WB-JEE-2017 (Mathematics)

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1. Transforming to parallel axes through a point (p, q), the equation
   $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ becomes $2x^2 + 3xy + 4y^2 = 1$. Then
   a. $p = -2, q = 3$
   b. $p = 2, q = -3$
   c. $p = 3, q = -4$
   d. $p = -4, q = 3$

2. Let A(2, –3) and B (–2, 1) be two angular points of $\triangle ABC$. If the centroid of the triangle
   moves on the line $2x + 3y = 1$, then the locus of the angular point C is given by
   a. $2x + 3y = 9$
   b. $2x - 3y = 9$
   c. $3x + 2y = 5$
   d. $3x - 2y = 3$

3. The point P (3, 6) is first reflected on the line y = x and then the image point Q is again
   reflected on the line y = – x to get the image point Q'. Then the circumcentre of the $\triangle PQQ'$
   is
   a. $(6, 3)$
   b. $(6, -3)$
   c. $(3, -6)$
   d. $(0, 0)$

4. Let d_1 and d_2 be the lengths of the perpendiculars drawn from any point of the line 7x – 9y
   + 10 = 0 upon the lines 3x + 4y = 5 and 12x + 5y = 7 respectively. Then
   a. $d_1 > d_2$
   b. $d_1 = d_2$
   c. $d_1 < d_2$
   d. $d_1 > 2d_2$

5. The common chord of the circles $x^2 + y^2 – 4x – 4y = 0$ and $2x^2 + 2y^2 = 32$ subtends at the
   origin an angle equal to
   a. $\dfrac{\pi}{3}$
   b. $\dfrac{\pi}{4}$
   c. $\dfrac{\pi}{6}$
   d. $\dfrac{\pi}{2}$

6. The locus of the mid-points of the chords of the circle $x^2 + y^2 + 2x - 2y - 2 = 0$ which make an angle of 90º at the centre is

   a. $x^2 + y^2 - 2x - 2y = 0$ b. $x^2 + y^2 - 2x + 2y = 0$

   c. $x^2 + y^2 + 2x - 2y = 0$ d. $x^2 + y^2 + 2x - 2y - 1 = 0$

7. Let P be the foot of the perpendicular from focus S of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ on the line bx – ay = 0 and let C be the centre of hyperbola. Then the area of the rectangle whose sides are equal to that of SP and CP is

   a. $2ab$ b. ab

   c. $\dfrac{\left(a^2 + b^2\right)}{2}$ d. $\dfrac{a}{b}$

8. B is an extremity of the minor axis of an ellipse whose foci are S and S'. If ∠SBS' is a right angle, then the eccentricity of the ellipse is

   a. $\dfrac{1}{2}$ b. $\dfrac{1}{\sqrt{2}}$

   c. $\dfrac{2}{3}$ d. $\dfrac{1}{3}$

9. The axis of the parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$ is

   a. $x + y = 0$ b. $x + y - 1 = 0$

   c. $x - y + 1 = 0$ d. $x - y = \dfrac{1}{\sqrt{2}}$

10. The line segment joining the foci of the hyperbola $x^2 - y^2 + 1 = 0$ is one of the diameters of a circle. The equation of the circle is

    a. $x^2 + y^2 = 4$ b. $x^2 + y^2 = \sqrt{2}$

    c. $x^2 + y^2 = 2$ d. $x^2 + y^2 = 2\sqrt{2}$

11. The equation of the plane through (1, 2, –3) and (2, –2, 1) and parallel to X-axis is
    a. $y - z + 1 = 0$ b. $y - z - 1 = 0$
    c. $y + z - 1 = 0$ d. $y + z + 1 = 0$

12. Three lines are drawn from the origin O with direction cosines proportional to (1, –1, 1), (2 –3, 0) and (1, 0, 3). The three lines are

   a. not coplanar
                   b. coplanar

   c. perpendicular to each other
      d. coincident

13. Consider the non-constant differentiable function f of one variable which obeys the

relation $\dfrac{f(x)}{f(y)} = f(x-y)$ If f'(0) = p and f'(5)=q, then f'(–5) is

   a. $\dfrac{p^2}{q}$
                   b. $\dfrac{q}{p}$

   c. $\dfrac{p}{q}$
                   d. q

14. If f(x) = $\log_5 \log_3 x$, then f'(e) is equal t

   a. $e \log_e 5$
                 b. b. $e \log_e 3$

   c. $\dfrac{1}{e \log_e 5}$
             d. $\dfrac{1}{e \log_e 3}$

15. Let F(x) = e^x, G(x) = e^{-x} and H(x) = G(F(x)), where x is a real variable. Then $\dfrac{dH}{dx}$ at x = 0 is

   a. 1
                 b. –1

   c. $-\dfrac{1}{e}$
             d. –e

16. If f'(0) = k, k ≠ 0, then the value of $\lim\limits_{x\to\infty} \dfrac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

   a. K
                 b. 2k
   c. 3k
                 d. 4k

17. If $y = e^{m\sin^{-1}x}$, then $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - ky = 0$

   a. m^2
                 b. 2
   c. -1
                 d. $-m^2$

18. The chord of the curve $y = x^2 + 2ax + b$, joining the points where $x = \alpha$ and $x = \beta$, is parallel to the tangent to the curve at abscissa $x =$

    a. $\dfrac{a+b}{2}$

    b. $\dfrac{2a+b}{3}$

    c. $\dfrac{2\alpha+\beta}{3}$

    d. $\dfrac{\alpha+\beta}{2}$

19. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$. Then $f(x) = 0$ has

    a. 13 real roots

    b. only one positive and only two negative real roots

    c. not more than one real root

    d. has two positive and one negative real r

20. Let $f(x) = \begin{cases} \dfrac{x^p}{(\sin x)^q}, & \text{If } 0 < x \le \dfrac{\pi}{2} \\ 0 & \text{If } x = 0 \end{cases}$, $(p, q, \in R)$. Then Lagrange's mean value theorem is applicable to $f(x)$ in closed interval $[0, x]$

    a. for all p, q

    b. only when p > q

    c. only when p < q

    d. for no value of p, q

21. $\lim\limits_{x \to \infty} (\sin x)^{2\tan x}$

    a. is 2

    b. is 1

    c. is 0

    d. does not exist

22. $\int \cos(\log x) dx = F(x) + c$, where c is an arbitrary constant. Here $F(x) =$

    a. $x\left[\cos(\log x) + \sin(\log x)\right]$

    b. $x\left[\cos(\log x) - \sin(\log x)\right]$

    c. $\dfrac{x}{2}\left[\cos(\log x) + \sin(\log x)\right]$

    d. $\dfrac{x}{2}\left[\cos(\log x) - \sin(\log x)\right]$

23. $\int \dfrac{x^2-1}{x^4+3x^2+1} dx \,(x>0)$ is

    a. $\tan^{-1}\left(x+\dfrac{1}{x}\right)+c$

    b. $\tan^{-1}\left(x-\dfrac{1}{x}\right)+c$

    c. $\log_e\left(\dfrac{x+\dfrac{1}{x}-1}{x+\dfrac{1}{x}-1}\right)+c$

    d. $\log_e\left(\dfrac{x-\dfrac{1}{x}-1}{x-\dfrac{1}{x}+1}\right)+c$

24. Let $I=\displaystyle\int_{10}^{19}\dfrac{\sin x}{1+x^8} dx.$ Then

    a. $|I|<10^{-9}$

    b. $|I|<10^{-7}$

    c. $|I|<10^{-5}$

    d. $|I|>10^{-7}$

25. Let $I_1=\displaystyle\int_0^n [x] dx$ and $I_2=\displaystyle\int_0^n \{x\} dx$, where [x] and {x} are integral and fractional parts of x and

    $n\in N-\{1\}$. Then I_1/I_2 is equal to

    a. $\dfrac{1}{n-1}$

    b. $\dfrac{1}{n}$

    c. n

    d. n −1

26. The value of $\displaystyle\lim_{x\to\infty}\left[\dfrac{n}{n^2+1^2}+\dfrac{n}{n^2+2^2}+......+\dfrac{1}{2n}\right]$ is

    a. $\dfrac{n\pi}{4}$

    b. $\dfrac{\pi}{4}$

    c. $\dfrac{\pi}{4n}$

    d. $\dfrac{\pi}{2n}$

27. The value of the integral $\displaystyle\int_0^1 e^{x^2} dx$

    a. is less than 1

    b. is greater than 1

    c. is less than or equal to 1

    d. lies in the closed interval [1,e]

28. $\int\limits_{0}^{100} e^{x-[x]}dx =$

    a. $\dfrac{e^{100}-1}{100}$

    b. $\dfrac{e^{100}-1}{e-1}$

    c. $100(e-1)$

    d. $\dfrac{e-1}{100}$

29. Solution of $(x+y)^2 \dfrac{dy}{dx} = a^2$ (a' being a constant) is

    a. $\dfrac{(x+y)}{a} = \tan\dfrac{y+c}{a}$, c is an arbitrary constant

    b. $xy = a\tan cx$, c is an arbitrary constant

    c. $\dfrac{x}{a} = \tan\dfrac{y}{c}$, c is an arbitrary constant

    d. $xy = \tan(x+c)$, c is an arbitrary constant

30. The integrating factor of the first order differential equation

    $x^2(x^2-1)\dfrac{dy}{dx} + x(x^2+1) = x^2-1$ is

    a. e^x

    b. $x-\dfrac{1}{x}$

    c. $x+\dfrac{1}{x}$

    d. $\dfrac{1}{x^2}$

31. In a G.P. series consisting of positive terms, each term is equal to the sum of next two terms. Then the common ratio of this G.P. series is

    a. $\sqrt{5}$

    b. $\dfrac{\sqrt{5}-1}{2}$

    c. $\dfrac{\sqrt{5}}{2}$

    d. $\dfrac{\sqrt{5}+1}{2}$

32. If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals
    a. 125
    b. 25
    c. 5/3
    d. 243

33. The expression $\dfrac{(1+i)^n}{(1-i)^{n-2}}$ equals

   a. $-i^{n+1}$

   b. $-i^{n+1}$

   c. $-2i^{n+1}$

   d. 1

34. Let z = x + iy, where x and y are real. The points (x, y) in the X-Y plane for which $\dfrac{z+i}{z-i}$ is

   purely imaginary lie on

   a. a straight line

   b. an ellipse

   c. a hyperbola

   d. a circle

35. If p, q are odd integers, then the roots of the equation $2px^2 + (2p + q)x + q = 0$ are

   a. rational

   b. irrational

   c. non-real

   d. equal

36. Out of 7 consonants and 4 vowels, words are formed each having 3 consonants and 2 vowels. The number of such words that can be formed is

   a. 210

   b. 25200

   c. 2520

   d. 302400

37. The number of all numbers having 5 digits, with distinct digits is

   a. 99999

   b. $9 \times {}^9P_4$

   c. ${}^{10}P_5$

   d. ${}^{10}P_4$

38. The greatest integer which divides $(p + 1)(p + 2)(p + 3)......(p + q)$ for all $p \in N$ and fixed $q \in N$ is

   a. p!

   b. q!

   c. p

   d. q

39. Let $((1 + x) + x^2)^9 = a_0 + a_1x + a_2x^2 + ... + a_{18}x^{18}$. Then

   a. $a_0 + a_2 + ... + a_{18} = a_1 + a_3 + ... + a_{17}$

   b. $a_0 + a_2 + ... + a_{18}$ is even

   c. $a_0 + a_2 + ... + a_{18}$ is divisible by 9

   d. $a_0 + a_2 + ... + a_{18}$ is divisible by 3 but not by 9

40. The linear system of equations $\begin{aligned} 8x - 3y - 5z &= 0 \\ 5x - 8y + 3z &= 0 \\ 3x + 5y - 8z &= 0 \end{aligned}$ has

   a. only 'zero solution'

   b. only finite number of non-zero solutions

   c. no non-zero solution

   d. infinitely many non-zero solutions

41. Let P be the set of all non-singular matrices of order 3 over □ and Q be the set of all orthogonal matrices of order 3 over □ .Then,

   a. P is proper subset of Q

   b. Q is proper subset of P

   c. Neither P is proper subset of Q nor Q is proper subset of P

   d. $P \cap Q = \phi$ the void set

42. Let $A = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}, B = \begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$. Then all solutions of the equation det (AB) = 0 is

   a. 1, –1, 0, 2

   b. 1, 4, 0, –2

   c. 1, –1, 4, 3

   d. –1, 4, 0, 3

43. The value of det A, where $A = \begin{Bmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{Bmatrix}$ lies

   a. in the closed interval [1, 2]

   b. in the closed interval [0, 1]

   c. in the open interval (0, 1)

   d. in the open interval (1, 2)

44. Let f : $\square \to \square$ be such that f is injective and f(x) f(y) = f(x + y) for $\forall x, y \in \square$. If f(x), f(y), f(z) are in G.P, then x, y, z, are in
    a. A.P always
    b. G.P always
    c. A.P depending on the value of x, y, z
    d. G.P depending on the value of x, y, z

45. On the set $\square$ of real numbers we define xPy if and only if xy $\geq$ 0. Then the relation P is
    a. reflexive but not symmetric
    b. symmetric but not reflexive
    c. transitive but not reflexive
    d. reflexive and symmetric but not transitive

46. On $\square$, the relation ρ be defined by 'x ρ y holds if and only if x – y is zero or irrational'. Then
    a. ρ $\square$is reflexive and transitive but not symmetric.
    b. ρ $\square$is reflexive and symmetric but not symmetric
    c. $\square\rho$ is symmetric and transitive but not reflexive.
    d. $\square\rho$ is equivalence relation

47. Mean of n observations $x_1, x_2, ..., x_n$ is $\bar{x}$. If an observation x_q is replaced by x_q' then the new mean is
    a. $\bar{x} - x_q + x_q'$
    b. $\dfrac{(n-1)\bar{x} + x_q'}{n}$
    c. $\dfrac{(n-1)\bar{x} - x_q'}{n}$
    d. $\dfrac{n\bar{x} - x_q + x_q'}{n}$

48. The probability that a non leap year selected at random will have 53 Sundays is
    a. 0
    b. 1/7
    c. 2/7
    d. 3/7

49. The equation sin x (sin x + cos x) = k has real solutions, where k is a real number. Then
    a. $0 \leq k \leq \dfrac{1+\sqrt{2}}{2}$
    b. $2 - \sqrt{3} \leq k \leq 2 + \sqrt{3}$
    c. $0 \leq k \leq 2 - \sqrt{3}$
    d. $\dfrac{1-\sqrt{2}}{2} \leq k \leq \dfrac{1+\sqrt{2}}{2}$

50. The possible values of x, which satisfy the trigonometric equation

    $\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right) = \dfrac{\pi}{4}$ are

    a. $\pm\dfrac{1}{\sqrt{2}}$

    b. $\pm\sqrt{2}$

    c. $\pm\dfrac{1}{2}$

    d. ± 2

51. On set A = {1, 2, 3}, relations R and S are given by

    R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

    S = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)} Then

    a. R ∪ S is an equivalence relation

    b. R ∪ S is reflexive and transitive but not symmetric

    c. R ∪ S is reflexive and symmetric but not transitive

    d. R ∪ S is symmetric and transitive but not reflexive

52. If one of the diameters of the curve $x^2 + y^2 - 4x - 6y + 9 = 0$ is a chord of a circle with centre (1, 1), the radius of this circle is

    a. 3

    b. 2

    c. $\sqrt{2}$

    d. 1

53. Let A (−1, 0) and B (2, 0) be two points. A point M moves in the plane in such a way that ∠MBA = 2∠MAB. Then the point M moves along

    a. a straight line

    b. a parabola

    c. an ellipse

    d. a hyperbola

54. If $f(x) = \int_{-1}^{x} |t|\,dt$, then for any $x \geq 0$, $f(x)$ is equal to

    a. $\dfrac{1}{2}\left(1 - x^2\right)$

    b. $1 - x^2$

    c. $\dfrac{1}{2}\left(1 + x^2\right)$

    d. $1 + x^2$

55. Let for all x > 0, $f(x) = \lim\limits_{x \to \infty} n\left(x^{\frac{1}{n}} - 1\right)$, then

a. $f(x) + f\left(\dfrac{1}{x}\right) = 1$

b. $f(xy) = f(x) + f(y)$

c. $f(xy) = x\, f(y) + y f(x)$

d. $f(xy) = x\, f(x) + y\, f(y)$

56. Let $I = \int\limits_{0}^{100\pi} \sqrt{(1 - \cos 2x)}\ \ dx$, then

a. $I = 0$

b. $I = 200\sqrt{2}$

c. $I = \pi\sqrt{2}$

d. $I = 100$

57. The area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is

a. $\dfrac{4}{3}$ square units

b. $\dfrac{2}{3}$ square units

c. $\dfrac{3}{7}$ square units

d. $\dfrac{6}{7}$ square units

58. Tangents are drawn to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$ at the ends of both latus rectum. The area of

the quadrilateral so formed is

a. 27 sq. units

b. $\dfrac{13}{2}$ sq. units

c. $\dfrac{15}{4}$ sq. units

d. 45 sq. units

59. The value of K in order that $f(x) = \sin x - \cos x - Kx + 5$ decreases for all positive real

values of x is given by

a. $K < 1$

b. $K \geq 1$

c. $K > \sqrt{2}$

d. $K < \sqrt{2}$

60. For any vector $\vec{x}$, the value of $\left(\vec{x} \times \hat{i}\right)^2 + \left(\vec{x} \times \hat{j}\right)^2 + \left(\vec{x} \times \hat{k}\right)^2$ is equal to

   a. $|\vec{x}|^2$

   b. $2|\vec{x}|^2$

   c. $3|\vec{x}|^2$

   d. $4|\vec{x}|^2$

61. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

   a. $\sqrt{2}$ units

   b. 2 units

   c. $\sqrt{3}$ units

   d. $\sqrt{5}$ units

62. Let α and β be the roots of $x^2 + x + 1 = 0$. If n be positive integer, then $\alpha^n + \beta^n$ is

   a. $2\cos\dfrac{2n\pi}{3}$

   b. $2\sin\dfrac{2n\pi}{3}$

   c. $2\cos\dfrac{n\pi}{3}$

   d. $2\sin\dfrac{n\pi}{3}$

63. For real x, the greatest value of $\dfrac{x^2 + 2x + 4}{2x^2 + 4x + 9}$ is

   a. 1

   b. -1

   c. $\dfrac{1}{2}$

   d. $\dfrac{1}{4}$

64. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Then for positive integer n, A^n is

a. $\begin{pmatrix} 1 & n & n^2 \\ 0 & n^2 & n \\ 0 & 0 & n \end{pmatrix}$

b. $\begin{pmatrix} 1 & n & n\left(\dfrac{n+1}{2}\right) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$

c. $\begin{pmatrix} 1 & n^2 & n \\ 0 & n & n^2 \\ 0 & 0 & n^2 \end{pmatrix}$

d. $\begin{pmatrix} 1 & n & 2n-1 \\ 0 & \dfrac{n+1}{2} & n^2 \\ 0 & 0 & \dfrac{n+1}{2} \end{pmatrix}$

65. Let a, b, c be such that $b(a+c) \neq 0$

If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$, then the value of n is

a. any integer

b. zero

c. any even integer

d. any odd integer

WB-JEE-2017 (Mathematics)

CATEGORY - III (Q66 to Q75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answer marked ÷ actual number of correct answers.

66. Let f : R->R be twice continuously differentiable. Let f(0) = f(1) = f'(0) = 0. Then

    a. $f''(x) \neq 0$ for all x

    b. $f''(c) = 0$ for some $c \in R$

    c. $f''(x) \neq 0$ if $x \neq 0$

    d. f'(x) > 0 for all x

67. If f(x) = x^n, n being a non-negative integer, then the values of n for which

    $f'(\alpha + \beta) = f'(\alpha) + f'(\beta)$ for all $\alpha, \beta > 0$ is

    a. 1

    b. 2

    c. 0

    d. 5

68. Le f be a non-constant continuous function for all $x \geq 0$. Let f satisfy the relation f(x) f(a–x)

    = 1 for some $a \in R^+$. Then $I = \int_0^a \dfrac{dx}{1 + f(x)}$ is equal to

    a. a

    b. $\dfrac{a}{4}$

    c. $\dfrac{a}{2}$

    d. f(a)

69. If the line ax + by + c = 0, $ab \neq 0$, is a tangent to the curve xy = 1–2x, then

    a. a > 0, b < 0

    b. a > 0, b > 0

    c. a < 0, b > 0

    d. a < 0, b < 0

70. Two particles move in the same straight line starting at the same moment from the same point in the same direction. The first moves with constant velocity u and the second starts from rest with constant acceleration f. Then

    a. they will be at the greatest distance at the end of time $\dfrac{u}{2f}$ from the start

    b. they will be at the greatest distance at the end of time $\dfrac{u}{f}$ from the start

    c. their greatest distance is $\dfrac{u^2}{2f}$

    d. their greatest distance is $\dfrac{u^2}{f}$

71. The complex number z satisfying the equation $|z-i| = |z+1| = 1$ is

    a. 0
    b. 1 + i

    c. −1 + i
    d. 1 – i

72. On $\mathbb{R}$, the set of real numbers, a relation ρ is defined as 'a ρ b' if and only if $1 + ab > 0$'.
    Then
    a. ρ is an equivalence relation
    b. ρ is reflexive and transitive but not symmetric
    c. ρ is reflexive and symmetric but not transitive
    d. ρ is only symmetric

73. If a, b $\in \{1, 2, 3\}$ and the equation $ax^2 + bx + 1 = 0$ has real roots, then
    a. a > b
    b. a ≤ b
    c. number of possible ordered pairs (a, b) is 3
    d. a < b

74. If the tangent to $y^2 = 4ax$ at the point $(at^2, 2at)$ where $|t|>1$ is a normal to $x^2 – y^2 = a^2$ at the point (a sec θ , a tan θ), then
    a. t = − cosec θ
    b. t = − sec θ
    c. t = 2 tan θ
    d. t = 2 cot θ

75. The focus of the conic $x^2 – 6x + 4y + 1 = 0$ is
    a. (2, 3)
    b. (3, 2)
    c. (3, 1)
    d. (1, 4)

WB-JEE-2017 (Mathematics)

ANSWER KEY

1. (b)	2. (a)	3. (d)	4. (b)	5. (d)	6. (c)	7. (b)	8. (b)	9. (a)	10. (c)
11. (d)	12. (b)	13. (a)	14. (c)	15. (c)	16. (c)	17. (a)	18. (d)	19. (c)	20. (b)
21. (b, d)	22. (c)	23. (a)	24. (b)	25. (d)	26. (b)	27. (b)	28. (c)	29. (a)	30. (b)
31. (b)	32. (a)	33. (c)	34. (d)	35. (a)	36. (b)	37. (b)	38. (b)	39. (b)	40. (d)
41. (b)	42. (b)	43. (a)	44. (a)	45. (d)	46. (b)	47. (d)	48. (b)	49. (d)	50. (a)
51. (c)	52. (a)	53. (d)	54. (c)	55. (b)	56. (b)	57. (a)	58. (a)	59. (c)	60. (b)
61. (c)	62. (a)	63. (c)	64. (b)	65. (c)	66. (b)	67. (b, c)	68. (c)	69. (b,d)	70. (b, c)
71. (a, c)	72. (c)	73. (c, d)	74. (a, c)	75. (c)					

SOLUTIONS

1. (b)

   Given $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ ...(i)

   After transforming eq.(i) through (p, q) it becomes

   $2x^2 + 3xy + 4y^2 = 1$ ...(ii)

   Then, the point (p, q) satisfies (i) and (ii)

   Differentiate (i) with respect to x and y respectively, we get

   $4x + 3y + 1 = 0$ ...(iii)

   $3x + 8y + 18 = 0$ ...(iv)

   $\because$ (p, q) satisfies (i) and (ii)

   $\therefore$ Point (p, q) satisfies (iii) and (iv)

   So, we get

   $4p + 3q + 1 = 0$ ...(v)

   $3p + 8q + 18 = 0$ ...(vi)

   Solving eq.(v) and (vi) simultaneously we get

   $p = 2, q = -3$
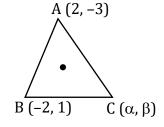
2. (a)

Given: A(2, –3), B(–2, 1) of ∆ABC

Centroid moves on 2x + 3y = 1

Let C(α, β) be the third point on ∆ABC

Let G → Centroid of ∆ABC.

Then, G = $\left(\dfrac{2-2+\alpha}{3}, \ \dfrac{-3+1+\beta}{3} \right) \Rightarrow \left(\dfrac{\alpha}{3}, \ \dfrac{\beta-2}{3} \right)$ ...(i)

Since, G moves on the line 2x + 3y = 1

Points on this line area of the $\left(x, \ \dfrac{1-2x}{3} \right)$

$\Rightarrow$ G = $\left(t, \ \dfrac{1-2t}{3} \right)$

From eq.(i) $\dfrac{\alpha}{3}$ = t and $\dfrac{\beta-2}{3}$ = $\dfrac{1-2t}{3}$

$\Rightarrow \alpha$ = 3t and β = 3 – 2t

$\therefore$ C ≡ (3t, 3 – 2t)

Locus of point C is given by,

$\Rightarrow (3 - 2t)^2 + (3 - 2t + 3)^3 = (3t + 2)^2 + (3 - 2t - 1)^2$

$\Rightarrow 9t^2 + 4 - 12t + 36 + 4t^2 - 24t = 9t^2 + 4 + 12t + 4 + 4t^2 - 8t$

$\Rightarrow 40t = 32$

$\Rightarrow$ t = $\dfrac{32}{40}$ = $\dfrac{4}{5}$

C ≡ $\left(\dfrac{12}{5}, \ \dfrac{7}{5} \right)$ so the equation of Locus of C is given by 2x + 3y = d

Substituting C in the given equation 2x + 3y = d we get d = 9.
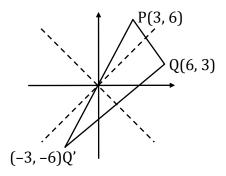
Thus, the Locus of C is 2x + 3y = 9

3. (d)

Given P(3, 6) an y = x, Q → y = –x to get Q' Circumcenter of ΔPQQ' = ?

When the point P(3, 6) is reflected on the line y = x the x and y coordination interchange, we get point Q(6, 3). Again reflecting Q' on the line y = –x we get the image point Q'(–3, –6).

To find out the circumcenter, we need to solve any two bisector equation and then point out the intersection point.

So, midpoint of PQ' = $\left(\dfrac{3-3}{2}, \dfrac{6-6}{2} \right)$ = (0, 0)

Slope of PQ' is $m_1 = \dfrac{-6-6}{-3-3} = 2$

Slope of bisector is the negative reciprocal of the given slope so, slope (m_1) of bisector of PQ' is $-\dfrac{1}{2}$

Equation of PQ' with slope (m_1) $-\dfrac{1}{2}$ and point (0, 0) is given by

$\Rightarrow y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = -\dfrac{1}{2}(x - 0)$

$\Rightarrow y = -\dfrac{1}{2}x$

$\Rightarrow 2x + y = 0$ \qquad ...(1)

Now, mid-point of PQ = $\left(\dfrac{3+6}{2}, \dfrac{6+3}{2} \right) = \left(\dfrac{9}{2}, \dfrac{9}{2} \right)$

Slope of PQ is $m_2 = \dfrac{3-6}{6-3} = \dfrac{-3}{3} = -1$

Slope of bisector of PQ is 1.

$\therefore$ Equation of PQ with slope (m_2) = 1 and point $\left(\dfrac{9}{2}, \dfrac{9}{2} \right)$ is given by

$$y - \frac{9}{2} = 1\left(x - \frac{9}{2} \right)$$

$\Rightarrow y = x$

$\Rightarrow x - y = 0$ \qquad\qquad\qquad ...(2)

Solving equation (1) & (2), we get

$\qquad x = 0 \ \& \ y = 0$

Therefore, circumcenter of $\Delta PQQ'$ is (0, 0)

4. (b)

Given: Perpendiculars drawn from any point of

$\qquad\qquad 7x - 9y + 10 = 0$ \qquad\qquad ...(i)

$\qquad\qquad 3x + 4y = 5$ \qquad\qquad\qquad ...(ii)

$\qquad\qquad 12x + 5y = 7$ \qquad\qquad\qquad ...(iii)

Any point on the line (i) will be of the form $\left(x, \ \dfrac{7x + 10}{9} \right)$

d_1 and d_2 are the lengths at any point of eq.(i) upon eq.(ii) and eq.(iii) distance from $\left(a, \ \dfrac{7a + 10}{9} \right)$ to eq.(ii) is

$$d_1 \quad = \quad \frac{\left| 3a + 4\left(\dfrac{7a + 10}{9} \right) - 5 \right|}{\sqrt{3^2 + 4^2}}$$

$$= \quad \frac{\left| \dfrac{27a + 28a + 40 - 45}{9} \right|}{\sqrt{25}}$$

$$= \quad \frac{\left| 11a - 1 \right|}{9}$$

Distance from $\left(a, \dfrac{7a+10}{9}\right)$ to eq.(iii) is

$$d_2 = \frac{\left|12a + 5\left(\dfrac{7a+10}{9}\right) - 7\right|}{\sqrt{12^2 + 5^2}}$$

$$= \frac{|143a - 13|}{9 \times 13}$$

$$= \frac{|11a - 1|}{9}$$

So, $d_1 = d_2$

5. (d)

Given: Circles are $x^2 + y^2 - 4x - 4y = 0$ ...(a)

                        $2x^2 + 2y^2 = 32$ ...(b)

To find the common chord we need to find the intersection points of the circles.

So, let's subtract eq.(b) from eq.(a), we get

$\Rightarrow -4x - 4y = -16$

i.e. $x + y = 4$ ...(c)

Substitute $x = y - 4$ in (b), we get

   $(y - 4)^2 + y^2 = 16$

$\Rightarrow y^2 - 8y + 16 + y^2 = 16$

$\Rightarrow y(y - 4) = 0$

$\Rightarrow y = 0, 4$

At $y = 0$, $x = 4$ and at $y = 4$, $x = 0$

So the points of intersection of these two circles is (4, 0) and (0, 4) thus, the chord drawn from (0, 4) to (4, 0).

Thus, the angle subtended at the origin by this chord is a right angle.
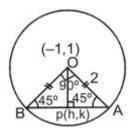
Hence, the chord subtends an angle of $\dfrac{\pi}{2}$ at the origin.

6. (c)

Given Circles is $x^2 + y^2 + 2x - 2y - 2 = 0$

Which can be written as $(x + 1)^2 + (y - 1)^2 = 4$, so C(–1, 1) and radius is 2.

Let O be the center and AB be the chord, angle = 90°

Draw OA and OB

∴ ∠AOB = 90°

Now OA and OB are radius of the circle.

∴ OA = OB

⇒ ∠OAB = ∠OBA [Angle opposite to equal sides are equal]

But   ∠OAB + ∠OBA + ∠AOB = 180°

⇒ 2∠OAB = 180 – 90

⇒ ∠OAB = 45° = ∠OBA

OP⊥C so P(h, k) is the mid-point of chord AB.

Join OP which makes an angle of 90° with the chord AB.

Now, ΔAPO is a right angled triangle such that ∠APO = 90°

$\Rightarrow \sin 45° = \dfrac{OP}{OA} = \dfrac{OP}{2} \Rightarrow OP = \sqrt{2}$

But OP = $\sqrt{(h+1)^2 + (k-1)^2}$ $\left\{\because \sqrt{(h+1)^2 + (k-1)^2} = \sqrt{2}\right.$
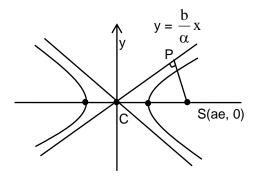
⇒ $h^2 + 2h + 1 + k^2 - 2k + 1 = 2$

⇒ $h^2 + k^2 + 2h - 2k = 0$

Locus of P is $x^2 + y^2 + 2x - 2y = 0$

7. (b)

Given hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, Center (C) = (0, 0)

Let the coordinates of the focus s be (s, 0) and assume that a > b

$\therefore$ s = $\sqrt{a^2 + b^2}$

Also, perpendicular from focus to asymptote [bx – ay = 0]

y = $\dfrac{b}{a}$ × has slope $\dfrac{-a}{b}$ we have equation of normal

As y = $\dfrac{-a}{b}$ x + K

$\Rightarrow$ by + ax = bk $\Rightarrow$ a$\sqrt{a^2 + b^2}$ (passing through the focus)

$\Rightarrow$ SP = $\sqrt{\left(\sqrt{a^2+b^2} - \dfrac{a^2}{\sqrt{a^2+b^2}}\right) + \left(0 - \dfrac{ab}{\sqrt{a^2+b^2}}\right)^2}$

$\Rightarrow$ SP = $\sqrt{\dfrac{b^4}{a^2+b^2} + \dfrac{a^2b^2}{a^2+b^2}} = \sqrt{b^2\left(\dfrac{b^2+a^2}{a^2+b^2}\right)}$

$\Rightarrow$ SP = b

Also, CP = $\sqrt{\left(0 - \dfrac{a^2}{\sqrt{a^2+b^2}}\right)^2 + \left(0 - \dfrac{ab}{\sqrt{a^2+b^2}}\right)^2}$
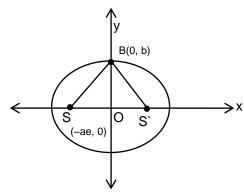
CP = $\sqrt{a^2\left(\dfrac{a^2+b^2}{a^2+b^2}\right)}$

CP = a

Hence, area of rectangle with sides CP and SP is ab

8. b

Given: $\angle SBS'$ is a night angle

$\therefore$ By Pythagoras theorem

$\Rightarrow SS'^2 = SB^2 + S'B^2$

$\Rightarrow (ae + ae)^2 = (ae^2) + b^2 + (ae)^2 + b^2$

$\Rightarrow (2ae)^2 = 2(ae)^2 + 2b^2$

$\Rightarrow = 4(ae)^2 = 2(ae)^2 + 2b^2$

$\Rightarrow ae = b$

$\Rightarrow e = \dfrac{b}{a}$

But $e^2 = 1 - \dfrac{b^2}{a^2}$

$\Rightarrow e^2 = 1 - e^2$

$\Rightarrow 2e^2 = 1$

$\Rightarrow e = \sqrt{\dfrac{1}{2}}$

$\Rightarrow e = \dfrac{1}{\sqrt{2}}$

9. a.

Given parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$

$\Rightarrow (x + y)^2 = 5x - 5y + 5 \Rightarrow (x + y)^2 = 5 (x - y + 1)$

For a parabola $(Ax + Cy)^2 + Dx + Ey + F = 0$

The axis is $Ax + Cy + \dfrac{AD + CE}{2(A^2 + C^2)} = 0$

$\Rightarrow$ axis is $(x + y) + \dfrac{-5 + 5}{2(1 + 1)} = 0$

$\therefore$ axis is $x + y = 0$

10. c

Given hyperbola $x^2 - y^2 + 1 = 0$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = 0$

Foci of the given hyperbola = $(0, \pm\sqrt{2})$

C (0, 0) diameter of the circle is the distance b + w foci

Diameter (D) = $\sqrt{(0-0)^2 + (\sqrt{2} + \sqrt{2})^2} = 2\sqrt{2}$

Centre of the circle will be same as that of hyperbola centre (c) = (0, 0) and Radius

(R) = $\sqrt{2}$

Equation of the circle is $x^2 + y^2 = 2$

11. d

Given equation plane (1, 2, –3) and (2, –2, 1) points

$$\Rightarrow \vec{u} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 4 \end{pmatrix}$$

A vector direction for the x-axis is

$$\Rightarrow \vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

A normal vector to the plane is

$\vec{n} = \vec{u} \times \vec{v}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & 4 \\ 1 & 0 & 0 \end{vmatrix}$$

$\Rightarrow \hat{i}(0 - 0) - \hat{j}(0 - 4) + \hat{k}(0 + 4)$

$\Rightarrow (< 0, 4, 4 >)$

$\Rightarrow 0(x - 1) + 4(y - 2) + 4(z + 3) = 0$
$\Rightarrow 4y - 8 + 4z + 12 = 0$
$\Rightarrow 4y + 4z + 4 = 0$
$\Rightarrow y + z + 1 = 0$

12. b

Given: The lines are drawn from origin 0 with dcs proportional to (1, –1, 1) (2, –3, 0) and (1, 0, 3)

Consider $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \\ 1 & 0 & 3 \end{vmatrix}$

$\Rightarrow 1(-9-0) + 1(6-0) + 1(0+3)$

$\Rightarrow -9 + 9 = 0$

$\therefore \Delta = 0$ Hence, the lines are co-planar

13. a

$f(x) = a^{kx}$ is non-constant differentiable function.

Also, $\dfrac{f(x)}{f(y)} = \dfrac{a^{kx}}{a^{ky}} = a^{kx}\, a^{-ky} = a^{k(x-y)} = f(x-y)$

$\therefore f(x) = a^{kx}$ is the required function

$\Rightarrow f'(x) = ka^{kx} \ln a$

Given $f'(0) = P$ and $f'(5) = 9$

$\Rightarrow a^{sk} = \dfrac{9}{p}$

$\therefore f'(-5) = k.a^{-5k} \ln a \Rightarrow k \ln a^{-5k} = p \times \dfrac{p}{9} = \dfrac{p^2}{9}$

Hence, $f'(-5) = \dfrac{p^2}{9}$

14. c

Given $f(x) = \log_5 \log_3 x$

$\Rightarrow f(x) = \dfrac{\log \log_3 x}{\log 5}$

$= \dfrac{1}{\log 5} \times \log \log_3 x$

$= \dfrac{1}{\log 5} \times \log\left(\dfrac{\log x}{\log 3}\right)$

$= \dfrac{1}{\log 5} \times [\log(\log x) - \log(\log 3)]$

$\therefore f'(x) = \dfrac{1}{\log 5} \times \left[\dfrac{1}{\log x} \cdot \dfrac{1}{x} - 0\right]$

$\Rightarrow f'(x) = \dfrac{1}{x \log x \log 5}$

$\therefore f'(e) = \dfrac{1}{e \log e \log 5} = \dfrac{1}{e \log_e 5}$

Hence, the answer is $\dfrac{1}{e \log_e 5}$

15. c.
$f(x) = e^x$
$G(x) = e^{-x}$
$G(f(x)) = e^{-(f(x))}$
$H(x) = e^{-ex}$

$\Rightarrow \log(H(x)) = \log_e e^{-e^x} = e^{-x}$ $\Rightarrow \dfrac{d}{dx}\log(1 + (x)) = \dfrac{-d}{dx}e^x = -e^x$

$\dfrac{1}{H(x)}\dfrac{dH(x)}{dx} = -e^x$

$\dfrac{dH}{dx}(x) = -e^x e^{-ex}$

$\dfrac{d(H(0))}{dx} = -e^0 e^{-e^0}$

$\dfrac{d(H(0))}{dx} = -1e^{-1}$

$H'(0) = \dfrac{-1}{e}$

16. c

Given, $\lim_{x \to 0} \dfrac{2f(x) - 3f(2x) + f(4x)}{x^2}$

$\Rightarrow \dfrac{2f(0) - 3f(0) + f(0)}{0} = \dfrac{0}{0}$

Applying L-Hospital rule

$\Rightarrow \lim_{x \to 0} \dfrac{2f'(x) - 3 \times 2f'(2x) + 4f'(4x)}{2x}$

$\Rightarrow \lim_{x \to 0} \dfrac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x} = \dfrac{0}{0}$

Again L-Hospital

$\Rightarrow \lim_{x \to 0} \dfrac{2f'(x) - 6.2f''(2x) + 4.4f''(4x)}{2}$

$\Rightarrow \dfrac{2f''(0) - 12f''(0) + 16f''(0)}{2}$

$\Rightarrow \dfrac{4f''(0) + 2f''(0)}{2} \Rightarrow \dfrac{6f''(0)}{2} = 3f''(0) = 3$ k.

17. a

Given, $y = e^{m\sin^{-1}x}$ then $(1 - x^2) \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} - ky = 0$

Now we have $y = e^{m\sin^{-1}x}$ ....(1)

Eq (1) apply chain rule then we get

$y' = e^{m\sin^{-1}x} . \dfrac{1}{\sqrt{1-x^2}}$

$y' = m \dfrac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}$ ....(2)

Again differentiate equation (2) and applying the quotient rule, along with the chain rule we get.

$$y'' = \frac{\left(\sqrt{1-x^2}\right)\left(\frac{d}{dx}me^{m\sin^{-1}x}\right) - \left(me^{m\sin^{-1}x}\right)\left(\frac{d}{dx}\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}$$

$$\Rightarrow \frac{m^2e^{m\sin^{-1}x} + \dfrac{mxe^{m\sin^{-1}x}}{\sqrt{1-x^2}}}{1-x^2}$$

$$\Rightarrow \frac{m^2y + xy'}{1-x^2}$$

$\therefore (1 - x^2)y'' = m^2y + xy'$

$\therefore (1 - x^2)y'' - xy' = m^2y$

$k = m^2$

18. **d**

Given curve is $y = x^2 + 2ax + b$ differentiate above equation with respect to x we get.

$$\Rightarrow \frac{dy}{dx} = 2x + 2a \qquad\qquad ....(1)$$

Equation (1) of tangent to the curve

But tangent to the curve is parallel to the chord of the curve which joints the points x = α and x = β

$\therefore$ Tangent to this curve = $(\alpha + \beta) + 2a$

$\therefore 2x + 2a = (\beta + \alpha) + 2a$

$$x = \frac{\alpha \times \beta}{2}$$

19. **c**

Given $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19 = 0$

$\Rightarrow f'(x) = 13x^{12} + 11x^{10} + 9x^8 + 7x^6 + 5x^4 + 3x^2 + 1$

Since $f'(x)$ consists of all even powers of x therefore

$f'(x) \geq 0$

That $f(x)$ increasing function. Hence, it interests the x-axis at only one point. Therefore $f(x)$ has atmost one real root.

20. b

Given $f(x) = \begin{cases} \dfrac{x^p}{(\sin x)^q} \;; & \text{if } 0 < x \leq \dfrac{\pi}{2} \\ \quad 0 \quad\; ; & \text{if } x = 0 \end{cases}$

Lagrange's mean theorem apply.

f, it is continuous at (a, b) and differential with in the closed interval [a, b]

i.e. $\lim_{x \to a} f(x) = f(a)$

Here, $f(a) = f(0) = 0$

$f(x)$ is differentiable at (0, x)

Consider $\lim_{x \to 0} f(x)$

$\Rightarrow \lim_{x \to 0} \dfrac{x^p}{(\sin x)^q}$

$\Rightarrow \lim_{x \to 0} \dfrac{x^p \cdot x^q \cdot x^{-q}}{(\sin x)^q} \Rightarrow \lim_{x \to 0} \dfrac{x^q \cdot x^{p-q}}{(\sin x)^q}$

$\Rightarrow \lim_{x \to 0} \dfrac{x^q}{(\sin x)^q} \times \lim_{x \to 0} x^{p-q} \Rightarrow \lim_{x \to 0} \left(\dfrac{x}{\sin x} \right)^q \times \lim_{x \to 0} x^{p-q}$

$\Rightarrow \lim_{x \to 0} x^{p-q} \ldots\ldots \lim_{x \to 0} \dfrac{x}{\sin x} = 1$

Now $\lim_{x \to 0} x^{p-q} = \begin{cases} 0 & \text{if } p > q \\ \infty & \text{if } p < q \\ 1 & \text{if } p = q \end{cases}$

$\therefore \lim_{x \to 0} f(x) = 0$ only when $p > q$

Hence, Lagrange's mean value theorem is applicable to $f(x)$ in [0, x] only when $p > q$.

21. b, d

Given $y = (\sin x)^{2\tan x}$

$\Rightarrow \log y = 2 \tan x . \log (\sin x)$

$\Rightarrow \lim\limits_{x \to 0} 2 \tan x \log(\sin x) \Rightarrow \lim\limits_{x \to 0} 2 \dfrac{\log(\sin x)}{\dfrac{1}{\tan x}}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{2.\dfrac{d}{dx}\log \sin x}{\dfrac{d}{dx}\cot x} \Rightarrow \lim\limits_{x \to 0} \dfrac{2\dfrac{1}{\sin x}.\cos x}{-\cosec^2 x}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\dfrac{2}{\tan x}}{-\dfrac{1}{\sin^2 x}} \Rightarrow \lim\limits_{x \to 0} - \dfrac{2\dfrac{d}{dx}\cot x}{\dfrac{d}{dx}\cosec^2 x}$

$\Rightarrow \lim\limits_{x \to 0} \left[\dfrac{-2\cosec^2 x}{-2\cosec^2 x \cot x} \right] = \tan x$

$\Rightarrow \lim\limits_{x \to 0} \tan x$

$\Rightarrow \tan x = 0$

$\log y = 0$

$y = e^o \Rightarrow 1$

$\lim\limits_{x \to 0} (\sin x)^{2\tan x} = 1$

22. c

Given $\int \cos(\log x) dx$

Let $\log x = t \Rightarrow x = e^t$

$\dfrac{1}{x} dx = dt \Rightarrow dx = x dt$

$dx = e^t dt$

$I \Rightarrow \int \cos(\log x) dx \Rightarrow \int e^t \cos t \, dt$

$$\begin{bmatrix} e^{ax} \cos bx \, dx \\ \because \dfrac{e^{ax}}{a^2 + b^2}(a \sin bx + b \cos bx) \end{bmatrix}$$

$a = 1, b = 1$

$I = \dfrac{e^t}{1^2 + 1^2}(\sin t + \cos t) + c$

$I \Rightarrow \dfrac{x}{2} \left[\sin(\log x) + \sin(\log x) \right] + c$

23. a

Given equation $\int \dfrac{x^2 - 1}{(x^4 + 3x^2 + 1)} dx$ $(x > 0)$

Dividing by x^2 numerator or denominator

$\Rightarrow \int \dfrac{\left(1 - \dfrac{1}{x^2} dx\right)}{\left(x^2 + \dfrac{1}{x^2} + 3\right)} dx$ $\Rightarrow \int \dfrac{\left(1 - \dfrac{1}{x^2}\right) dx}{\left(x + \dfrac{1}{x}\right)^2 + 1}$

Let, $x + \dfrac{1}{x} = t$ $\left(1 - \dfrac{1}{x^2}\right) dx = dt$

$\Rightarrow \int \dfrac{dt}{t^2 + 1} = \tan^{-1}(x) + C$

$\Rightarrow \tan^{-1}\left(x + \dfrac{1}{x}\right) + C$

24. b

Given $I = \int\limits_{10}^{19} \dfrac{\sin x}{1 + x^8} dx$ $\therefore |\sin x| \le |$

$\Rightarrow \left|\dfrac{\sin x}{1 + x^8}\right| \le \left|\dfrac{1}{1 + x^8}\right|$ ....(1)

WB-JEE-2017 (Mathematics)

$$\begin{cases} \Rightarrow 10 \le x \le 19 \\ \quad 10^8 \le x^8 \le 19^8 \\ 1+x^8 \ge x^8 \ge 10^8 \end{cases}$$

$$\Rightarrow \frac{1}{1+x^8} < \frac{1}{x^8} \le \frac{1}{10^8} \Rightarrow \left|\frac{1}{1+x^8}\right| \le \left|\frac{1}{10^8}\right|$$

So, $\left|\dfrac{\sin x}{1+x^8}\right| \le \dfrac{1}{10^8}$

$$\Rightarrow \int_{10}^{19} \left|\frac{\sin x}{1+x^8}\right| dx \le \int_{10}^{19} 10^{-8} dx$$

$$\Rightarrow \int_{10}^{19} \left|\frac{\sin x}{1+x^8}\right| dx \le (19-10) \times 10^{-8}$$

$$\Rightarrow \int_{10}^{19} \left|\frac{\sin x}{1+x^8}\right| dx \le (10-1) \times 10^{-8}$$

$$\Rightarrow \int_{10}^{19} \left|\frac{\sin x}{1+x^8}\right| dx \le 10^{-7} - 10^{-8}$$

$$\Rightarrow \int_{10}^{19} \left|\frac{\sin x}{1+x^2}\right| dx < 10^{-7} \Rightarrow |I| < 10^{-7}$$

Approximately $[10^{-8}]$

25. d

$$I_1 \int_0^n [x]dx = \int_0^1 odx + \int_1^2 1dx + \int_2^3 2dx + \int_3^4 3dx + ... + \int_{n-1}^n (n-1)dx$$

$$\Rightarrow 0 + 1(2-1) + 2[3-2] + 3[4-3] + ... + n-1[n-(n-1)]$$

$$\Rightarrow 1 + 2 + 3 + 4 + ... + (n-1)$$

$$I_1 = \frac{(n-1)(n)}{2} = \frac{n(n-1)}{2}$$

$$I_2 = \int_0^n \{x\}dx \Rightarrow \int_0^n (x-[x])dx$$

$$I_2 = \int_0^n x\,dx - \int_0^n [x]\,dx$$

$$I_2 = \left[\frac{x^2}{2}\right]_0^n - \frac{n(n-1)}{2}$$

$$I_2 = \frac{n^2}{2} - \frac{n(n-1)}{2} \Rightarrow \frac{n}{2}[n-(n-1)] = \frac{n}{2}$$

$$\frac{I_1}{I_2} = \frac{\dfrac{n(n-1)}{2}}{\dfrac{n}{2}} \Rightarrow (n-1)$$

26. b

$$\lim_{n\to\infty} \frac{1}{n}\left[\frac{n^2}{n^2+1^2} + \frac{n^2}{n^2+2^2} + \ldots + \frac{n}{2n}\right]$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n}\left[\frac{n^2}{n^2+1^2} + \frac{n^2}{n^2+2^2} + \ldots + \frac{n^2}{n^2+n^2}\right]$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{\dfrac{n^2+1^2}{n^2}} + \frac{1}{\dfrac{n^2+2^2}{n^2}} + \ldots + \frac{1}{\dfrac{n^2+n^2}{n^2}}\right]$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{1+\left(\dfrac{1}{n}\right)^2} + \frac{1}{1+\left(\dfrac{2}{n}\right)^2} + \ldots + \frac{1}{1+\left(\dfrac{n}{n}\right)^2}\right]$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{1}{1+\left(\dfrac{r}{n}\right)^2} \qquad \begin{cases} \dfrac{1}{n} = h \\ nh = 1 \\ h \to \infty \\ n \to 0 \end{cases}$$

$$\Rightarrow \lim_{n\to 0} h\sum_{r=1}^{n} \frac{1}{1+(rh)^2} \qquad \Rightarrow \int_0^1 \frac{dx}{1+x^2}$$

$$\Rightarrow \tan^{-1} x\Big|_0^1 = \tan^{-1}1 - \tan^{-1}0 \qquad = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

27. b

$$\int_0^1 e^{x^2} dx$$

$$\left\{ m(b-a) < \int_0^1 e^{x^2} dx < m(b-a) \right. \qquad \qquad \ldots\ldots(1)$$

$a = 0, \; b = 1$

$f(x) = e^{x^2}$

$m = f(0) = e^0 = 1$

$m\, f(1) = e^{1^2} = e^1 = e$

So, equation (1)

$$1(1-0) < \int_0^1 e^{x^2} dx < e(1-0)$$

$$\Rightarrow 1 < \int_0^1 e^{x^2} dx < e$$

$$\Rightarrow \int_0^1 e^{x^2} dx \text{ lies in } (l, e)$$

28. c

$$\int_0^{100} e^{x-[x]} dx \Rightarrow \int_0^{100} e^{[x]} dx \qquad \qquad \left\{ \begin{array}{l} x = [x] + \{x\} \\ x - [x] = \{x\} \end{array} \right.$$

$0 \le \{x\} < 1 \; \text{period} = 1$

$$\int_0^{mT} f(x) dx = m \int_0^T f(x) dx \qquad \qquad T \rightarrow \text{time period}$$

$$\Rightarrow \int_0^{100} e^x \, dx \Rightarrow 100 [e^x]_0^1$$

$$\Rightarrow 100 \, (e^1 - e^0) \Rightarrow 100 \, (e-1)$$

29. a

$$(x + y)^2 \frac{dy}{dx} = a^2$$

$$\text{Let} \begin{cases} x + y = t \\ 1 + \dfrac{dy}{dx} = \dfrac{dt}{dx} \\ \dfrac{dy}{dx} = \dfrac{dt}{dx} - 1 \end{cases}$$

$$\Rightarrow t^2 \left(\frac{dt}{dx} - 1 \right) = a^2$$

$$\Rightarrow t^2 \frac{dt}{dx} - t^2 = a^2$$

$$\Rightarrow t^2 \frac{dt}{dx} = a^2 + t^2$$

$$\Rightarrow \int \frac{t^2 + a^2 - a^2}{t^2 + a^2} dt = \int dx$$

$$\Rightarrow \int \left(1 - \frac{a^2}{(t^2 + a^2)} \right) dt = x + c$$

$$\Rightarrow \int dt - \int \frac{a^2}{(t^2 + a^2)} dt = x + c$$

$$\Rightarrow t - a^2 \int \frac{dt}{t^2 + a^2} dt = x + c$$

$$\Rightarrow t - a^2 \left[\frac{1}{a} \tan^{-1} \left(\frac{t}{a} \right) \right] = x + c$$

$$\Rightarrow x + y - a \tan^{-1} \left(\frac{x + y}{a} \right) = x + c$$

$$\Rightarrow \frac{y - c}{a} = \tan^{-1} \left(\frac{x + y}{a} \right)$$

$$\Rightarrow \frac{x + y}{a} = \tan \left(\frac{y - c}{a} \right)$$

30. b

$$x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = x^2-1$$

$$\Rightarrow \frac{dy}{dx} + \frac{(x^2+1)}{x(x^2-1)}y = \frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} + py = \theta$$

$$\Rightarrow \text{I.F.} = e^{\int pdx} = e^{\int \frac{(x^2+1)}{x(x^2-1)}dx}$$

$$\Rightarrow I = \int \frac{x^2+1}{x(x^2-1)}dx = \int \frac{(x^2-1)+1+1}{x(x^2-1)}dx$$

$$\Rightarrow I = \int \frac{1}{x}dx = \int \frac{2}{x(x-1)(x+1)}dx$$

$$\begin{cases} A = \dfrac{2}{(-1)(1)} = -2 \\ B = \dfrac{2}{1 \times 2} = 1 \\ C = \dfrac{2}{(-1)(-2)} = 1 \end{cases}$$

$$\Rightarrow I = \ell nx + \int \left(\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \right)dx$$

$$\Rightarrow I = \ell nx + \int \left(\frac{-2}{x} + \frac{1}{x-1} + \frac{1}{x+1} \right)dx$$

$$\Rightarrow I = \ell nx - 2\,\ell nx + \ell n\,(x-1) + \ell n(x+1)$$

$$\Rightarrow \ell nx - \ell nx^2 + \ell n(x-1) + \ell n\,(x+1)$$

$$\Rightarrow \ell n\,x\,\frac{(x^2-1)}{x^2} \Rightarrow \ell n\frac{x^2-1}{x}$$

$$\text{I.F.} = e^{\ell n_e}\left(\frac{x^2-1}{x} \right) \Rightarrow \frac{x^2-1}{x}$$

$$\text{I.F.} = x - \frac{1}{x}$$

31. b

Each term is sum of next two terms

$\Rightarrow t_n = t_{n+1} + t_{n+2}$

$\Rightarrow ar^{n-1} = ar^n + ar^{n+1}$

$\Rightarrow 1 = r + r^2$

$\Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \dfrac{-1 \pm \sqrt{1 - 4(-1)}}{2(1)}$

$\Rightarrow r = \dfrac{-1 \pm \sqrt{5}}{2}$

$r = \dfrac{\sqrt{5} - 1}{2}$ or $r = \dfrac{-\sqrt{5} - 1}{2}$

But common ration can be negative

$r = \dfrac{\sqrt{5} - 1}{2}$

32. a

$(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$

$\Rightarrow \left(\dfrac{\log x}{\log 5}\right)\left(\dfrac{\log 3x}{\log x}\right)\left(\dfrac{\log y}{\log 3}\right) = \dfrac{\log x^3}{\log x}$

$\Rightarrow \dfrac{\log x \cdot \log 3x \cdot \log y}{\log 5 \cdot \log x \cdot \log 3x} = \dfrac{3 \log x}{\log x}$

$\Rightarrow \dfrac{\log y}{\log 5} = 3$

$\Rightarrow \log y = 3 \log 5$

$\qquad y = 5^3 = 125$

33. c

$$\frac{(1+i)^n}{(1-i)^{n-2}} \Rightarrow \frac{(1+i)^n}{\dfrac{(1-i)^n}{(1-i)^2}}$$

$$\Rightarrow \frac{(1+i)^n(1-i)^2}{(1-i)^n} \Rightarrow \frac{\left(\dfrac{1}{\sqrt{2}}+\dfrac{i}{\sqrt{2}}\right)^n(\sqrt{2})^n(1-i)^2}{\left(\dfrac{1}{\sqrt{2}}-\dfrac{i}{\sqrt{2}}\right)^n(\sqrt{2})^n}$$

$$\Rightarrow \frac{\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)^n(1+i^2-2i)}{\left(\cos\dfrac{\pi}{4}-i\sin\dfrac{\pi}{4}\right)^n} \qquad \therefore \begin{cases} \cos\theta+i\sin\theta=e^{i\theta} \\ \cos\theta-i\sin\theta=e^{-i\theta} \end{cases}$$

$$\Rightarrow \frac{\left(e^{i\pi/4}\right)^n(1-1-2i)}{(e^{-i\pi/4})^n} \Rightarrow \frac{e^{in\pi/4}(-2i)}{e^{-in\pi/4}}$$

$$\Rightarrow e^{in\,\pi/4}\cdot e^{in\,\pi/4}(-2i)$$

$$\Rightarrow e^{in\left(\frac{\pi}{4}+\frac{\pi}{4}\right)}(-2i)$$

$$\Rightarrow e^{in\,\pi/2}(-2i)$$

$$\Rightarrow \left[\cos\left(\frac{n\pi}{2}\right)+i\sin\left(\frac{n\pi}{2}\right)\right](-2i)$$

34. d

Let z = x + iy

$$\therefore \left(\frac{z+i}{z-i}\right) = \frac{x+iy+i}{x+iy-i}$$

$$\Rightarrow \frac{(x+i(y+1))}{(x-i(1-y))}\times\frac{(x+i(1-y))}{(x+i(1-y))}$$

$$\Rightarrow \frac{x^2+(y^2-1)+2ix}{x^2+(1-y)^2}$$

Since, $\dfrac{z+i}{z-i}$ should be purely imaginary

$\therefore R_e\left(\dfrac{z+i}{z-i}\right)=0$

$\Rightarrow \dfrac{x^2+(y^2-1)}{x^2+(1-y)^2}=0$

$\Rightarrow x^2+y^2-1=0$

$x^2+y^2=1$ is Circle.

35. a

Given equation $2px^2+(2p+q)x+q=0$ \qquad .....(1)

Now $D=(2p+q)^2-8pq$

$\Rightarrow 4p^2+4pq+q^2-8pq$

$\Rightarrow 4p^2-4pq+q^2$

$\Rightarrow (2p-q)^2$ \qquad always a perfect square

$\therefore$ Roots are given by

$x=\dfrac{-(2p-q)+\sqrt{(2p+q)^2-8pq}}{4p}$

$x=\dfrac{-2p-q\pm(2p-q)}{4p}$

$\qquad x=\dfrac{-2p-q+2p-q}{4p}=-\dfrac{q}{2p}$ \qquad (rational p, q are odd integers)

Or \qquad $x=\dfrac{-2p-q-2p+q}{4p}=\dfrac{-4p}{4p}=-1$ which is also rational.

36. b

No of ways of selecting $= 7C_3 \times 4C_2 = 210$

No. of group, each heavy 3 consonants and 2 vowels = 210

Each group contains 5 letters.

No of ways of arranging 5 letters = 5! = 210

Required no. of ways = 210 × 120 = 25200

37. b

5 digits   9 9 8 7 6

Total no. (1 to 9)

Formula = $^n p_r = \dfrac{n!}{(n-n)!}$ $\because$ 0 can't be used at first place

$9 \times \dfrac{(9 \times 8 \times 7 \times 6 \times 5!)}{5!} \Rightarrow 9 \times \dfrac{9!}{5!}$

Apply formula $\dfrac{9 \times 9!}{(9-4)!} = 9 \times {}^9 p_4$

38. b

$(p + 1), (p + 2), (p + 3).....(p + q)$ are q consecutive numbers.

$(p + 1)(p + 2)(p + 3) .... (p + q)$ is product of 'q' consecutive natural numbers

The product will be divisible by all its factors and since it is a product of consecutive natural numbers, so it will always be divisible by q!

Also q! is the greatest integer amongst all their divisors.

Hence, the greatest integer which divides the product is q!

39. b

Given, $((1+x)+x^2)^9 = a_0 + a_1 x + a_2 x^2 + ...+ a_{18} x^{18}$ ....(1)

Put x = 1 in eq. (1), we get.
$\Rightarrow (1+1+1)^9 = a_0 + a_1 + a_2 + ...+ a_{18}$
$\Rightarrow (3)^9 = a_0 + a_1 + a_2 + ... + a_{18}$ ....(2)
Put x = –1 in eq. (1) we get.
$\Rightarrow (1)^9 = a_0 - a_1 + a_2 - a_3 + ...a_{17} + a_{18}$ ....(3)
Adding eq. (2) & (3), we get.
$\Rightarrow 3^9 + 1 = a_0 + a_1 + ... + a_{18} + a_0 - a_1 + a_2 - .... - a_{17} + a_{18}$
$\Rightarrow 2a_0 + 2a_2 + 2a_4 + ... + 2a_{18}$
$\Rightarrow 2(a_0 + a_2 + a_4 + ...+ a_{18}) = 3^9 + 1$
$\Rightarrow a_0 + a_2 + ...a_{18} = \dfrac{3^9 + 1}{2} \rightarrow$ even

Hence, $a_0 + a_2 + a_4 + ... + a_{18}$ is even

40. d

Given $8x - 3y - 5z = 0$

$$5x - 8y + 3z = 0$$

$$3x + 5y - 8z = 0$$

Use Cramer's rule

$x = \dfrac{\Delta x}{\Delta}$, $y = \dfrac{\Delta y}{\Delta}$, $z = \dfrac{\Delta z}{\Delta}$

$\Delta x = \Delta y = \Delta z$

$$D = \begin{vmatrix} 8 & -3 & -5 \\ 5 & -8 & 3 \\ 3 & 5 & -8 \end{vmatrix}$$

$D = (512 - 125 - 27) - (120 + 120 + 120)$

$D = 360 - 360 = 0$

So $\dfrac{\Delta x}{\Delta} = \dfrac{0}{0}, \dfrac{\Delta y}{\Delta} = \dfrac{0}{0}, \dfrac{0z}{\Delta} = \dfrac{0}{0}$

So infinitely many non-zero solutions

41. b

$P = \{P_1, P_2, P_3 \ldots P_n\}$ (non –singular matrix)

$Q = \{Q_1, Q_2, Q_3 \ldots Q_n\}$ (orthogonal matrix)

$Q, Q^T = I$

Det $(Q) = \pm 1$

P contains matrix whose value is non zero
Q is proper sub-sets of P.

42. b

Given $A = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}$, $B = \begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$

det $|AB| = \begin{vmatrix} x^2 + 17x & 3x^2 + 6x \\ 8x + 10 & (x+2)^2 \end{vmatrix} = 0$

$\Rightarrow x(x + 2)(x - 4)(x - 1) = 0$
$\Rightarrow x = 0, -2, 1, 4$

43. a

Given A = $\begin{pmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{pmatrix}$

$\Rightarrow R_1 \to R_1 + R_3$

$|A| \Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$

$|A| \Rightarrow 1\ (\cos^2\theta + 1)$

$\begin{cases} -1 \leq \cos\theta \leq 1 \\ 0 \ \leq \cos^2\theta \leq 1 \\ 1 \ \leq \cos^2\theta + 1 \leq 2 \end{cases}$

$|A| \Rightarrow [1, 2]$

44. (a)

Given: $f : R \to R$ injective

   $f(x)f(y) = f(x + y)$

Let $f(x) = a^x$ then $f(x)\ f(y) = a^x\ a^y = a^{x+y}$

Also $f(x) = f(y) \Rightarrow a^x = a^y$

   $\Rightarrow a^{x-y} = a^0$

   $\Rightarrow x - y = 0 \quad \Rightarrow x = y$

$\therefore\ a^n$ is injective

Now a^x, a^y, a^z are in G.P.

$\Rightarrow a^y = a^x \cdot r$ where r is common ratio

Also $a^z = a^y r$

$\Rightarrow \dfrac{a^y}{a^z} = \dfrac{a^x}{a^y} \Rightarrow a^{2y} = a^{x+z}$

   $y = \dfrac{x+z}{2}$

$\therefore$ x, y, z are in A.p.

45. d

Let p be the relation on the set real number R such that xpy if an only if $xy \geq 0$

(a) we know that, for any real number x, $x^2 \geq 0$

$\Rightarrow xx \geq 0 \Rightarrow xpx$

$\therefore$ p 15 reflexive

(b) let $(x, y) \in p$ i.e. xpy

$\Rightarrow xy \geq 0 \ yx \geq 0 \Rightarrow ypx$

$\therefore$ p is symmetric

(c) Let xpy and ypz

$\Rightarrow xy \geq 0$ and $yz \geq 0$

But from this, we can't conclude $x Z \geq 0$

For example

(–1, 0), (0, 2) satisfies the relation $xy \geq 0$ but (–1, 2)

Doesn't satisfy relation $xy \geq 0$

Thus p is not transitive

Hence p is reflexive symmetric but not transitive

46. B

$xRy \Rightarrow x - y$ is zero or irrational.

$xRx \Rightarrow 0 \ \therefore$ Reflective

if $xRy \Rightarrow x - y$ is zero or irrational.

$\Rightarrow y - x$ is zero or irrational

$\therefore yR x$ symmetric xRy

$\Rightarrow x - y$ is 0 or irrational yRz

$\Rightarrow y - z$ is 0 or irrational then $(x - y)+(y-z)=(x-2)$ may be rational

$\therefore$ it is not transitive

47. d

Given: mean of n observation x_1, x_2 ....x_n is $\bar{x}$

Mean = $\dfrac{\displaystyle\sum_{i=1}^{n} x_i}{n} = \bar{x}$ ....(1)

New mean = $\dfrac{x_1 + ...x_{q-1} + x_{q^1} + x_{q+1} + ...x_n}{n}$

$= \dfrac{x_1 + ...x_{q^1} + ... + x_n + x_q - x_q}{n}$

$= \dfrac{\displaystyle\sum_{i=1}^{n} x_i - x_q + x_q'}{n}$

$= \dfrac{n\bar{x} - x_q + x_q'}{n}$ ....from eq. (1)

48. b

We know non leap year – 365 days.

365 days, number of weeks = 52 and 1 day remaining 52 weeks, there will be 52 Sundays, 1 day remaining can be either Sunday, Mon, Tue, Wed, Thus, Fri & Saturday out of these 7 total out comes, the favorable outcomes are 1.

Hence the probability of getting 53 Sundays = $\dfrac{1}{7}$

49. d

Given: sin × (sin x + cos x) = k

$\Rightarrow \sin^2 x + \sin x \cos x = k$

$\Rightarrow \dfrac{1 - \cos 2x}{2} + \dfrac{\sin 2x}{2} = k$

$\Rightarrow 1 - \cos 2x + \sin 2x = 2k$

$\Rightarrow \sin 2x - 2\cos 2x = 2k - 1$

$\Rightarrow -\sqrt{2} \le 2k - 1 \le \sqrt{2}......[\because -\sqrt{2} \le \sin 2x - \cos 2x \le \sqrt{2}]$

$\Rightarrow 1 - \sqrt{2} \le 2k \le 1 + \sqrt{2}$ $\qquad \Rightarrow \dfrac{1 - \sqrt{2}}{2} \le k \le \dfrac{1 + \sqrt{2}}{2}$

50. a

Given: $\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right) = \dfrac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\dfrac{\dfrac{x-1}{x-2} + \dfrac{x+1}{x+2}}{1 - \dfrac{x-1}{x-2}\cdot\dfrac{x+1}{x+2}}\right) = \dfrac{\pi}{4}$ {using formula of $\tan^{-1} a + \tan^{-1} b$}

$\Rightarrow \left(\dfrac{\dfrac{x-1}{x-2} + \dfrac{x+1}{x+2}}{1 - \dfrac{x-1}{x-2}\cdot\dfrac{x+1}{x+2}}\right) = \tan\dfrac{\pi}{4}$

$\Rightarrow \dfrac{x-1}{x-2} + \dfrac{x+1}{x+2} = 1 - \dfrac{x-1}{x-2}\cdot\dfrac{x+1}{x+2}$ ....∴ {$\tan\dfrac{\pi}{4} = 1$}

$\Rightarrow \dfrac{x^2 + x - 2 + x^2 - x - 2}{x^2 - 4} = 1 - \dfrac{x^2 - 1}{x^2 - 4}$

$\Rightarrow 2x^2 - 4 = x^2 - 4 - x^2 + 1$

$\Rightarrow 2x^2 = 1 \Rightarrow x = \pm\dfrac{1}{\sqrt{2}}$

51. (c)

Given A = {1, 2, 3}
R = {(1,1), (2,2), (3,3), (1,2), (2,1)}
S = {(1,1), (2,2), (3,3), (1,3), (3,1)}
(i) Check reflexive
    Them (a,a) ∈ R for every a ∈ {1, 2, 3}
    Since R∪S = {(1,1), (2,2), (3,3), (1,2), (2,1), (1, 3), (3,1)}
        R is reflexive
(ii) Check symmetric
    If (a,b) ∈ R & S, then (b, a) ∈ R & S
    Here (1,2) ∈ R, but (2, 1) ∈ R
    ∴ (1, 3) ∈ R, but (3, 1) ∈ S
    ∴ R & S symmetric
(iii) Check transitive
        IF (a,b) ∈ R & S & (b, c) ∈ R & S, then (a, c) ∈ R & S
        Here R & S not transitive
        R∪S is reflexive and symmetric but not transitive.

WB-JEE-2017 (Mathematics)

52. (a)

Given equation of circle is

$x^2 + y^2 - 4x - 6y + 9 = 0$

$\{ x^2 + y^2 + 2gx + 2fy + c = 0 \}$

So, g = –2, f = –3, c = 9

Centre (–g, –f) $\Rightarrow$ (2, 3)

Radius r = $\sqrt{f^2 + g^2 - c}$

r = $\sqrt{(-3)^2 + (-2)^2 - 9}$

r = $\sqrt{9 + 4 - 9}$

r = $\sqrt{4}$ = 2

Chord (0', 0) = $\sqrt{(1-2)^2 + (1-3)^2}$

= $\sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

We apply Pythagoras theorem
In $\triangle OO'A \Rightarrow O'O^2 + O'A^2 = OA^2 = R^2$
R = $\sqrt{(\sqrt{5})^2 + (2)^2} = \sqrt{5 + 4}$
R = 3

53. (d)
Given A(–1, 0), B (2, 0)
We know that $\angle$ MBA = 2$\angle$MAB

Also tan θ = $\left| \dfrac{m_2 - m_1}{1 + m_1 m_2} \right|$

Where m_1 & m_2 are the slopes of two lines and θ is the angle between two line.
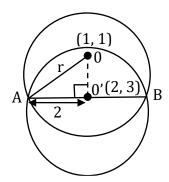Given θ = 2ϕ $\Rightarrow$ tanθ 2ϕ

$\Rightarrow$ tanθ = $\dfrac{2\tan\phi}{1 - \tan^2\phi}$ $\Rightarrow$ $\dfrac{-y_0}{x_0 - 2} = \dfrac{2y_0(x_0 + 1)}{(x_0 + 1)^2 - y_0^2}$
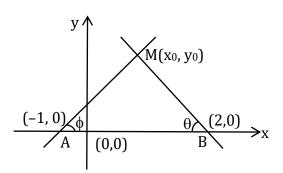
$\Rightarrow \dfrac{-1}{x_0 - 2} = \dfrac{2(x_0 + 1)}{(x_0 + 1)^2 - y_0^2}$

$\Rightarrow 3x_0^2 - y_0^2 = 3$

$\therefore$ Locus of M is hyperbola $3x^2 - y^2 = 3$
Now verify alternatives.

54. (c)

Given $f(x) = \int\limits_{-1}^{x} |t| \, dt$, $x \geq 0$

$$\left\{ \begin{array}{c} \text{Formula apply} \\ \int\limits_{a}^{b} f(x) dx \\ \Rightarrow \int\limits_{a}^{c} f(x) dx + \int\limits_{c}^{b} f(x) dx \end{array} \right\}$$

$\Rightarrow f(x) = \int\limits_{-1}^{0} -t \, dt + \int\limits_{0}^{x} t \, dt$

$\Rightarrow f(x) = \left[\dfrac{-t^2}{2} \right]_{-1}^{0} + \left[\dfrac{t^2}{2} \right]_{0}^{x}$

$f(x) \Rightarrow 0 - \left(-\dfrac{1}{2} \right) + \dfrac{x^2}{2} - 0 \Rightarrow \dfrac{1}{2} + \dfrac{x^2}{2} = \dfrac{1}{2}(1 + x^2)$

55. (b)

$f(x) = \lim\limits_{x \to \infty} n \left(x^{\frac{1}{x}} - 1 \right)$ $\qquad$ Let $n = \dfrac{1}{t}$ $\quad$ $\begin{array}{l} n \to \infty \\ t \to 0 \end{array}$

$\Rightarrow \lim\limits_{t \to 0} \dfrac{1}{t}(x^t - 1)$

$\Rightarrow \lim\limits_{t \to 0} \dfrac{x^t - 1}{t} = \dfrac{x^0 - 1}{0} = \dfrac{0}{0}$

L-Hospital

$\lim\limits_{t \to 0} \dfrac{x^t \ln x - 0}{1} = x^0 \ln x = 1 \ln x$

$f(x) = \ln x$

$\Rightarrow f(xy) = \ln(xy) = \ln x + \ln y$

$\Rightarrow f(xy) = f(x) + f(y)$

56. (b)

Given $I = \int\limits_{0}^{100\pi} \sqrt{(1-\cos 2x)dx}$

$\cos 2x = 1 - 2\sin^2 x$

$1 - \cos 2x = 2\sin^2 x$

$\Rightarrow \int\limits_{0}^{nT} f(x)dx = n\int\limits_{0}^{T} f(x)dx$

$I = \int\limits_{0}^{100\pi} \sqrt{2\sin^2 x}\,dx$

$\Rightarrow 100\int\limits_{0}^{\pi} \sqrt{2}\sin x\,dx$

$\Rightarrow 100\sqrt{2}\int\limits_{0}^{\pi} \sin x\,dx$

$\Rightarrow 100\sqrt{2}[-\cos x]_{0}^{\pi}$

$\Rightarrow 100\sqrt{2}[-\cos x + \cos 0]$

$\Rightarrow 100\sqrt{2}[1+1]$

$\Rightarrow 200\sqrt{2}$

57. (a)

$x = -2y^2$ .....(1)

$x = 1 - 3y^2$ .....(2)

Equation (1) & (2)

$\Rightarrow -2y^2 = 1 - 3y^2$

$\Rightarrow -2y^2 + 3y^2 = 1$
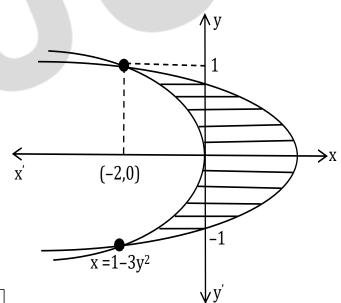
$\Rightarrow y^2 = 1$

$\Rightarrow y = \pm 1$

So area of region bounded by

$A => 2\int\limits_{0}^{1}[(1-3y^2)-(-2y^2)]dy$

$\Rightarrow 2\int\limits_{0}^{1}(1-y^2)dy$

$\Rightarrow 2\left[y-\dfrac{y^3}{3}\right]_{0}^{1} \Rightarrow 2\left[\left(1-\dfrac{1}{3}\right)-\left(0-\dfrac{0}{3}\right)\right]$

$\Rightarrow 2\left[\dfrac{1}{1}-\dfrac{1}{3}\right] \Rightarrow 2\left[\dfrac{2}{3}\right] = \dfrac{4}{3}$ square units

58. (a)

$$\frac{x^2}{9} + \frac{y^2}{5}$$

Now $e^2 - 1 - \frac{5}{9} = \frac{4}{9}$ $\therefore$ e = $\frac{2}{3}$

Equation of tangent at [2, (5/3)] is given as

$\Rightarrow \left(\frac{2x}{9}\right) + \left(\frac{4}{3}\right) = 1$

$\Rightarrow$ Let F and f' be foci

$\Rightarrow$ area of $\triangle$CPQ = $\left(\frac{1}{2}\right) \times \left(\frac{9}{2}\right) \times 3 = \frac{27}{4}$

$\Rightarrow$ area of $\square$PQRS = $4 \times \frac{27}{4}$ = 27 sq. units.

59. (c)

Given: f(x) = sin x – cos x – kx + 5

f(x) decrease for all x if f'(x) < 0

f'(x) = cos x + sin x – k < 0

$\therefore$ k > cos x + sin x

We know, cos x + sin x = $\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$

$\Rightarrow \sqrt{2}\left(\sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x\right) = \sqrt{2}\left(\sin\left(\frac{\pi}{4}+x\right)\right)$ and

$-1 \le \sin\left(\frac{\pi}{4}+x\right) \le 1 \Rightarrow -\sqrt{2} \le \sqrt{2}\sin\left(\frac{\pi}{4}+x\right) \le \sqrt{2}$

$\therefore -\sqrt{2} \le \cos x + \sin x \le \sqrt{2}$

Max (cos x + sin x) = $\sqrt{2}$

$\therefore$ k > $\sqrt{2}$

60. (b)

Let $x = x_1i + x_2j + x_3k$

Then $x \times i = -x_2k + x_3j$

$x \times j = x_1k - x_3i$

$x \times k = -x_1j + x_2i$

$\therefore (x \times i)^2 + (x \times j)^2 + (x \times k)^2$

$\Rightarrow x_2^2 + x_3^2 + x_1^2 + x_3^2 + x_1^2 + x_2^2$

$\Rightarrow 2(x_1^2 + x_2^2 + x_3^2) \Rightarrow 2x^2$

$\Rightarrow 2|x|^2$

61. (c)

Let $\vec{a}, \vec{b}$ be two unit vectors whose sum is also

a unit vector $\vec{c}$

$\vec{a} + \vec{b} = \vec{c}$ .....(1)

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Now, magnitude of sum of $\vec{a}$ and $\vec{b}$ is

$\Rightarrow |\vec{a} + \vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 (\vec{a} . \vec{b})$

$\Rightarrow 1 = 1 + 1 + 2(\vec{a} . \vec{b})$ ......(2)

$\Rightarrow \vec{a} . \vec{b} = \dfrac{1}{2}$

Magnitude of their difference is given by

$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \vec{b}$

$= 1 + 1 + 2 \times \dfrac{1}{2} = 3$

$\Rightarrow |a - b| = \sqrt{3}$

62. (a)

$x^2 + x + 1 = 0$ ....(1)

Roots of this equation is

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

α and β are the roots of eq.(1)

So $\alpha = \frac{-1 + \sqrt{3}i}{2}$ and $\beta = \frac{-1 - \sqrt{3}i}{2}$

Consider, $\alpha^n + \beta^n$

$$= \left(\frac{-1 + \sqrt{3}i}{2}\right)^n + \left(\frac{-1 - \sqrt{3}i}{2}\right)^n$$

$$= \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^n + \left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)^n$$

$$= \left(e^{i\frac{2\pi}{3}}\right)^n + \left(e^{-i\frac{2\pi}{3}}\right)^n \ldots\ldots \left\{ \begin{array}{l} \therefore \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \text{ and} \\ \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2} \end{array} \right\}$$

$$= e^{i\frac{2n\pi}{3}} + e^{-i\frac{2n\pi}{3}}$$

$$= \cos\left(\frac{2n\pi}{3}\right) + i\sin\left(\frac{2n\pi}{3}\right) + \cos\left(\frac{2n\pi}{3}\right) - i\sin\left(\frac{2n\pi}{3}\right)$$

$$= 2\cos\left(\frac{2n\pi}{3}\right)$$

Hence $\alpha^n + \beta^n = 2\cos\left(\frac{2n\pi}{3}\right)$

63. (c)

$y = \dfrac{x^2 + 2x + y}{2x^2 + 4x + 9}$

$\Rightarrow 2x^2 y + 4xy + 9y = x^2 + 2x + 4$

$\Rightarrow x^2(2y-1) + x(4y-2) + 9y-4 = 0$

$\Delta \geq 0$

$\Rightarrow (4y-2)^2 - 4(2y-1)(9y-4) \geq 0$

$\Rightarrow 4(2y-1)^2 - 4(2y-1)(9y-4) \geq 0$

$\Rightarrow 4(2y-1)[2y-1-9y+4] \geq 0$

$\Rightarrow 4(2y-1)(3-7y) \geq 0$

$\Rightarrow 4(2y-1)(7y-3) \leq 0$

$\Rightarrow \dfrac{3}{7} \leq y \leq \dfrac{1}{2}$

$[y_{greatest} = \dfrac{1}{2}]$

64. (b)

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & n & n\left(\dfrac{n+1}{2}\right) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ The statement is demoted by P(n)

$P(n) : A^n = \begin{bmatrix} 1 & n & \dfrac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore P(1) : A^1 \begin{bmatrix} 1 & 1 & 1\dfrac{(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore P(1)$ is true

Again $P(2) : A^2 = A \times A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 & \dfrac{2(2+1)}{2} \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore P(2)$ is true.

Let $P(m)$ b true

$P(m) : A^m = \begin{bmatrix} 1 & m & m(m+1) \\ 0 & 1 & m \\ 0 & 0 & 1 \end{bmatrix}$

$P(m+1) = A^{m+1} = A^m . A = \begin{bmatrix} 1 & m & \dfrac{m(m+1)}{2} \\ 0 & 1 & m \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1+m & 1+m+\dfrac{m(m+1)}{2} \\ 0 & 1 & 1+m \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & (m+1) & \dfrac{(m+1)(m+2)}{2} \\ 0 & 1 & m+1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{m+1} = \begin{bmatrix} 1 & (m+1) & \dfrac{(m+1)\{(m+1)+1\}}{2} \\ 0 & 1 & (m+1) \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ P(m + 1) is true

$\Rightarrow$ P(1) & P(2) is true and p(m + 1) is true when p(m) is true

$\Rightarrow$ By mathematical induction method, we can say that p(n) is also true i.e.

$$A^n = \begin{bmatrix} 1 & n & \dfrac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

65. (d)

The 2nd det $D_2 = \begin{bmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^n a & -(-1)^n b & (-1)^n c \end{bmatrix}$

$$= (-1)^n \begin{bmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{bmatrix}$$

$= (-1)^n D$

$\therefore R_{13}, R_{23}$ and taking transpose

$\therefore (1+(-1)^n)D_1 = 0$ for any odd integer

$\because D_1 \neq 0$ since $b(a + c) \neq 0$

66. (b)

f(x) is continuous and differentiable

f(0) = f(1) = 0 by Rolle's theorem

f '(a) = 0, a belongs to (0, 1)

Given f'(0) = 0 by Rolle's theorem

f''(0) = 0 for some, c belongs to (0, a)

Therefore answer is f''(c) = 0 for some $C \in R$

67. (b, c)

We have $f(x) = x^n$

$\Rightarrow f'(x) = nx^{n-1}$

$\therefore f'(\alpha + \beta) = f'(\alpha) + f'(\beta)$ for all $\alpha, \beta > 0$

$\Rightarrow n(\alpha + \beta)^{n-1} = n\alpha^{n-1} + n\beta^{n-1}$ for all $\alpha, \beta > 0$

$\Rightarrow (\alpha + \beta)^{n-1} = \alpha^{n-1} + \beta^{n-1}$ for all $\alpha, \beta > 0$

$\Rightarrow n - 1 = 1 \Rightarrow n = 2$

Also, for n = 0, we have

f(x) = 1 for all x
$\Rightarrow f'(x) = 0$ for all x
$\therefore f'(\alpha + \beta) = f'(\alpha) + f'(\beta)$
Hence, n = 0, 2

68. (c)
Given: f(x) = f(a – x) = 1
Let $I = \int\limits_0^a \dfrac{dx}{1 + f(x)}$ $\qquad \Rightarrow \int\limits_0^a \dfrac{f(x)dx}{1 + f(x)}$

$\Rightarrow \int\limits_0^a \dfrac{1 + f(x) - 1}{1 + f(x)} dx \qquad \Rightarrow \int\limits_0^a dx - \int\limits_0^a \dfrac{dx}{1 + f(x)}$

$\Rightarrow 2I = \int\limits_0^a dx$

$\Rightarrow 2I = |x|_0^a$

$\Rightarrow I = \dfrac{a}{2}$

69. (b, d)

Given: ax + by + c = 0

By = – ax – c

$y = -\dfrac{a}{b}x - \dfrac{c}{b}$

$m = \text{slope} = -\dfrac{a}{b}$

Tangent slope $\left(\dfrac{dy}{dx}\right)$

$-\dfrac{a}{b} = -\dfrac{1}{x^2}$

$x^2 = \dfrac{b}{a}$ $x^2 > 0$

$\dfrac{b}{a} > 0$ if (1)   b > 0, a > 0

& (2)   b < 0, a < 0

$xy = 1 - 2x \rightarrow xy + 2x = 1$	
$x\dfrac{dy}{dx} + y = -2$	$x(y+2) = 1$
$x\dfrac{dy}{dx} = -(y+2)$	$y + 2 = \dfrac{1}{x}$
$\dfrac{dy}{dx} = \dfrac{-1}{x^2}$	

70. (b, c)

Let $S \rightarrow$ distance at time t

Particle 1 is $x_1 = u$

Particle 2 is $x_2 = \dfrac{1}{2}ft^2$

Then $S = x_1 - x_2$

$= u + (-\dfrac{1}{2}ft^2\ )$ .... (1)

For s being maximum

$\dfrac{ds}{dt} = 0$ $\Rightarrow u - ft = 0$ $\Rightarrow t = \dfrac{u}{f}$

(b) Option correct.

Substituting t in eq^n(1) we get

$S = \dfrac{u}{2f}$ this is greatest distance

(c option correct)

71. (a, c)

   Given: $|Z - i| = |Z + 1| = 1$

   Let $Z = x + iy$

   $|x + iy - i| = 1$

   $\Rightarrow x^2 + (y - 1)^2 = 1$

   $\Rightarrow x^2 + y^2 - 2y + 1 = 1$

   $x^2 + y^2 - 2y + 1 = 1$

   $x^2 + y^2 - 2y = 0$ ....(1)

   Also $|z + 1| = 1$

   $\Rightarrow |x + iy + 1| = 1$

   $\Rightarrow (x + 1)^2 + y^2 = 1$

   $\Rightarrow x^2 + y^2 + 2x = 0$ ......(2)

   Eq.(1) and (2) we get

   $-y = x$

   For $y = 1$, $x = 1$

   $\therefore z = -1 + i$ (c option is correct)

   Also $y = 0 \Rightarrow x = 0$

   $\therefore z = 0$ (a option is correct)

72. (c)

   Reflexive: if $(x, x) \in R$, where x is domain

       Relation = $\{1 + ab > 0$ where $a, b \in R\}$

       For any $a \in R$, $a^2 \geq 0$

       So, $1 + a^2 \geq 1$

       Or, $1 + a^2 > 0$

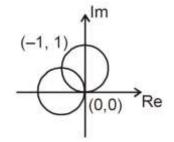 $\Rightarrow 1 + a \cdot a > 0$

       $\Rightarrow (a,a) \in R$ its relation is Reflexive

   Symmetric: if $(y,x) \in R$ where $(x,y) \in R$.

       Relation $(1 + ab > 0$ where $a,b \in R$

       Let $(a,b) \in R$ $a,b \in R$

So, $1 + ab > 0$

$\Rightarrow 1 + ba > 0$

$\Rightarrow (b,a) \in R$ is symmetric

Transitive :- if $(x,z) \in R$, where $(x,y) \in R$ and $(y,z) \in R$

Relation $\{1 + ab > 0 \text{ where } a,b \in R\}$

Let's take $(-2, 0)$ and $(0, 1)$ such that $1 + -2 \times 0 > 0$

And $1 + 0 \times 1 > 0$ so, $(-2, 0)$ and $(0, 1)$ belongs to R.

But $(-2, 1)$ doesn't belongs to R because $1 + -2 \times 1 < 0$

This doesn't follow transitive.

Relation is not transitive.

73. (c, d)

$ax^2 + bx + 1 = 0$

$a = a, b = b, c = 1$

$D = b^2 - 4a$

$D \geq 0$

$b^2 - 4a \geq 0$ $\qquad \{\therefore a, b \in (1, 2, 3)\}$

$b^2 \geq 4a$

$(a, b) = (1, 2)\ (2, 3)\ (1, 3)$ possible order pairs 3

And $\qquad a < b$

74. (a, c)

$y^2 = 4ax$

Tangent $y = \dfrac{x}{t} + at$ and normal $x\cos\theta + y\cot\theta = 2a$

So $(at^2, 2at)$ must satisfy equation of normal

$\Rightarrow t^2\cos\theta + 2 + x\cot\theta - 2 = 0$

So, the roots are $t = -\operatorname{cosec}\theta$ and $t = 2\tan\theta$

75. (c)

Given conic is $x^2 - 6x + 4y + 1 = 0$

$\Rightarrow x^2 - 6x + 9 - 9 + 4y + 1 = 0$

$\Rightarrow x^2 - 3x - 3x + 9 - 9 + 4y + 1 = 0$

$\Rightarrow x(x - 3) - 3(x - 3) + 4y - 8 = 0$

$\Rightarrow (x - 3)^2 + 4y - 8 = 0$

$\Rightarrow (x - 3)^2 = 4(-1)(y - 2)$

Thus, the focus is $(3, 1)$