KCET-2019 (Mathematics)

1. If $\sqrt[3]{y}\sqrt{x}=\sqrt[6]{(x+y)^5}$, then $\frac{dy}{dx}=$

   a. $\frac{x}{y}$
   b. x+y
   c. x-y
   d. $\frac{y}{x}$.

2. Rolle's theorem is not applicable in which one of the following cases?

   a. $f(x)=x^2-4x+5$ in [1,3]
   b. $f(x)=x^2-x$ in [0,1]
   c. $f(x)=|x|$ in [-2,2]
   d. $f(x)=[x]$ in [2.5,2.7]

3. The interval in which the function f(x) $=x^3-6x^2+9x+10$ is increasing in

   a. $(-\infty,1)\cup(3,\infty)$
   b. $(-\infty,-1]\cup(-3,\infty)$
   c. [1,3]
   d. $(-\infty,-1)\cup[3,\infty)$

4. The side of an equilateral triangle are increasing at the rate of 4cm/sec. The rate at which its area is increasing, when the side is 14 cm

   a. $10\sqrt{3}\,cm^2$ / sec
   b. $14\sqrt{3}\,cm^2$ / sec
   c. $42\,cm^2$ / sec
   d. $14\,cm^2$ / sec

5. The value of $\sqrt{24.99}$ is

   a. 4.999
   b. 4.899
   c. 5.001
   d. 4.897

6. If $|3x-5|\le 2$ then

   a. $-1\le x\le\frac{7}{3}$
   b. $1\le x\le\frac{7}{3}$
   c. $1\le x\le\frac{9}{3}$
   d. $-1\le x\le\frac{9}{3}$

7. A random variable 'X' has the following probability distribution:

X	1	2	3	4	5	6	7
P(X)	k-1	3k	k	$3k$	$3k^2$	k^2	k^2+k

   a. $\frac{1}{5}$
   b. -2
   c. $\frac{2}{7}$
   d. $\frac{1}{10}$

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8. If A and B are two events of a sample space S such that P(A) =0.2, P(B) =0.6 and P(A|B)=0.5 then P(A'|B)=
   a. $\frac{3}{10}$
   b. $\frac{2}{3}$
   c. $\frac{1}{2}$
   d. $\frac{1}{3}$

9. If 'X' has a binomial distribution with parameters n=6, p and P(X=2) = 12, P(X=3) =5 then P=
   a. $\frac{5}{12}$
   b. $\frac{16}{21}$
   c. $\frac{1}{2}$
   d. $\frac{5}{16}$

10. A man speaks truth 2 out of 3 times. He picks one of the natural numbers in the set $S=\{1,2,3,4,5,6,7\}$ and reports that it is even. The probability that it is actually even is
   a. $\frac{2}{5}$
   b. $\frac{1}{5}$
   c. $\frac{1}{10}$
   d. $\frac{3}{5}$

11. If U is the universal set with 100 elements; A and B are two sets such that n(A) = 50, n(B) =60, $n(A\cap B)=20$ then n $n(A'\cap B')=?$
   a. 40
   b. 20
   c. 90
   d. 10

12. The domain of the function $f:R\to R$ defined by $f(x)=\sqrt{x^2-7x+12}$
   a. $(-\infty,3]\cup[4,\infty)$
   b. $(-\infty,3]\cup(4,\infty)$
   c. $(-\infty,3]\cap[4,\infty)$
   d. (3,4)

13. If cos x =|sin x| then, the general solution is
   a. $x=n\pi\pm\frac{\pi}{4},n\in Z$
   b. $x=2n\pi\pm\frac{\pi}{4},n\in Z$
   c. $x=n\pi+(-1)^n\frac{\pi}{4},n\in Z$
   d. $x=(2n+1)\pi\pm\frac{\pi}{4},n\in Z$

14. $\sqrt{3}\ \cos ec\,20^0-\sec 20^0=$
   a. 2
   b. 3
   c. 4
   d. 1

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15. If $P(n): 2^n < n!$ then the smallest positive integer for which P(n) is true, is

a. 2
b. 3
c. 4
d. 5

16. Foot of the perpendicular drawn from the point (1,3,4) to the plane 2x-y+z+3=0 is

a. (-1,4,3)
b. (0,-4,-7)
c. (1,2,-3)
d. (-3,5,2)

17. Acute angle between the line $\frac{x-5}{2}=\frac{y+1}{-1}=\frac{z+4}{1}$ And the plane 3x-4y-z+5=0 is

a. $\cos^{-1}\left(\frac{9}{\sqrt{364}}\right)$
b. $\sin^{-1}\left(\frac{9}{\sqrt{364}}\right)$
c. $\cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$
d. $\sin^{-1}\left(\frac{5}{2\sqrt{13}}\right)$

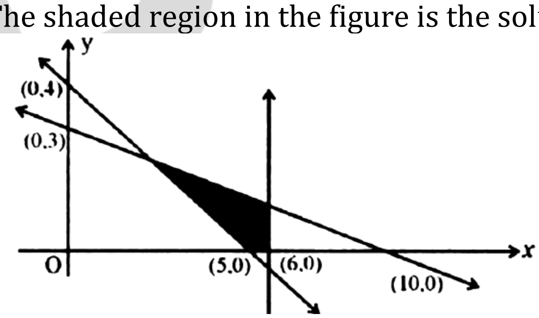
18. The distance of the point (1,2,1) form the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$ is

a. $\frac{2\sqrt{3}}{5}$
b. $\frac{2\sqrt{5}}{3}$
c. $\frac{\sqrt{5}}{3}$
d. $\frac{20}{3}$

19. XY-plane divides the line joining the points A(2,3,-5) and B(-1,-2,-3) in the ratio

a. 2:1 internally
b. 3:2 externally
c. 5:3 internally
d. 5:3 externally

20. The shaded region in the figure is the solution set of the inequations



a. $4x+5y \ge 20, \quad 3x+10y \le 30, x \le 6, \quad x, y \ge 0$.
b. $4x+5y \ge 20, \quad 3x+10y \le 30, x \ge 6, \quad x, y \ge 0$.
c. $4x+5y \le 20, \quad 3x+10y \le 30, x \le 6, \quad x, y \ge 0$.
d. $4x+5y \le 20, \quad 3x+10y \le 30, x \ge 6, \quad x, y \ge 0$.

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21. The order of the differential equation $y = C_1 e^{C_2 + x} + C_3 e^{C_4 + x}$ is

a. 1
b. 2
c. 3
d. 4

22. If $|\vec{a}| = 16, |\vec{b}| = 4$ then, $\sqrt{|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2} =$

a. 4
b. 8
c. 16
d. 64

23. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$ and the projection of $\vec{a}$ in the direction of $\vec{b}$ is -2, then $|\vec{a}| =$

a. 4
b. 3
c. 2
d. 1

24. A unit vector perpendicular to the plane containing the vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-2\hat{i} + \hat{j} + 3\hat{k}$ is

a. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$
b. $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$
c. $\frac{-\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$
d. $\frac{-\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

25. $\left[\vec{a} + 2\vec{b} - \vec{c}, \vec{a} - \vec{b}, \vec{a} - \vec{b} - \vec{c}\right] =$

a. 0
b. $\left[\vec{a}, \vec{b}, \vec{c}\right]$
c. $2\left[\vec{a}, \vec{b}, \vec{c}\right]$
d. $3\left[\vec{a}, \vec{b}, \vec{c}\right]$

26. $\int_{-3}^{3} \cot^{-1} x \, dx =$

a. 3π
b. 0
c. 6π
d. 3

27. $\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx =$

a. $2\log\left(\sqrt{x} + 1\right) + C$
b. $\frac{1}{2}\tan^{-1}\sqrt{x} + C$
c. $\tan^{-1}\sqrt{x} + C$
d. $2\tan^{-1}\sqrt{x} + C$

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28. $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx = A \log|x-1| + B \log|x+2| + C \log|x-3| + K$, then A, B ,C are respectively

a. $\frac{-1}{6}, \frac{1}{3}, \frac{-1}{2}$
b. $\frac{1}{6}, \frac{1}{3}, \frac{-1}{5}$
c. $\frac{1}{6}, \frac{-1}{3}, \frac{1}{3}$
d. $\frac{-1}{6}, \frac{-1}{3}, \frac{1}{2}$

29. $\int_0^2 \left[x^2\right] dx =$

a. $5-\sqrt{2}-\sqrt{3}$
b. $5+\sqrt{2}-\sqrt{3}$
c. $5-\sqrt{2}+\sqrt{3}$
d. $-5-\sqrt{2}-\sqrt{3}$

30. $\int_0^1 \sqrt{\frac{1+x}{1-x}} dx =$

a. $\frac{\pi}{2}-1$
b. $\frac{\pi}{2}+1$
c. $\frac{\pi}{2}$
d. $\frac{1}{2}$

31. If α and β are roots of the equation $x^2+x+1=0$ then $\alpha^2+\beta^2$ is

a. 1
b. $\frac{-1+i\sqrt{3}}{2}$
c. $\frac{-1-i\sqrt{3}}{2}$
d. -1

32. The number of 4-digit numbers without repetition that can be formed using the digits 1,2,3,4,5,6,7 in which each number has two odd digits and two even digits is

a. 432
b. 436
c. 450
d. 454

33. The number of terms in the expansion of $\left(x^2+y^2\right)^{25}-\left(x^2-y^2\right)^{25}$ after simplification is

a. 0
b. 13
c. 26
d. 50

34. The third term of a G.P. is 9. The product of its first five terms is

a. 3^5
b. 3^9
c. 3^{10}
d. 3^{12}

35. A line cuts off equal intercepts on the co-ordinate axes. The angle made by this line with the positive direction of X-axis is

a. 45^0
b. 90^0
c. 120^0
d. 135^0

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36. $\int x^3 \sin 3x \, dx =$

a. $-\frac{x^3\cos 3x}{3}-\frac{x^2\sin 3x}{3}+\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

b. $\frac{x^3\cos 3x}{3}+\frac{x^2\sin 3x}{3}-\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

c. $-\frac{x^3\cos 3x}{3}+\frac{x^2\sin 3x}{3}+\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

d. $-\frac{x^3\cos 3x}{3}+\frac{x^2\sin 3x}{3}-\frac{2x\cos 3x}{9}-\frac{2\sin 3x}{27}+C$

37. The area of the region above X-axis included between the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2x$ in square units is

a. $\frac{\pi}{4}-\frac{3}{2}$

b. $\frac{3}{2}-\frac{\pi}{4}$

c. $\frac{2}{3}-\frac{\pi}{4}$

d. $\frac{\pi}{4}-\frac{2}{3}$

38. The area of the region bounded by Y-axis, y = cos x and y = sin x; $0 \le x \le \frac{\pi}{2}$ is

a. $\sqrt{2}-1$ Sq. units

b. $\sqrt{2}$ Sq. units

c. $\sqrt{2}+1$ Sq. units

d. $2-\sqrt{2}$ Sq. units

39. The integrating factor of the differential equation $(2x + 3y^2)\, dy = y\, dx$ $(y > 0)$ is

a. $e^{\frac{1}{y}}$

b. $-\frac{1}{y^2}$

c. $\frac{1}{x}$

d. $\frac{1}{y^2}$

40. The equation of the curve passing through the point (1,1) such that the slope of the tangent at any point (x,y) is equal to the product of its coordinates is

a. $2\log x = y^2 - 1$

b. $2\log y = x^2 + 1$

c. $2\log y = x^2 - 1$

d. $2\log x = y^2 + 1$

41. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is

a. $\frac{4}{5}$

b. $\frac{3}{5}$

c. $\frac{3}{4}$

d. $\frac{9}{16}$

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42. $\sum_{r=1}^{n}(2r-1)=x$ then $\lim_{n\to\infty}\left[\frac{1^3}{x^2}+\frac{2^3}{x^2}+\frac{3^3}{x^2}+.........+\frac{n^3}{x^2}\right]=$

a. $\frac{1}{2}$
b. $\frac{1}{4}$
c. 1
d. 4

43. The negation of the statement ''All continuous functions are differentiable.''

a. All continuous functions are not differentiable
b. Some continuous functions are differentiable.
c. Some continuous functions are not differentiable.
d. All differentiable functions are continuous

44. Mean and standard deviation of 100 items are 50 and 4 respectively. The sum of all squares of the items is

a. 251600
b. 256100
c. 266000
d. 261600

45. Two letters are chosen from the letters of the word 'EQUATIONS'. The probability that one is vowel and the other is consonant is

a. $\frac{8}{9}$
b. $\frac{4}{9}$
c. $\frac{3}{9}$
d. $\frac{5}{9}$

46. The constant term in the expansion of $\begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$ is

a. 0
b. 2
c. -10
d. 6

47. If [x] represents the greatest integer function and f(x) = x - [x] – cos x then $f'\left(\frac{\pi}{2}\right)=$

a. 0
b. 1
c. 2
d. does not exist

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48. If $f(x)=\begin{cases}\frac{\sin 3x}{e^{2x}-1} & ; \; x\neq 0\\ k-2 & ; \; x=0\end{cases}$ *is*

Continuous at x=0, then k=

a. $\frac{3}{2}$

b. $\frac{9}{5}$

c. $\frac{1}{2}$

d. $\frac{2}{3}$

49. If f(x) = $\sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$, then f'(0) =

a. 2 log 2

b. log 2

c. $\frac{2\log 2}{5}$

d. $\frac{4\log 2}{5}$

50. If x= a $\sec^2\theta, y=a\tan^2\theta$ then $\frac{d^2y}{dx^2}=$

a. 2a

b. 1

c. 0

d. 4

51. The inverse of the matrix $\begin{bmatrix}2 & 5 & 0\\ 0 & 1 & 1\\ -1 & 0 & 3\end{bmatrix}$ *is*

a. $\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$

b. $\begin{bmatrix}3 & -5 & 5\\ -1 & -6 & -2\\ 1 & -5 & 2\end{bmatrix}$

c. $\begin{bmatrix}3 & -15 & 5\\ -1 & 6 & -2\\ 1 & -5 & 2\end{bmatrix}$

d. $\begin{bmatrix}3 & -15 & 5\\ -1 & 6 & -2\\ 1 & -5 & -2\end{bmatrix}$

52. If P and Q are symmetric matrices of the same order then PQ-QP is

a. Identity matrix

b. Symmetric matrix

c. zero matrix

d. Skew symmetric matrix

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53. If 3A+4B' = $\begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix}$ and 2B-3A' = $\begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}$ then B=

a. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$

c. $\begin{bmatrix} -1 & -18 \\ 4 & -16 \\ -5 & -7 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & -4 \end{bmatrix}$

54. If $A=\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}, B=\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, then |ABB'|=

a. 50

b. -250

c. 100

d. 250

55. If the value of a third order determinant is 16, then the value of the determinant formed by replacing each of its elements by its cofactor is

a. 96

b. 48

c. 256

d. 16

56. f : R $\rightarrow$R and g: $[0,\infty)\rightarrow R$ is defined by f(x) = x^2 and g(x) = $\sqrt{x}$. Which one of the following is not true ?

a. gof (4) = 4

b. Fog (-4)=4

c. Fog (2) =2

d. gof (-2) =2

57. $A=\{x \mid x\in N, x\le 5\}, B=\{x \mid x\in Z, x^2-5x+6=0\}$, then the number of onto functions from A to B is

a. 2

b. 23

c. 30

d. 32

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58. On the set of positive rationals, a binary operation * is defined by a*b=$\frac{2ab}{5}$. If 2* x=3^{-1} then x=

a. $\frac{1}{6}$

b. $\frac{5}{12}$

c. $\frac{2}{5}$

d. $\frac{125}{48}$

59. $\cos\left[2\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right]=$

a. $\frac{-3}{4}$

b. $\frac{3}{4}$

c. $\frac{3}{5}$

d. does not exist

60. If a+$\frac{\pi}{2} < 2\tan^{-1}x + 3\cot^{-1}x < b$ then 'a' and 'b' are respectively.

a. 0 and π

b. $\frac{\pi}{2}$ and 2π

c. 0 and 2π

d. $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

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ANSWER KEYS

1. (d)	2. (c)	3. (b)	4. (G)	5. (a)	6. (b)	7. (a)	8. (c)	9. (G)	10. (d)
11. (d)	12. (a)	13. (b)	14. (c)	15. (c)	16. (a)	17. (c)	18. (b)	19. (d)	20. (a)
21. (a)	22. (d)	23. (a)	24. (c)	25. (d)	26. (a)	27. (d)	28. (d)	29. (a)	30. (b)
31. (d)	32. (a)	33. (b)	34. (c)	35. (d)	36. (c)	37. (d)	38. (a)	39. (d)	40. (c)
41. (a)	42. (b)	43. (c)	44. (a)	45. (d)	46. (d)	47. (c)	48. (G)	49. (b)	50. (c)
51. (c)	52. (d)	53. (a)	54. (b)	55. (c)	56. (b)	57. (c)	58. (d)	59. (a)	60. (b)

* G – Indicates One GRACE MARK awarded for the question number

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KCET-2019 (Mathematics)

SOLUTIONS

1. (d)

$\sqrt[3]{y} \cdot \sqrt{x} = \sqrt[6]{(x+y)^5}$

Apply log both side

$\Rightarrow \log (y^{1/3} \cdot x^{1/2}) = \log (x + y)^{5/6}$

$\Rightarrow \frac{1}{3} \log y + \frac{1}{2} \log x = \frac{5}{6} \log (x + y)$

$\Rightarrow \frac{1}{3y} . \frac{dy}{dx} + \frac{1}{2x} = \frac{5}{6(x+y)} . \left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \frac{1}{3y} \frac{dy}{dx} - \frac{5}{6(x+y)} \frac{dy}{dx} = \frac{5}{6(x+y)} - \frac{1}{2x}$

$\Rightarrow \frac{dy}{dx} \left[\frac{1}{3y} - \frac{5}{6(x+y)}\right] = \frac{5x-3(x+y)}{6x(x+y)}$

$\Rightarrow \frac{dy}{dx} = \left[\frac{2x-3y}{6x(x+y)}\right] \left[\frac{6y(x+y)}{2x-3y}\right]$

$\frac{dy}{dx} = \frac{y}{x}$

2. (c)

Check option:

Option (a):

(1) f(x) is continuous function in [1, 3]

(2) f(x) is differentiable in (1, 3)

Option (b):

(1) f(x) is continuous function in [0, 1]

(2) f(x) is differentiable in (0, 1)

Option (c):

(1) f(x) is continuous function in [–2, 2]

(2) f(x) is not differentiable at point x = 0 in (–2, 2)

Option (d):

(1) f(x) is continuous function in [2.5, 2.7]

(2) f(x) is differentiable in (2.5, 2.7)

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3. (b)

$F(x) = x^3 - 6x^2 + 9x + 10$

Differentiate with respect to

$F(x) = 3x^2 - 12x + 9$

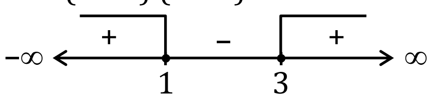
$f'(x) \geq 0$

$\Rightarrow \quad 3x^2 - 12x + 9 \geq 0$

$\Rightarrow \quad x^2 - 4x + 3 \geq 0$

$\Rightarrow \quad x^2 - 3x - x + 3 \geq 0$

$\Rightarrow \quad (x-3)(x-1) \geq 0$



$x \in (-\infty, 1] \cup [3, \infty)$

4. (G)

Bonus

5. (a)

$f(x) = \sqrt{x}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$

$\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x \qquad \ldots (1)$

Here $x = 25$ and $\Delta x = 0.1$

$\Delta y = \sqrt{x} - \sqrt{(x - \Delta x)}$

$\dfrac{dy}{dx}(\Delta x) = \sqrt{25} - \sqrt{25 - 0.1}$

$\sqrt{24.99} = 5 - \dfrac{1}{2 \times 5} \times 0.1$

$= 5 - 0.01$

$= 4.99$

6. (b)

$|3x - 5| \leq 2$

$\Rightarrow \quad -2 \leq 3x - 5 \leq 2$

$\Rightarrow \quad 3 \leq 3x \leq 7$

$\Rightarrow \quad 1 \leq x \leq \dfrac{7}{3}$

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7. (a)

$\Sigma P(K) = 1$

$\Rightarrow K - 1 + 3K + K + 3K + 3K^2 + K^2 + K^2 + K = 1$

$\Rightarrow 5K^2 + 9K - 2 = 0$

$\Rightarrow 5K^2 + 10K - K - 2 = 0$

$\Rightarrow (5K - 1)(K + 2) = 0$

$5K - 1 = 0$	$K + 2 = 0$
$K = \frac{1}{5}$	$K = -2$ (not possible)

8. (c)

Given that

$P(A) = 0.2$

$P(B) = 0.6$

$P(A|B) = 0.5$

Now

$P(A'|B) = 1 - P(A|B)$

$= 1 - 0.5$

$= 0.5 = \frac{1}{2}$

9. (G)

Bonus

10. (d)

$S = \{1, 2, 3, 4, 5, 6, 7\}$

Total number = 7

$P(S_1)$ = Probability of even number $= \frac{3}{7}$

$P(S_2)$ = Probability of odd number $= \frac{4}{7}$

$P(E|S_1) = \frac{2}{3}$

$P(E|S_2) = 1 - \frac{2}{3} = \frac{1}{3}$

$$P(S_1|E) = \frac{P(S_1)P(E|S_1)}{P(S_1)P(E|S_1) + P(S_2)P(E|S_2)}$$

$$= \frac{\frac{3}{7}\times\frac{2}{3}}{\frac{3}{7}\times\frac{2}{3}+\frac{4}{7}\times\frac{1}{3}} = \frac{3}{5}$$

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11. (d)

Given that

$n(A) = 50$

$n(B) = 60$

$n(A \cap B) = 20$

Now,

$n(A' \cap B') = P(A \cup B)'$

$= 100 - P(A \cup B)$

$= 100 - [P(A) + P(B) - P(A \cap B)]$

$= 100 - [50 + 60 - 20]$

$= 100 - 90$

$= 10$

12. (a)

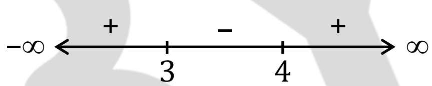
$f(x) = \sqrt{x^2 - 7x + 12}$

Now

$x^2 - 7x + 12 \geq 0$

$x^2 - 4x - 3x + 12 \geq 0$

$(x - 4)(x - 3) \geq 0$



$x \in (-\infty, 3] \cup [4, \infty)$

13. (b)

$\cos x = |\sin x|$

Case-1:

$\cos x = \sin x$ for $x \in [0, \pi]$

$\tan x = 1$ for $x = \frac{\pi}{4}$

Case-2:

$\cos x = -\sin x$ for $x \in [\pi, 2\pi]$

$\tan x = -1$ for $x \in \frac{7\pi}{4}$

Combining both the cases, we get the general is solution $2n\pi \pm \frac{\pi}{4}$

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14. (c)

$\sqrt{3}$ cosec 20° – sec 20°

$\Rightarrow \dfrac{\sqrt{3}}{\sin 20°} - \dfrac{1}{\cos 20°}$

$\Rightarrow \dfrac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$

$\Rightarrow \dfrac{4[\sin 60° \cos 20° - \cos 60° \sin 20°]}{2\sin 20° \cos 20°}$

$\Rightarrow \dfrac{4\sin(60° - 20°)}{\sin 40°}$ {$\because$ 2sinAcosA = sin2A & sinAcosB – cosAsinB = sin(A – B)}

$\Rightarrow \dfrac{4\sin 40°}{\sin 40°} = 4$

15. (c)

P(n): $2^n < n!$

Since

P(1): 2 < 1 is false.

P(2): $2^2 < 2!$ is false.

P(3): $2^3 < 3!$ is false.

P(4): $2^4 < 4!$ is true.

P(5): $2^5 < 5!$ is true.

So, smallest positive integer is 4.

16. (a)

The Dr's of PA are

$x_1 - 1, y_1 - 3, z_1 - 4$

The Dr's $\vec{n}$ are 2, –1, 1

The Dr's of PA and $\vec{n}$ are parallel

$\therefore \dfrac{x_1 - 1}{2} = \dfrac{y_1 - 3}{-1} = \dfrac{z_1 - 4}{1} = \lambda$

$x_1 = 2\lambda + 1, y_1 = -\lambda + 3, z_1 = \lambda + 4$

$\therefore$ A = $(2\lambda + 1, -\lambda + 3, \lambda + 4)$ lies on plane.

$\Rightarrow 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) = 0$

$6\lambda + 6 = 0$

$\lambda = -1$

A = [(2(–1) + 1), (+1+3), (–1+4)]

A = (–1, 4, 3)

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17. (c)

L: $\frac{x-5}{2} = \frac{y+1}{-1} = \frac{z+4}{1}$

Dr's of line: 2, –1, 1

P: 3x – 4y – z + 5 = 0

Dr's of normal to plane: –3, 4, 1

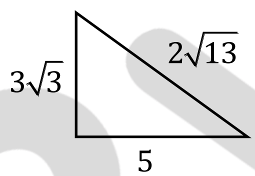
$\sin\theta = \left|\frac{-6-4+1}{\sqrt{4+1+1}\sqrt{9+16+1}}\right| = \left|\frac{-9}{\sqrt{156}}\right|$

$\sin\theta = \frac{9}{\sqrt{156}}$

$\theta = \sin^{-1}\left(\frac{9}{\sqrt{156}}\right)$

$\theta = \sin^{-1}\left(\frac{9}{2\sqrt{39}}\right)$

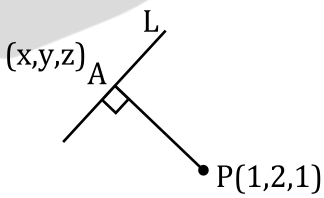
$\theta = \cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$



18. (b)

Let

L: $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$



A = [x = 2λ + 1, y = λ + 2, z = 2λ + 3]

$\overrightarrow{PA}$ = Position vector of A – position of vector of P

$= [(2\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (2\lambda + 3)\hat{k}] - [\hat{i} + 2\hat{j} + \hat{k}]$

$= (2\lambda)\hat{i} + (\lambda)\hat{j} + (2\lambda + 2)\hat{k}$

$\because \overrightarrow{PA} \perp L$

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$\Rightarrow \quad \overrightarrow{PA} \cdot L = 0$

$\Rightarrow \quad [2\lambda\hat{i} + \lambda\hat{j} + (2\lambda + 2)\hat{k}] \cdot [2\hat{i} + \hat{j} + 2\hat{k}] = 0$

$\Rightarrow \quad 4\lambda + \lambda + 4\lambda + 4 = 0$

$\lambda = -\frac{4}{9}$

$$A = \left[2\left(\frac{-4}{9}\right)+1,\ \frac{-4}{9}+2,\ 2\left(\frac{-4}{9}\right)+3\right] = \left[\frac{1}{9}, \frac{-14}{9}, \frac{19}{9}\right]$$

$$\text{Distance} = \sqrt{\left(1-\frac{1}{9}\right)^2+\left(2-\frac{14}{9}\right)^2+\left(1-\frac{19}{9}\right)^2}$$

$$= \sqrt{\frac{64}{81}+\frac{16}{81}+\frac{100}{81}} = \frac{2\sqrt{5}}{3}$$

19. (d)

A(2,3,–5), B(–1,–2,–3)

The ratio that xy plane divide line joining the points (x_1,y_1,z_1) & $(x_2,y_2,z_2) = -z_1:z_2$

If result is positive, it divides internally otherwise externally.

The ratio that xy plane divides the points (2,3–5) & (–1,–2,–3)

= – (–5) : (–3)

= –5 : 3

So, 5 : 3 externally in the ratio.

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20. (a)

(1) Since region lies above x-axis

$\therefore \quad y \geq 0$

(2) Shaded region is on right side of y-axis

$\therefore \quad x \geq 0$

(3) Equation of line passing through (5, 0) & (0, 4) is

$\Rightarrow \quad (y-0) = \frac{4}{-5}(x-5)$

$\Rightarrow \quad -5y = 4x - 20$

$\Rightarrow \quad 4x + 5y = 20$

(4) Equation of line passing through (10, 0) & (0, 3) is

$\Rightarrow \quad (y-0) = \frac{3}{-10}(x-10)$

$\Rightarrow \quad -10y = 3x - 30$

$\Rightarrow \quad 3x + 10y = 30$

(5) Shaded region is on left of x = 6

$\therefore \quad x \leq 6$

Consider the figure and observe the shaded region the inequalities can be written as

$x \geq 0$,

$y \geq 0$,

$x \leq 6$

$4x + 5y \geq 20$,

$3x + 10y \leq 30$

21. (a)

$y = C_1 e^{C_2+x} + C_3 e^{C_4+x}$

Differentiate with respect to

$\frac{dy}{dx} = C_1 e^{C_2+x} + C_3 e^{C_4+x}$

$\frac{dy}{dx} = y$

$\frac{dy}{dx} - y = 0$

Therefore, order is 1.

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22. (d)

$|\vec{a}| = 16$

$|\vec{b}| = 4$

Now,

$\sqrt{|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2} = \sqrt{|\vec{a}|^2|\vec{b}|^2\sin^2\theta + |\vec{a}|^2|\vec{b}|^2\cos^2\theta}$

$= |\vec{a}||\vec{b}|\sqrt{\sin^2\theta + \cos^2\theta}$

$= (16)\,(4)\,(1)$

$= 64$

23. (a)

Given, angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$

And $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = -2$

$\Rightarrow \frac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{b}|} = -2 \qquad \{\because \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta\}$

$\Rightarrow |\vec{a}|\cos\frac{2\pi}{3} = -2 \qquad \{\because \theta = \frac{2\pi}{3}\}$

$\Rightarrow |\vec{a}|\left(-\frac{1}{2}\right) = -2$

$\Rightarrow |\vec{a}| = 4$

24. (c)

The vector which is perpendicular to two vectors is given by the cross product of the two vectors.

$\vec{P}\times\vec{Q} = (-2\hat{i} + \hat{j} + 3\hat{k}) \times (\hat{i} + 2\hat{j} + \hat{k})$

$= -5\hat{i} + 5\hat{j} - 5\hat{k}$

$|\vec{P}\times\vec{Q}| = \sqrt{5^2+5^2+5^2} = 5\sqrt{3}$

Unit vector perpendicular to P and Q is given by

$\vec{C} = \frac{\vec{P}\times\vec{Q}}{|\vec{P}\times\vec{Q}|}$

$\vec{C} = \frac{-5\hat{i}+5\hat{j}-5\hat{k}}{5\sqrt{3}}$

$\vec{C} = \frac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

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25. (d)

$[\vec{a} + 2\vec{b} - \vec{c}, \vec{a} - \vec{b}, \vec{a} - \vec{b} - \vec{c}]$

Let $x = \vec{a} + 2\vec{b} - \vec{c}, y = \vec{a} - \vec{b}, z = \vec{a} - \vec{b} - \vec{c}$

Using properties $\{x \to x - z\}$

$\Rightarrow [3\vec{b}, \vec{a} - \vec{b}, \vec{a} - \vec{b} - \vec{c}]$

$\Rightarrow [3\vec{b}, \vec{a}, \vec{a} - \vec{b} - \vec{c}] + [3\vec{b}, -\vec{b}, \vec{a} - \vec{b} - \vec{c}]$

$\Rightarrow [3\vec{b}, \vec{a}, \vec{a} - \vec{b} - \vec{c}] + 3\vec{b}[(-\vec{b}) \times (\vec{a} - \vec{b} - \vec{c})]$

$\Rightarrow [3\vec{b}, \vec{a}, \vec{a} - \vec{b} - \vec{c}] + 0$

$\Rightarrow [3\vec{b}, \vec{a}, \vec{a}] + [3\vec{b}, \vec{a}, -\vec{b}] + [3\vec{b}, \vec{a}, -\vec{c}]$

$\Rightarrow 0 + 0 + [3\vec{b}, \vec{a}, -\vec{c}]$

$\Rightarrow [3\vec{b}, \vec{a}, -\vec{c}]$

Using interchange property

$\Rightarrow -[\vec{a}, 3\vec{b}, -\vec{c}]$

$\Rightarrow [\vec{a}, 3\vec{b}, \vec{c}]$

$\Rightarrow 3[\vec{a}, \vec{b}, \vec{c}]$

26. (a)

Let $I = \int_{-3}^{3} \cot^{-1} x \, dx$

Using by parts property

$I = \left[\cot^{-1} x(x)\right]_{-3}^{3} - \int_{-3}^{3} \left(\frac{-1}{x^2+1}\right)(x)\, dx$

$I = \left[x\cot^{-1} x\right]_{-3}^{3} + \int_{-3}^{3} \frac{x}{x^2+1} dx$

$\{\text{Let } x^2 + 1 = t \quad \Rightarrow 2x = \frac{dt}{dx} \quad \Rightarrow x\,dx = \frac{1}{2}dt\}$

$I = \left[x\cot^{-1} x\right]_{-3}^{3} + \int_{-3}^{3} \frac{1}{t} dt$

$I = \left[x\cot^{-1} x\right]_{-3}^{3} + \left[\ln|x^2+1|\right]_{-3}^{3}$

$I = [3\cot^{-1} 3 + 3\cot^{-1}(-3)] + [\ln 10 - \ln 10]$

$I = [3\cot^{-1} 3 + 3(\pi - \cot^{-1} 3)] + 0$

$I = 3\pi$

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27. (d)

Let $I = \int \frac{1}{\sqrt{x} + x\sqrt{x}} dx$

$I = \int \frac{1}{\sqrt{x}(x+1)} dx$

Let $\sqrt{x} = t$

$x = t^2$

$dx = 2t\, dt$

So, $I = \int \frac{1}{t(1+t^2)} (2t\, dt)$

$I = 2\int \frac{1}{1+t^2} dt$

$I = 2 \tan^{-1} t + C$

$I = 2 \tan^{-1} \sqrt{x} + C$

28. (d)

Let $I = \int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

Now,

$$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x-3)}$$

$\Rightarrow 2x - 1 = A(x + 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x + 2)$

Putting x = 3, we get

$6 - 1 = C(3 - 1)(3 + 2)$

$C = \frac{1}{2}$

Putting x = 1, we get

$2 - 1 = A(1 + 2)(1 - 3)$

$A = -\frac{1}{6}$

Putting x = −2, we get

$-4 - 1 = B(-2 - 1)(-2 - 3)$

$B = -\frac{1}{3}$

$I = -\frac{1}{6}\int \frac{1}{x-1} dx - \frac{1}{3}\int \frac{1}{x+2} dx + \frac{1}{2}\int \frac{1}{x-3} dx$

$= -\frac{1}{6} \log|x - 1| - \frac{1}{3} \log|x + 2| + \frac{1}{2} \log|x - 3| + C$

A, B, C are $-\frac{1}{6}, -\frac{1}{3}, \frac{1}{2}$

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29. (a)

$I = \int_0^2 [x^2]dx$

$I = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$

$I = 0 + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$

$I = (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$

$I = \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$I = 5 - \sqrt{2} - \sqrt{3}$

30. (b)

$I = \int_0^1 \sqrt{\frac{1+x}{1-x}}\, dx$

$= \int_0^1 \sqrt{\frac{1+x}{1-x} \times \frac{1+x}{1+x}}\, dx$

$= \int_0^1 \frac{1+x}{\sqrt{1-x^2}}\, dx$

$= \int_0^1 \frac{1}{\sqrt{1-x^2}}\, dx + \int_0^1 \frac{x}{\sqrt{1-x^2}}\, dx$

$= \left[\sin^{-1} x\right]_0^1 - \left[\sqrt{1-x^2}\right]_0^1 + C$

$= \frac{\pi}{2} - 0 - [0 - 1]$

$= \frac{\pi}{2} + 1$

31. (d)

Q.E. $x^2 + x + 1$

$\alpha + \beta = -1$

$\alpha\beta = 1$

Now

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2(1)$

$= 1 - 2$

$= -1$

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32. (a)

Given digits are 1, 2, 3, 4, 5, 6, 7

Two even digits can be selected in $^3C_2$

Two odd digits can be selected in $^4C_2$ ways

These selected 4 digits can be arranged in 4! ways

$\therefore$ Total number of ways = $^4C_2 \cdot {}^3C_2 \cdot 4!$

$= 6 \times 3 \times 24$

$= 432$

33. (b)

$(x^2 + y^2)^{25} - (x^2 - y^2)^{25}$

$(a + b)^n - (a - b)^n$ if n is odd

On simplification we get $\frac{n+1}{2}$ terms

i.e. $\frac{25+1}{2} = \frac{26}{2} = 13$

34. (c)

The third terms of G.P. is 9

$ar^2 = 9 \quad \ldots (1)$

Now, product of five terms = $T_1T_2T_3T_4T_5$

$= (a)(ar)(ar^2)(ar^3)(ar^4)$

$= a^5r^{10}$

$= (ar^2)^5$

$= (9)^5$

$= 3^{10}$

35. (d)

We know that equation of a line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\because$ Here, both intercept are equal

$\Rightarrow \quad \frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \quad y = -x + a$

Now comparing it with

$y = mx + c$

We get m = −1 = slope

We know that

Slope = tan θ

tan θ = −1

θ = 135°

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36. (c)

$I = \int x^3 \cdot \sin 3x \, dx$

Using by part property

$$I = x^3\left[\frac{-\cos 3x}{3}\right] + \int 3x^2 \cdot \frac{\cos 3x}{3} dx$$

$$I = \frac{-x^3\cos 3x}{3} + \left[x^2\int \cos 3x \, dx - \int 2x \cdot \frac{\sin 3x}{3} \, dx\right]$$

$$I = \frac{-x^3\cos 3x}{3} + \frac{x^2 \sin 3x}{3} - \frac{2}{3}\int x \cdot \sin 3x \, dx$$

$$I = \frac{-x^3\cos 3x}{3} + \frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[x\int \sin 3x \, dx + \int (1) \cdot \frac{\cos 3x}{3} \, dx\right]$$

$$I = \frac{-x^3\cos 3x}{3} + \frac{x^2 \sin 3x}{3} + \frac{2x\cos 3x}{9} - \frac{2}{27}\sin 3x + C$$

37. (d)

$y^2 = x$ ... (1)

$x^2 + y^2 = 2x$ ...(2)

$\Rightarrow (x - 1)^2 + (y - 0)^2 = 1$

Equation (2) is a circle with centre (1, 0) and radius 1.

Solving eq. (1) and eq. (2)

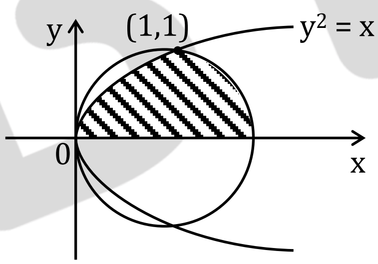
$\Rightarrow (x - 1)^2 + x = 1$

$x^2 - x = 0$

$x(x - 1) = 0$

$x = 0, 1$

We get the points of intersection (0, 0) and (1, 1)



$$\text{Area} = \int_0^1 \left(\sqrt{1-(x-1)^2} - \sqrt{x}\right) dx$$

$$= \left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^1 - \left[\frac{x^{3/2}}{3/2}\right]_0^1$$

$$= \left[0 + \frac{\pi}{4}\right] - \frac{2}{3}$$

$$= -\frac{2}{3} + \frac{\pi}{4}$$

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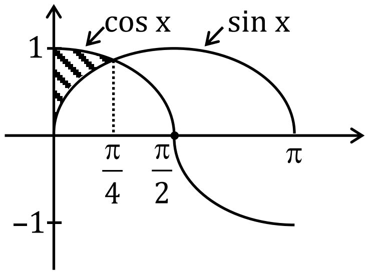
38. (a)



Required area = $\int_0^{\pi/4}(\cos x - \sin x)dx$

$= [\cos x - \sin x]_0^{\pi/4}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right)$

$= (\sqrt{2} - 1)$ sq. units

39. (d)

$(2x + 3y^2)\, dy = y\, dx$

$\Rightarrow \frac{2x+3y^2}{y} = \frac{dx}{dy}$

$\Rightarrow \frac{dx}{dy} - \frac{2}{y}\cdot x = 3y$

I.F. $\Rightarrow e^{\int -\frac{2}{y}dy} = e^{-2\log y} = y^{-2} = \frac{1}{y^2}$

40. (c)

$\frac{dy}{dx} = xy$

$\frac{1}{y}dy = x\, dx$

Integrate

$\log y = \frac{x^2}{2} + C \qquad ...(1)$

Equation (1) passing through (1, 1)

$\log(1) = \frac{1^2}{2} + C$

$C = -\frac{1}{2}$

Put in eq.(1)

$\Rightarrow \log y = \frac{x^2}{2} - \frac{1}{2} \qquad \Rightarrow \qquad 2\log y = x^2 - 1$

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41. (a)

$9x^2 + 25y^2 = 225$

$\frac{x^2}{(5)^2} + \frac{y^2}{(3)^2} = 1$

Eccentricity (e) = $\frac{\sqrt{a^2 - b^2}}{a}$

$= \frac{\sqrt{25-9}}{5} = \frac{4}{5}$

42. (b)

$\sum_{r=1}^{n}(2r-1) = x$

$\Rightarrow\ 2\sum_{r=1}^{n} r - \sum 1 = x$

$\Rightarrow\ n(n + 1) - n = x$

$\Rightarrow\ n^2 = x$

Now,

$\Rightarrow\ \lim_{n\to\infty}\left[\frac{1^3+2^3+3^3+......+n^3}{x^2}\right]$

$\Rightarrow\ \lim_{n\to\infty}\left[\left(\frac{n(n+1)}{2}\right)^2 \times \frac{1}{n^4}\right]$

$\Rightarrow\ \lim_{n\to\infty}\frac{1}{4}\left[\frac{n^2(n+1)^2}{n^4}\right]$

$\Rightarrow\ \frac{1}{4} \times 1 = \frac{1}{4}$

43. (c)

Always all differentiable functions are continuous. But all continuous functions are not differentiable. So, negation of the given statement is some continuous functions are not differentiable. Ex: |x|

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44. (a)

Here n = 100, A = 50, σ = 4

Now, $A = \frac{\Sigma x}{n} \Rightarrow \Sigma x = nA$

$\Rightarrow \Sigma x = 100 \times 50$

$\Rightarrow \Sigma x = 5000$

Again, from the formula

$\Rightarrow \sigma^2 + A^2 = \frac{\Sigma x^2}{n}$

$\Rightarrow \Sigma x^2 = n(\sigma^2 + A^2)$

$= 100(16 + 2500)$

$= 251600$

45. (d)

Vowel: E, A, U, I, O $\Rightarrow P(v) = {}^5C_1$

Consonant: Q, T, N, S $\Rightarrow P(c) = {}^4C_1$

$$\text{Probability} = \frac{{}^5C_1 \times {}^4C_1}{{}^9C_2}$$

$$= \frac{5 \times 4}{36} = \frac{5}{9}$$

46. (d)

$$\Delta = \begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$$

Put x = 0

$$\Delta = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ -2 & 1 & -1 \end{vmatrix}$$

$\Delta = 1(-3) + 1(3) + 2(-1 + 4)$

$\Delta = -3 + 3 + 6$

$\Delta = 6$

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47. (c)

$f(x) = x - [x] - \cos x$

$f(x) = \{x\} - \cos x \qquad \{\because x = [x] + \{x\}\}$

$f'(x) = 1 + \sin x$

$f'\left(\frac{\pi}{2}\right) = 1 + \sin\frac{\pi}{2}$

$= 1 + 1 = 2$

48. (G)

Bonus

49. (b)

$f(x) = \sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$

$f(x) = \sin^{-1}\left[\frac{2\cdot 2^x}{1+(2^x)^2}\right]$

$f(x) = 2\tan^{-1}(2^x)$

Differentiating both side w.r.t. x

$f'(x) = \frac{2}{1+4^x}(2^x \log 2)$

$f'(0) = \frac{2}{1+1}(\log 2)$

$f'(0) = \log 2$

50. (c)

$x = a\sec^2\theta$

$y = a\tan^2\theta$

$y = a(\sec^2\theta - 1)$

$y = a(x - 1)$

$\frac{dy}{dx} = a$

$\frac{d^2y}{dx^2} = 0$

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51. (c)

Let $A = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$

$|A| = 2(3 - 0) - 5(0 + 1) + 0$

$|A| = 1$

$A^{-1} = \dfrac{\text{adj } A}{|A|}$

$A^{-1} = \dfrac{1}{|A|}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}^T$

$A^{-1} = \dfrac{1}{1}\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

52. (d)

Given $P = P'$ and $Q = Q'$

Now,

$(PQ - QP)' = (PQ)' - (QP)'$

$= Q'P' - P'Q'$

$= QP - PQ$

$= -[PQ - QP]$

$\therefore$ $(PQ - QP)$ is skew symmetric.

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53. (a)

$3A + 4B' = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix}$ ... (1)

$2B - 3A' = \begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}$ ... (2)

$\Rightarrow (2B - 3A')' = \begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}^T$

$\Rightarrow 2B' - 3A = \begin{bmatrix} -1 & 4 & -5 \\ 18 & 0 & -7 \end{bmatrix}$ ... (3)

Adding equation (1) & (3), we get

$\Rightarrow 6B' = \begin{bmatrix} 6 & -6 & 12 \\ 18 & 6 & 24 \end{bmatrix}$

$\Rightarrow B' = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

$\therefore B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$

54. (b)

$A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

Now,

$|ABB'| = |A||B||B'|$

$= \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$

$= (2 - 12)(4 + 1)(4 + 1)$

$= -10 \times 5 \times 5$

$= -250$

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55. (c)

Given that

$|A| = 16$

Order of determinant A = 3

Now,

$$|\text{adj } A| = |A|^2 = (16)^2 = 256$$

56. (b)

$f(x) = x^2$

$g(x) = \sqrt{x}$

Now,

$fog(x) = f[g(x)]$

$= f[\sqrt{x}]$

$= x$

$fog(2) = 2$

$fog(-4) = -4$

And

$gof(x) = g[f(x)]$

$gof(x) = g[x^2]$

$gof(-2) = g(4)$

$= \sqrt{4} = 2$

57. (c)

$A = \{x \mid x \in N, x \leq 5\}$

$A = \{1, 2, 3, 4, 5\}$

$B = \{x \mid x \in Z, x^2 - 5x + 6 = 0\}$

$B = \{2, 3\}$

Total number of onto function = $5 \begin{smallmatrix} 4 \\ 1 \end{smallmatrix} + 5 \begin{smallmatrix} 3 \\ 2 \end{smallmatrix}$

$$= \left(\frac{5!}{4!\ 1!} + \frac{5!}{3!\ 2!}\right) \times 2!$$

$$= (5 + 10) \times 2$$

$$= 30$$

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58. (d)

Given condition

$$a * b = \frac{2ab}{5}$$

Now,

$\Rightarrow$ $2 * x = 3^{-1}$

$\Rightarrow$ $\frac{2(2)(x)}{5} = \frac{1}{3}$

$\Rightarrow$ $4x = \frac{5}{3}$

$x = \frac{5}{12}$

59. (a)

$$\cos\left[2\sin^{-1}\frac{3}{4} + \cos^{-1}\frac{3}{4}\right]$$

$\Rightarrow$ $\cos\left[\sin^{-1}\frac{3}{4} + \sin^{-1}\frac{3}{4} + \cos^{-1}\frac{3}{4}\right]$

$\Rightarrow$ $\cos\left[\frac{\pi}{2} + \sin^{-1}\frac{3}{4}\right]$

$\Rightarrow$ $-\sin\left(\sin^{-1}\frac{3}{4}\right)$

$\Rightarrow$ $-\frac{3}{4}$

60. (b)

We know that

$\Rightarrow$ $0 < \cot^{-1} x < \pi$

$\Rightarrow$ $\pi + 0 < 2\left(\frac{\pi}{2}\right) + \cot^{-1} x < \pi + \pi$

$\Rightarrow$ $\pi < 2(\tan^{-1} x + \cot^{-1} x) + \cot^{-1} x < 2\pi$

$\Rightarrow$ $\pi < 2\tan^{-1} x + 3\cot^{-1} x < 2\pi$ ... (1)

Equation (1) compare to $a + \frac{\pi}{2} < 2\tan^{-1} x + 3\cot^{-1} x < b$

$a = \frac{\pi}{2}$

$b = 2\pi$

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