8$
   b. ${}^{16}C_7$
   c. ${}^{16}C_9$
   d. ${}^{16}C_{10}$

4. If P(n) : "$2^{2n} - 1$ is divisible by k for all $n \in N$" is true, then the value of 'k' is
   a. 6
   b. 3
   c. 7
   d. 2

5. The equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through $(-2, 3)$ is
   a. $3x - 4y + 18 = 0$
   b. $3x - 4y - 18 = 0$
   c. $3x + 4y + 18 = 0$
   d. $3x + 4y - 18 = 0$

6. If $\left(\frac{1-i}{1+i}\right)^{96} = a + ib$ then (a, b) is
   a. (1, 1)
   b. (1, 0)
   c. (0, 1)
   d. (0, –1)

7. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is
   a. $x^2 - y^2 = 32$
   b. $\frac{x^2}{4} - \frac{y^2}{9} = 1$
   c. $2x^2 - 3y^2 = 7$
   d. $y^2 - x^2 = 32$

8. The number of ways in which 5 girls and 3 boys can be seated in a row so that no two boys are together is
   a. 14040
   b. 14440
   c. 14000
   d. 14400

9. If a, b, c are three consecutive terms of an AP and x, y, z are three consecutive terms of a GP, then the value of $x^{b-c} . y^{c-a} . z^{a-b}$ is
   a. 0
   b. xyz
   c. -1
   d. 1

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10. The value of $\lim_{x \to 0} \frac{|x|}{x}$ is

a. 1
b. -1
c. 0
d. Does not exist

11. Let $f(x) = x - \frac{1}{x}$ then $f'(-1)$ is

a. 0
b. 2
c. 1
d. -2

12. The negation of the statement "72 is divisible by 2 and 3" is

a. 72 is not divisible by 2 or 72 is not divisible by 3
b. 72 is divisible by 2 or 72 is divisible by 3
c. 72 is divisible by 2 and 72 is divisible by 3
d. 72 is not divisible by 2 and 3

13. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is

a. 0.4
b. 0.5
c. 0.2
d. 0.9

14. In a simultaneous throw of a pair of dice, the probability of getting a total more than 7 is

a. $\frac{7}{12}$
b. $\frac{5}{36}$
c. $\frac{5}{12}$
d. $\frac{7}{36}$

15. If A and B are mutually exclusive events given that $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$, then P(A or B) is

a. 0.8
b. 0.6
c. 0.4
d. 0.2

16. Let $f, g : R \to R$ be two functions define as $f(x) = |x| + x$ and $g(x) = |x| - x \forall x \in R$. Then (fog)(x) for $x < 0$ is

a. 0
b. 4x
c. -4x
d. 2x

17. A is a set having 6 distinct elements. The number of distinct functions from A to A which are not bijections is

a. $6! - 6$
b. $6^6 - 6$
c. $6^6 - 6!$
d. $6!$

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18. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 2x & ; \quad x > 3 \\ x^2 & ; \quad 1 < x \le 3 \\ 3x & ; \quad x \le 1 \end{cases}$$

Then $f(-1) + f(2) + f(4)$ is

a. 9
b. 14
c. 5
d. 10

19. If $\sin^{-1} x + \cos^{-1} y = \frac{2\pi}{5}$, then $\cos^{-1} x + \sin^{-1} y$ is

a. $\frac{2\pi}{5}$
b. $\frac{3\pi}{5}$
c. $\frac{4\pi}{5}$
d. $\frac{3\pi}{10}$

20. The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

a. $2-\sqrt{5}$
b. $\sqrt{5}-2$
c. $\frac{\sqrt{5}-2}{2}$
d. $5-\sqrt{2}$

21. If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$, then $A^n = 2^k A$, Where k =

a. 2^{n-1}
b. n +1
c. n – 1
d. 2(n-1)

22. If $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, then the value of x and y respectively are

a. –3, –1
b. 1, 3
c. 3, 1
d. –1, 3

23. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then AA' =

a. A
b. Zero matrix
c. A'
d. I

24. If $x, y, z \in R$, then the value of determinant

$$\begin{vmatrix} (5^x+5^{-x})^2 & (5^x-5^{-x})^2 & 1 \\ (6^x+6^{-x})^2 & (6^x-6^{-x})^2 & 1 \\ (7^x+7^{-x})^2 & (7^x-7^{-x})^2 & 1 \end{vmatrix}$$ is

a. 10
b. 12

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c. 1 d. 0

25. The value of determinant

$$\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix} \text{ is}$$

a. $a^3 + b^3 + c^3$
b. 3abc
c. $a^3 + b^3 + c^3 - 3abc$
d. $a^3 + b^3 + c^3 + 3abc$

26. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle whose area is 'k' Square units,

then $\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2$ is

a. $32k^2$
b. $16k^2$
c. $64k^2$
d. $48k^2$

27. Let A be a square matrix of order 3 × 3, then |5A| =

a. 5|A|
b. 125|A|
c. 25|A|
d. 15|A|

28. If $f(x) = \begin{cases} \dfrac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if} \quad -1 \le x < 0 \\ \dfrac{2x+1}{x-1} & \text{if} \quad 0 \le x \le 1 \end{cases}$

is continuous at x = 0, then the value of k is

a. k = 1
b. k = –1
c. k = 0
d. k = 2

29. If cos y = x cos (a + y) with $\cos a \neq \pm 1$, then $\dfrac{dy}{dx}$ is equal to

a. $\dfrac{\sin a}{\cos^2(a+y)}$
b. $\dfrac{\cos^2(a+y)}{\sin a}$
c. $\dfrac{\cos a}{\sin^2(a+y)}$
d. $\dfrac{\cos^2(a+y)}{\cos a}$

30. If f(x) = |cos x – sin x|, then $f'\left(\dfrac{\pi}{6}\right)$ is equal to

a. $-\dfrac{1}{2}(1+\sqrt{3})$
b. $\dfrac{1}{2}(1+\sqrt{3})$
c. $-\dfrac{1}{2}(1-\sqrt{3})$
d. $\dfrac{1}{2}(1-\sqrt{3})$

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31. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$, then $\frac{dy}{dx} =$

a. $\frac{1}{y^2 - 1}$
b. $\frac{1}{2y + 1}$
c. $\frac{2y}{y^2 - 1}$
d. $\frac{1}{2y - 1}$

32. If $f(x) = \begin{cases} \frac{\log_e x}{x - 1} & ; \quad x \neq 1 \\ k & ; \quad x = 1 \end{cases}$ is

Continuous at x = 1, then the value of k is

a. e
b. 1
c. -1
d. 0

33. Approximate change in the volume V of a cube of side x metres caused by increasing the side by 3% is

a. $0.09\ x^3\ m^3$
b. $0.03\ x^3 m^3$
c. $0.06\ x^3 m^3$
d. $0.04\ x^3 m^3$

34. The maximum value of $\left(\frac{1}{x}\right)^x$ is

a. e
b. e^e
c. $e^{1/e}$
d. $\left(\frac{1}{e}\right)^{1/e}$

35. $f(x) = x^x$ has stationary point at

a. $x = e$
b. $x = \frac{1}{e}$
c. $x = 1$
d. $x = \sqrt{e}$

36. The maximum area of a rectangle inscribed in the circle $(x + 1)^2 + (y - 3)^2 = 64$ is

a. 64 sq. units
b. 72 sq. units
c. 128 sq. units
d. 8 sq. units

37. $\int \frac{1}{1 + e^x} dx$ is equal to

a. $\log_e\left(\frac{e^x + 1}{e^x}\right) + c$
b. $\log_e\left(\frac{e^x - 1}{e^x}\right) + c$
c. $\log_e\left(\frac{e^x}{e^x + 1}\right) + c$
d. $\log_e\left(\frac{e^x}{e^x - 1}\right) + c$

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38. $\int \frac{1}{\sqrt{3-6x+9x^2}} dx$ is equal to

a. $\sin^{-1}\left(\frac{3x+1}{2}\right) + c$

b. $\sin^{-1}\left(\frac{3x+1}{6}\right) + c$

c. $\frac{1}{3}\sin^{-1}\left(\frac{3x+1}{2}\right) + c$

d. $\sin^{-1}\left(\frac{2x+1}{3}\right) + c$

39. $\int e^{\sin x}.\left(\frac{\sin x+1}{\sec x}\right) dx$ is equal to

a. $\sin x.\, e^{\sin x} + c$

b. $\cos x.\, e^{\sin x} + c$

c. $e^{\sin x} + c$

d. $e^{\sin x}(\sin x+1) + c$

40. $\int_{-2}^{2} |x\cos \pi x|\, dx$ is equal to

a. $\frac{8}{\pi}$

b. $\frac{4}{\pi}$

c. $\frac{2}{\pi}$

d. $\frac{1}{\pi}$

41. $\int_{0}^{1} \frac{dx}{e^x + e^{-x}}$ is equal to

a. $\frac{\pi}{4} - \tan^{-1}(e)$

b. $\tan^{-1}(e) - \frac{\pi}{4}$

c. $\tan^{-1}(e) + \frac{\pi}{4}$

d. $\tan^{-1}(e)$

42. $\int_{0}^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ is equal to

a. $\frac{1}{\sqrt{2}}\tan^{-1}\sqrt{\frac{2}{3}}$

b. $\frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$

c. $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{3}{2}\right)$

d. $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

43. The area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

a. 1 sq. unit

b. 4 sq. units

c. 2 sq. units

d. 3 sq. units

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44. The area bounded by the line y = x, x-axis and ordinates x = –1 and x = 2 is

a. $\frac{3}{2}$

b. $\frac{5}{2}$

c. 2

d. 3

45. The degree and the order of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{1+\left(\frac{dy}{dx}\right)^2}$ respectively are

a. 2 and 3

b. 3 and 2

c. 2 and 2

d. 3 and 3

46. The solution of the differential equation $x\frac{dy}{dx} - y = 3$ represents a family of

a. straight lines

b. circles

c. parabolas

d. ellipses

47. The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

a. xe^x

b. $xe^{1/x}$

c. $\frac{e^x}{x}$

d. $\frac{x}{e^x}$

48. If $\left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}.\vec{b}\right|^2 = 144$ and $\left|\vec{a}\right| = 4$, then the value of $\left|\vec{b}\right|$ is

a. 1

b. 2

c. 3

d. 4

49. If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors, then $(3\vec{a}+2\vec{b}).(5\vec{a}-6\vec{b})$ =

a. 5

b. 3

c. 6

d. 12

50. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar $(a \neq b \neq c \neq 1)$, then the value of abc – (a + b + c) =

a. 2

b. -2

c. 0

d. -1

51. If $\vec{a} = \hat{i}+\lambda\hat{j}+2\hat{k}$; $\vec{b} = \mu\hat{i}+\hat{j}-\hat{k}$ are orthogonal and $\left|\vec{a}\right| = \left|\vec{b}\right|$ then (λ, μ) =

a. $\left(\frac{1}{4}, \frac{7}{4}\right)$

b. $\left(\frac{7}{4}, \frac{1}{4}\right)$

c. $\left(\frac{1}{4}, \frac{9}{4}\right)$

d. $\left(\frac{-1}{4}, \frac{9}{4}\right)$

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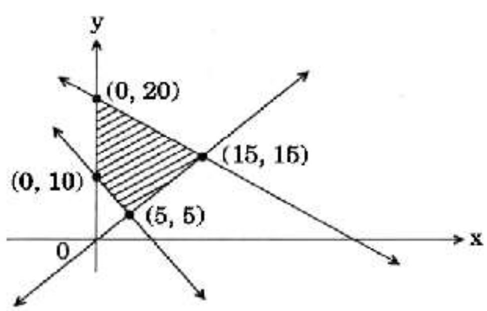
52. The image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is
   a. (1, 0, 7)
   b. (7, 0, 1)
   c. (2, 7, 0)
   d. (−1, −6, −3)

53. The angle between the lines 2x = 3y = −z and 6x = −y = −4z is
   a. 0^o
   b. 45^o
   c. 90^o
   d. 30^o

54. The value of k such that the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies on the plane 2x − 4y + z = 7 is
   a. −7
   b. 4
   c. −4
   d. 7

55. The locus represented by xy + yz = 0 is
   a. a pair of perpendicular lines
   b. a pair of parallel lines
   c. a pair of parallel planes
   d. a pair of perpendicular planes

56. The feasible region of an LPP is shown in the figure. If z = 3x + 9y, then the minimum value of z occurs at



   a. (5, 5)
   b. (0, 10)
   c. (0, 20)
   d. (15, 15)

57. For the LPP; maximize z = x + 4y subject to the constraints $x + 2y \le 2$, $x + 2y \ge 8$, $x, y \ge 0$
   a. $z_{max} = 4$
   b. $z_{max} = 18$
   c. $z_{max} = 16$
   d. Has no feasible solution

58. For the probability distribution given by

$X=x_i$	0	1	2
P_i	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

The standard deviation (σ) is
   a. $\sqrt{\frac{1}{3}}$
   b. $\frac{1}{3}\sqrt{\frac{5}{2}}$
   c. $\sqrt{\frac{5}{36}}$
   d. None of the above

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59. A bag contains 17 tickets numbered from 1 to 17. A ticket is drawn at random, then another ticket is drawn without replacing the first one. The probability that both the tickets may show even numbers is

a. $\frac{7}{34}$
b. $\frac{8}{17}$
c. $\frac{7}{16}$
d. $\frac{7}{17}$

60. A flashlight has 10 batteries out of which 4 are dead. If 3 batteries are selected without replacement and tested, then the probability that all 3 are dead is

a. $\frac{1}{30}$
b. $\frac{2}{8}$
c. $\frac{1}{15}$
d. $\frac{1}{10}$

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ANSWER KEYS

1. (c)	2. (b)	3. (a)	4. (b)	5. (a)	6. (b)	7. (a,d)	8. (d)	9. (d)	10. (d)
11. (b)	12. (a)	13. (c)	14. (c)	15. (a)	16. (c)	17. (c)	18. (a)	19. (b)	20. (b)
21. (d)	22. (d)	23. (d)	24. (d)	25. (G)	26. (c)	27. (b)	28. (b)	29. (b)	30. (a)
31. (d)	32. (b)	33. (a)	34. (c)	35. (b)	36. (c)	37. (c)	38. (c)	39. (a)	40. (a)
41. (b)	42. (a)	43. (c)	44. (b)	45. (b)	46. (a)	47. (c)	48. (c)	49. (b)	50. (b)
51. (a)	52. (a)	53. (c)	54. (d)	55. (d)	56. (a)	57. (d)	58. (b)	59. (a)	60. (a)

* G – Indicates One GRACE MARK awarded for the question number

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Solution

1. (c)
$\Rightarrow |x+5| \geq 10$
$\Rightarrow x+5 \leq -10$ or $x+5 \geq 10$
$\Rightarrow x \leq -15$ or $x \geq 5$
$\Rightarrow x \in (-\infty, -15] \cup [5, \infty)$

2. (b)
Let total no. of handshake = ${}^nC_2$
We know $\Rightarrow {}^nC_2 = 45$
$$\Rightarrow \frac{n!}{2!(n-2)!} = 45$$
$$\Rightarrow \frac{n(n-1)}{2} = 45$$
$\Rightarrow n^2-n-90 = 0$
$\Rightarrow n=10, -9$ (not possible)
Total no. of persons in room is 10.

3. (a)
Let T_{r+1} is constant term
$$\Rightarrow T_{r+1} = {}^{16}C_r(x^2)^{16-r}\left(\frac{1}{x^2}\right)^r$$
$$= {}^{16}C_r x^{32-4r}$$
For constant term $32-4r = 0$
$\Rightarrow 4r=32$
$\Rightarrow r=8$
$\Rightarrow T_{8+1} = {}^{16}C_8$

4. (b)
$P(1) = 2^{2\times1} - 1 = 4-1=3$
$P(2) = 2^{2\times2} - 1 = 16-1=15$
We can say 3 is only prime number which is divisible by 3 and 15.
So the value of K = 3

5. (a)
Let line which is parallel to given line is –
$\Rightarrow \quad 3x-4y+\lambda=0 \quad \ldots (1)$
This line is passing through (–2, 3)
$\Rightarrow \quad 3(-2)-4(3)+\lambda = 0$
$\Rightarrow \lambda=18$ {put in equation (1)}
$\Rightarrow$ Equation of line = $3x-4y+18 = 0$

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6. (b)

$$\Rightarrow \left(\frac{1-i}{1+i}\right)^{96}$$

$$\Rightarrow \left[\frac{(1-i)(1-i)}{(1+i)(1-i)}\right]^{96}$$

$$\Rightarrow \left[\frac{-2i}{2}\right]^{96}$$

$\Rightarrow [-i]^{96} = 1+0i = a+ib$

$\Rightarrow$ On comparing both sides

$\Rightarrow (a,b)= (1, 0)$

7. (a,d)

Distance b/w the foci of hyperbola

$\Rightarrow 2ae = 16$ $\{e=\sqrt{2}$ given$\}$

$\Rightarrow a = 4\sqrt{2}$

$\Rightarrow e^2 = 1+\frac{b^2}{a^2}$

$\Rightarrow b^2 = a^2 (e^2 -1)$

$\Rightarrow b^2 = 32(2-1) = 32$

$\Rightarrow b = 4\sqrt{2}$

Equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow \frac{x^2}{32}-\frac{y^2}{32}=1$

$\Rightarrow x^2 - y^2 = 32$

$\Rightarrow$ Equation of conjugate hyperbola is $x^2 - y^2 = -32$

Because eccentricity of C.H. is $\sqrt{2}$

8. (d)

No two are together $= 5! \times {}^6C_3 \times 3!$ (gap method)

$= 120 \times 20 \times 6$

$= 14400$

9. (d)

$2b = a+c$ ...(1)

$y^2 = xz$ ...(2)

$= x^{b-a} . y^{c-a} . z^{a-b}$

$= x^{a-b} . x^{\frac{c-a}{2}} . z^{\frac{c-a}{2}} . z^{a-b}$

$= x^{\frac{a-2b+c}{2}} . z^{\frac{a-2b+c}{2}}$

$= x^0 . z^0 = 1$

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10. (d)

LHL $\lim\limits_{x\to 0^-}\frac{-x}{x}=-1$

RHL $\lim\limits_{x\to 0^+}\frac{x}{x}=1$

$\Rightarrow$ LHL $\neq$ RHL

$\Rightarrow$ limit does not exist

11. (b)

f (x) = $x-\frac{1}{x}$

On differentiating

f ’(x) = $1+\frac{1}{x^2}$

f ‘ (–1) = $1+\frac{1}{1}=2$

12. (a)

p : 72 is divisible by 2

q : 72 is divisible by 3

$\sim$ p: 72 is not divisible by 2

$\sim$ q : 72 is not divisible by 3

$\Rightarrow$ $\sim (p\wedge q) = \sim p \vee \sim q$

$\Rightarrow$ 72 is not divisible by 2 or 72 is not divisible by 3

13. (c)

P(A) = 0.5

P(B) = 0.3

P $(A\cap B)$ = 0

$\Rightarrow$ P $(A\cup B)$ = P (A) + P (B)–P $(A\cap B)$

= 0.5+0.3–0 = 0.8

$\Rightarrow$ P $(A\cup B)'$ = 1–P $(A\cup B)$

= 1-0.8 = 0.2

14. (c)

No. of ways of getting total more than 7

{(6,6),(6,5),(5,6),(6,4),(4,6),(6,3),(3,6),(6,2),(2,6),(5,5),(5,4),(4,5),(5,3),(3,5),(4,4)}= 15

Probability of getting total more then 7 is = $\frac{15}{36}$ = $\frac{5}{12}$

15. (a)

P (A) = $\frac{3}{5}$

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P (B) = $\frac{1}{5}$

P $(A \cap B)$ = 0

$\Rightarrow$ P $(A \cup B)$ = P (A) + P (B) –P $(A \cap B)$

$= \frac{3}{5} + \frac{1}{5} + 0$

$= \frac{4}{5} = 0.8$

16. (c)

f(x) = |x|+x

g(x) = |x|–x

f[g(x)] = f[|x|–x]

= ||x|-x| + |x|-x [$\because$ x<0]

= |-2x|+(-2x)

= –2x–2x

= –4x

17. (c)

Total no. of functions from $A \rightarrow A$ is = 6^6

Total no. of bijective functions =6!

No. of not bijective functions = 6^6–6!

18. (a)

$\Rightarrow$ f(–1) + f(2)+ f(4)

$\Rightarrow$ 3(–1) + $(2)^2$ + 2×4

$\Rightarrow$ –3+4+8 = 9

19. (b)

$\Rightarrow \sin^{-1} x + \cos^{-1} y = \frac{2\pi}{5}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \sin^{-1} y = \frac{2\pi}{5}$

$\Rightarrow \cos^{-1} x + \sin^{-1} y = \pi - \frac{2\pi}{5} = \frac{3\pi}{5}$

20. (b)

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$$\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$$

Let $\theta = \frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}$ $\quad\because\left[\theta\in\left(0,\frac{\pi}{2}\right)\right]$

$$\Rightarrow \cos 2\theta = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \sqrt{5}-\sqrt{5}\tan^2\theta = 2+2\tan^2\theta$$

$$\Rightarrow \frac{\sqrt{5}-2}{\sqrt{5}+2} = \tan^2\theta$$

$$\Rightarrow \tan^2\theta = \left(\sqrt{5}-2\right)^2$$

$$\Rightarrow \tan\theta = \pm\left(\sqrt{5}-2\right) \qquad \because \theta\in\left(0,\frac{\pi}{2}\right)$$

$$\therefore \tan\theta = \sqrt{5}-2$$

21. (d)

$$A = 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^2 = 2^2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 2^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2^2 A$$

$A^3 = A.A^2$

$$= 2^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 2^4 A$$

Now check from option by putting n = 2 & 3

22. (d)

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$\Rightarrow x + y = 2$ ... (i)

$-x + y = 4$ ... (ii)

On solving equation (i) & (ii)

We get

$x = -1$

$y = 3$

23. (d)

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$$AA' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

24. (d)

$C_1 \to C_1 - C_2$

$$= \begin{vmatrix} 4 & (5^x - 5^{-x})^2 & 1 \\ 4 & (6^x - 6^{-x})^2 & 1 \\ 4 & (7^x - 7^{-x})^2 & 1 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & (5^x - 5^{-x})^2 & 1 \\ 1 & (6^x - 6^{-x})^2 & 1 \\ 1 & (7^x - 7^{-x})^2 & 1 \end{vmatrix} = 4 \times 0 = 0$$

25. (G) Bonus

26. (c)

$$2k = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2 = 16\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = 16 \times 4k^2 = 64k^2$$

27. (b)

$|5A| = 5^3\,|A| = 125\,|A|$

28. (b)

L.H.L

$$\lim_{x\to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \times \frac{\sqrt{1+kx} + \sqrt{1-kx}}{\sqrt{1+kx} + \sqrt{1-kx}}$$

$$\lim_{x\to 0^-} \frac{2kx}{x\left(\sqrt{1+kx} + \sqrt{1-kx}\right)} = \frac{2k}{2} = k$$

RHL

$$\lim_{x\to 0^+} \frac{2x+1}{x-1} = -1$$

$\Rightarrow$ L.H.L = R.H.L

$\Rightarrow k = -1$

29. (b)

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cosy =xcos(a+y)

$\Rightarrow x = \frac{\cos y}{\cos(a+y)}$

$\Rightarrow 1 = \frac{\left[-\cos(a+y)\sin y + \cos y \sin(a+y)\right]\frac{dy}{dx}}{\cos^2(a+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

30. (a)

f(x) = | cosx – sinx |

At $x = \frac{\pi}{6}$

f(x) = cosx–sinx

f'(x) = –sinx–cosx

$f'\left(\frac{\pi}{6}\right) = \frac{-1}{2} - \frac{\sqrt{3}}{2} = \frac{-1}{2}\left(1+\sqrt{3}\right)$

31. (d)

$\Rightarrow y = \sqrt{x+y}$

$\Rightarrow y^2 - y - x = 0$

$\Rightarrow 2y\frac{dy}{dx} - \frac{dy}{dx} - 1 = 0$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$

32. (b)

$\Rightarrow \lim_{x\to 1}\frac{\log_e x}{x-1}\left(\frac{0}{0}\right)$

Using L-Hospital Rule

$\Rightarrow \lim_{x\to 1}\frac{\frac{1}{x}}{1} = 1$

$\Rightarrow$LHL = RHL = f (1)

$\Rightarrow$1=k

33. (a)

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$$\Delta x = \frac{3}{100} \times x$$

$= .03x$

$$\Rightarrow \Delta v = \frac{dv}{dx} \times \Delta x$$

$\Delta v = 3x^2 \times .03x$

$= .09\ x^3$

34. (c)

$$y = \frac{1}{x^x}$$

$$\frac{dy}{dx} = -x^{-x}\left(1 + \log_e^x\right) = 0$$

$= x = 1/e$ (point of maxima)

$$y_{max} = (e)^{1/e}$$

35. (b)

$f(x) = x^x$

$f'(x) = x^x(1 + \log x) = 0$

$\Rightarrow x = 1/e$ (stationary point)

36. (c)

$(x+1)^2 + (y-3)^2 = 64$

$\Rightarrow x^2 + x^2 = (16)^2$

$\Rightarrow 2x^2 = 16.16$

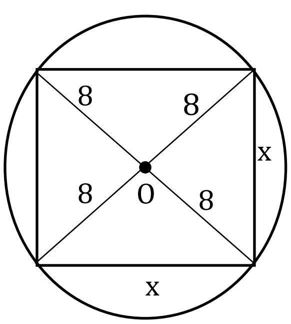
$\Rightarrow x^2 = 8.16$

$\Rightarrow x = 8\sqrt{2}$

Given rectangle is a square

$\Rightarrow$ Area of square is $= 8\sqrt{2} \times 8\sqrt{2}$

$= 128$ sq. units



37. (c)

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$\Rightarrow \int \frac{1}{1+e^x} dx$

$\Rightarrow \int \frac{e^{-x}}{1+e^{-x}} dx$

$\because$ Let $1+e^{-x} = t$

$-e^{-x}dx = dt$

$\Rightarrow -\int \frac{dt}{t}$

$\Rightarrow -\ell nt + c$

$\Rightarrow +\ell n\left|\frac{1}{1+e^{-x}}\right| + c$

$\Rightarrow \ell n\left(\frac{e^x}{1+e^x}\right) + c$

38. (c)

$\int \frac{1}{\sqrt{3-6x-9x^2}} dx$

$\Rightarrow \int \frac{1}{\sqrt{4-(3x+1)^2}}$

$\Rightarrow \frac{1}{3}\sin^{-1}\left(\frac{3x+1}{2}\right) + c$

39. (a)

$\int e^{\sin x}\left(\frac{\sin x+1}{\sec x}\right) dx$

Let $\sin x = t$

$\cos x dx = dt$

$\Rightarrow \int e^t(t+1)dt$

$\Rightarrow te^t + c$

$\Rightarrow (\sin x)e^{\sin x} + c$

40. (a)

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$$\int_{-2}^{2} |x\cos(\pi x)|\,dx$$

$$\Rightarrow 2\int_{0}^{2} |x\cos(\pi x)|\,dx \qquad \because f(-x) = f(x)$$

$$\Rightarrow f(x) = \begin{cases} x\cos(\pi x) & ; \quad 0 \le x \le \frac{1}{2} \\ -x\cos(\pi x) & ; \quad \frac{1}{2} < x \le \frac{3}{2} \\ x\cos(\pi x) & ; \quad \frac{3}{2} < x \le 2 \end{cases}$$

$$\Rightarrow 2\left[\int_{0}^{1/2} x\cos(\pi x)dx - \int_{1/2}^{3/2} x\cos(\pi x) - \int_{3/2}^{2} x\cos(\pi x)dx\right]$$

$$\Rightarrow 2\left[\frac{x}{\pi}\sin\pi x + \frac{1}{\pi^2}\cos\pi x\right]_0^{1/2} - 2\left[\frac{x}{\pi}\sin\pi x + \frac{1}{\pi^2}\cos\pi x\right]_{1/2}^{3/2} + 2\left[\frac{x}{\pi}\sin\pi x + \frac{1}{\pi^2}\cos\pi x\right]_{3/2}^{2}$$

$$\Rightarrow 2\left[\frac{1}{\pi} + \frac{3}{\pi}\right] = \frac{8}{\pi}$$

41. (b)

$$\Rightarrow \int_0^1 \frac{dx}{e^x + e^{-x}}$$

$$\Rightarrow \int_0^1 \frac{e^x dx}{1 + e^{2x}}$$

Let $e^x = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int_1^e \frac{dt}{1+t^2}$$

$$\Rightarrow \tan^{-1} t \,|_1^e$$

$$\Rightarrow \tan^{-1} e - \frac{\pi}{4}$$

42. (a)

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$$\Rightarrow \int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$$

$$\Rightarrow \text{Let } x = \frac{1}{t} \Rightarrow dx = \frac{-1}{t^2}dt$$

$$\Rightarrow -\int_{\infty}^{2} \frac{1/t^2 dt}{\dfrac{(1+t^2)\sqrt{t^2-1}}{t^3}}$$

$$\Rightarrow \int_2^{\infty} \frac{t\,dt}{(1+t^2)\sqrt{t^2-1}}$$

Let $t^2 - 1 = z^2 \Rightarrow t^2 = z^2 + 1$

$2tdt = 2zdz$

$$\Rightarrow \int_{\sqrt{3}}^{\infty} \frac{zdz}{(z^2+2)z}$$

$$\Rightarrow \int_{\sqrt{3}}^{\infty} \frac{dz}{2+z^2}$$

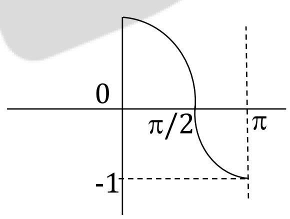
$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right)\Bigg|_{\sqrt{3}}^{\infty} =$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{\pi}{2} - \tan^{-1}\frac{\sqrt{3}}{\sqrt{2}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cot^{-1}\frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{2}}{2}\tan^{-1}\sqrt{\frac{2}{3}} \Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\sqrt{\frac{2}{3}}$$

43. (c)



$$\Rightarrow A = \int_0^{\pi/2} \cos x dx + \left|\int_{\pi/2}^{\pi} \cos x dx\right| \Rightarrow A = \sin x\Big|_0^{\pi/2} + \left|\sin x\Big|_{\pi/2}^{\pi}\right|$$

$\Rightarrow A = 1 + |-1| = 2$ sq. units

44. (b)

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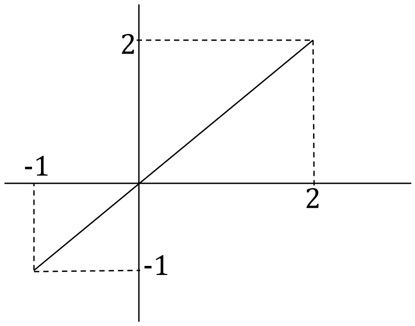

$\Rightarrow A = \left|\frac{1}{2}(-1)(-1)\right| + \frac{1}{2}\times 2\times 2$

$\Rightarrow A = \frac{1}{2} + 2$

$\Rightarrow A = \frac{5}{2}$

45. (b)

$\Rightarrow \frac{d^2y}{dx^2} = \left|1+\left(\frac{dy}{dx}\right)^2\right|^{1/3}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^3 = 1+\left(\frac{dy}{dx}\right)^2$

$\Rightarrow$ Order = 2

degree = 3

46. (a)

$\Rightarrow x\frac{dy}{dx} - y = 3$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = \frac{3}{x}$ (Linear equation)

I.F = $e^{-\int \frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$\Rightarrow y\times$ I.F = $\int \frac{3}{x}\times \text{I.F}dx$

$\Rightarrow \frac{y}{x} = \int \frac{3}{x^2}dx$

$\Rightarrow \frac{y}{x} = \frac{-3}{x} + c$

$\Rightarrow y = -3+xc$ (straight line)

47. (c)

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$\Rightarrow \frac{dy}{dx}+y=\frac{1+y}{x}$

$\Rightarrow \frac{dy}{dx}+y-\frac{y}{x}=\frac{1}{x}$

$\Rightarrow \frac{dy}{dx}+y\left(1-\frac{1}{x}\right)=\frac{1}{x}$

I.F = $e^{\int\left(1-\frac{1}{x}\right)dx}=e^{x-\ell nx}=e^{x}.\frac{1}{x}$

48. (c)

$\Rightarrow |\vec{a}\times\vec{b}|^2+|\vec{a}.\vec{b}|^2=144$

$\Rightarrow |\vec{a}|^2|\vec{b}|^2=144$

$\Rightarrow |\vec{b}|^2=\frac{144}{16}=9$

$\Rightarrow |\vec{b}| = 3$

49. (b)

Given –

$|\vec{a}|=1$

$|\vec{b}|=1$

$\vec{a}.\vec{b}=0$

$\Rightarrow (3\vec{a}+2\vec{b}).(5\vec{a}-6\vec{b})$

$\Rightarrow 15-18(\vec{a}.\vec{b})+10(\vec{a}.\vec{b})-12$

$\Rightarrow$15-12 = 3

50. (b)

$\because$ Vector one coplanar

$\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

$\Rightarrow$a(bc−1)−(c−1)+(1−b) = 0

$\Rightarrow$abc−(a+b+c) = −2

51. (a)

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$\Rightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow \mu + \lambda = 2 \qquad \ldots (1)$

$\Rightarrow |\vec{a}| = |\vec{b}|$

$\Rightarrow \sqrt{1+\lambda^2+4} = \sqrt{\mu^2+1+1}$

$\Rightarrow \lambda^2 + 3 = \mu^2 \qquad \ldots (2)$

Put equation (1) in equation (2)

$\Rightarrow \lambda^2 + 3 = (2-\lambda)^2$

$\Rightarrow 3 = 4 - 4\lambda$

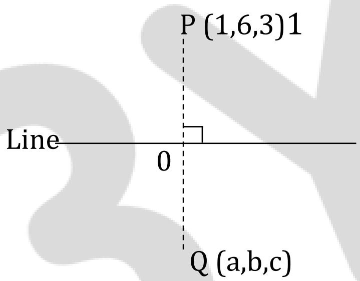
$\Rightarrow \lambda = \frac{1}{4}$

$\mu = 2 - \frac{1}{4} = \frac{7}{4}$ {using eq(1)}

52. (a)

$\overrightarrow{PQ}$ d.r is = ((a–1), (b–6), (c–3))

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = t$



x=t, y=2t+1, z=3t+2 $\qquad$ ...(1)

$\vec{O} \equiv \langle t, 2t+1, 3t+2 \rangle$

$\because \overrightarrow{OP} \perp^r$ dr of line

$\Rightarrow$ (1–t).1+2(6–2t–1)+3(3–3t–2)=0

$\Rightarrow$ 1–t + 10–4t+3-9t = 0

$\Rightarrow$ 14t = 14 $\Rightarrow$ t=1

$\vec{O} = \langle 1,3,5 \rangle$

$\vec{Q} = \langle 2-1, 6-6, 10-3 \rangle$

$= \langle 1,0,7 \rangle$

53. (c)

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$\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ ... (1)

$\frac{x}{-4}=\frac{y}{24}=\frac{z}{6}$ ... (2)

$\Rightarrow \cos\theta=\frac{-12+48-36}{\sqrt{9+4+36}\sqrt{16+24^2+36}}$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta=\frac{\pi}{2}$

54. (d)

Point (4,2,k) of line will be on plane. So point will satisfy the plane.

$\Rightarrow 2(4)-4(2)+k = 7$

$\Rightarrow k = 7$

55. (d)

$\Rightarrow y(x+z) = 0$

$\Rightarrow y=0$ or $x+z=0$

p_1: $y = 0$ $\Rightarrow \vec{n}_1=\hat{j}$

p_2: $x+z = 0$ $\Rightarrow \vec{n}_2=\hat{i}+\hat{k}$

$\Rightarrow \vec{n}_1.\vec{n}_2=\hat{j}\left(\hat{i}+\hat{k}\right)=0$

$\Rightarrow \vec{n}_1. \perp^r \vec{n}_2$

$\Rightarrow$ a pair of perpendicular planes.

56. (a)

Z=3x+9y

Z(0,10) = 90

Z(5,5) = 60

Z(0,20) = 180

Z(15,15) = 180

Z_{min} = 60 at (5,5)

57. (d)

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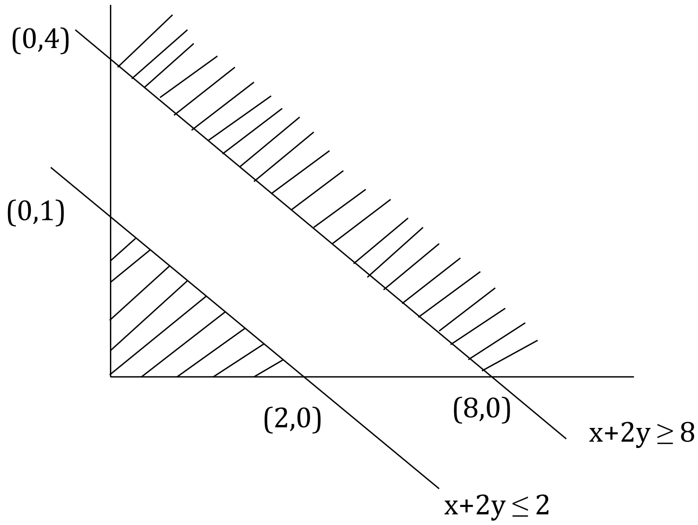
$L_1 = x+2y \leq 2$

$L_2 = x+2y \geq 8$

Has no feasible solution



58. (b)

$M(x) = \sum p_i x_i$

$= 0\times\frac{25}{36}+1\times\frac{5}{18}+2\times\frac{1}{36}=\frac{6}{18}=\frac{1}{3}$

$M(x^2) = \sum p_i x_i^2$

$=0^2\times\frac{25}{36}+1^2\times\frac{5}{18}+2^2\times\frac{1}{36}=\frac{7}{18}$

Variance (x) = $M(x^2)-[M(x)]^2$

$= \frac{7}{18}-\frac{2}{18}=\frac{5}{18}$

$\sigma=\sqrt{Var(x)}$

$\sigma=\sqrt{\frac{5}{18}}$

$\sigma=\frac{1}{3}\sqrt{\frac{5}{2}}$

59. (a)

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In bag $\rightarrow$ 1,2,3, ……………….17 $\leftarrow$ tickets

Even no $\rightarrow$ 2,4,6,8,10,14,16 $\leftarrow$8 even numbers

Probability at getting even no. at first time = $\frac{8}{17}$

Probability of getting even no. at second time = $\frac{7}{16}$

$\Rightarrow$Probability that both the ticket may show

Even numbers = $\frac{8}{17}\times\frac{7}{16}=\frac{7}{34}$

60. (a)

Total batteries =10

Dead batteries = 4

Probability of battery to be dead = $\frac{4}{10}$

Probability that all 3 batteries are dead

$= \frac{4}{10}\times\frac{{}^{3}C_{1}}{{}^{9}C_{1}}\times\frac{{}^{2}C_{1}}{{}^{8}C_{1}}$

$= \frac{4}{10}\times\frac{3}{9}\times\frac{2}{8}=\frac{1}{30}$

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