WBJEE-2018 (Mathematics)

1. If $(2 \le r \le n)$, then ${}^nC_r + 2.{}^nC_{r+1} + {}^nC_{r+2}$ is equal to

   a. $2.{}^nC_{r+2}$

   b. ${}^{n+1}C_{r+1}$

   c. ${}^{n+2}C_{r+2}$

   d. ${}^{n+1}C_r$

2. The number $(101)^{100} - 1$ is divisible by

   a. 10^4

   b. 10^6

   c. 10^8

   d. 10^{12}

3. If n is even positive integer, then the condition that the greatest term in the expansion of $(i+x)^n$ may also have the greatest coefficient is

   a. $\frac{n}{n+2} < x < \frac{n+2}{n}$

   b. $\frac{n}{n+1} < x < \frac{n+1}{n}$

   c. $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$

   d. $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

4. If $\begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = A$, then $\begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$ is

   a. A^2

   b. $A^2 - A + I_3$

   c. $A^2 - 3A + I_3$

   d. $3A^2 + 5A - 4I_3$

   I_3 denotes the det of the identity matrix of order 3

5. If $a_r = (\cos 2r\pi + i\sin 2r\pi)^{1/9}$, then the value of $\begin{vmatrix} a_i & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

   a. 1

   b. −1

   c. 0

   d. 2

6. If $Sr = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} S_r$ is independent of

a. x only
b. y only
c. n only
d. x,y,z and n

7. If the following three linear equations have a non –trivial solution, then x + 4ay +az =0,

x+ 3by +bz = 0, x + 2cy + cz =0

a. a,b,c are in A.P
b. a,b,c are in G.P.
c. a,b,c are in H.P
d. a + b + c = 0

8. On R, a relation ρ is defined by xρy if and only if x–y is zero or irrational.Then

a. ρ is equivalence relation
b. ρ is reflexive but neither symmetric nor transitive
c. ρ is reflexive & symmetric but not transitive
d. ρ is symmetric & transitive but not reflexive

9. On the set R of real numbers, the relation ρ is defined by xρy,(x,y) ∈ R.

a. if |x–y| < 2 then ρ is reflexive but neither symmetric nor transitive
b. if x–y < 2 then ρ is reflexive and symmetric but not transitive
c. if |x| ≥ y then ρ is reflexive and transitive but not symmetric
d. if x > |y| then ρ is transitive but neither reflexive nor symmetric

10. If f : R→R be defined by f(x) =e^x and g : R →R be defined by g(x) = x^2. The mapping g of : R →R be defined by (g of)x =g[f(x)] ∀ x ∈ R, then

a. g of is bijective but f is not injective
b. g of is injective and g is injective
c. g of is injective but g is not bijective
d. g of is surjective and g is surjective

11. In order to get a head at least once with probability ≥ 0.9, the minimum number of times a unbiased coin needs to be tossed is

a. 3
b. 4
c. 5
d. 6

12. A student appears for tests I,II and III. The student is successful if he passes in tests I, II or III.The probabilities of the student passing in tests I,II and III are respectively p,q and $\frac{1}{2}$. If the probability of the student to be successful is $\frac{1}{2}$. Then

a. $p(1 + q) = 1$
b. $q(1 + p) = 1$
c. $pq = 1$
d. $\frac{1}{p}+\frac{1}{q}=1$

13. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then general value of θ is
(n is integer)

a. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$
b. $\frac{n\pi}{4}, 4\pi \pm \frac{\pi}{6}$
c. $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$
d. $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

14. If $0 \leq A \leq \frac{\pi}{4}$, then $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$ is equal to

a. $\frac{\pi}{4}$
b. π
c. 0
d. $\frac{\pi}{2}$

15. Without changing the direction of the axes, the origin is transferred to the point (2, 3). Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

a. $x^2 + y^2 + 4 = 0$
b. $x^2 + y^2 = 4$
c. $x^2 + y^2 - 8x - 12y + 48 = 0$
d. $x^2 + y^2 = 9$

16. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin 2\alpha + 13\cos 2\alpha = 0$ is 2α. The equation of the locus of the point P is

a. $x^2 + y^2 + 4x + 6y + 9 = 0$
b. $x^2 + y^2 - 4x + 6y + 9 = 0$
c. $x^2 + y^2 - 4x - 6y + 9 = 0$
d. $x^2 + y^2 + 4x - 6y + 9 = 0$

17. The point Q is the image of the point P(1,5) about the line y = x and R is the image of the point Q about the line y =-x. The circumcentre of the ΔPQR is

a. (5,1)
b. (–5,1)
c. (1,–5)
d. (0,0)

18. The angular points of a triangle are A(–1,–7), B (5,1) and C (1,4).The equation of the dissector of the angle $\angle ABC$ is

a. $x = 7y + 2$
b. $7y = x + 2$
c. $y = 7x + 2$
d. $7x = y + 2$

19. If one the diameters of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is (2, –3), the radius of S is

a. $\sqrt{41}$ unit
b. $3\sqrt{5}$ unit
c. $5\sqrt{2}$ unit
d. $2\sqrt{5}$ unit

20. A chord AB is drawn from the point A(0,3) on the circle $x^2 + 4x + (y-3)^2 = 0$, and is extended to M such that AM=2AB. The locus of M is

a. $x^2 + y^2 - 8x - 6y + 9 = 0$
b. $x^2 + y^2 + 8x + 6y + 9 = 0$
c. $x^2 + y^2 + 8x - 6y + 9 = 0$
d. $x^2 + y^2 - 8x + 6y + 9 = 0$

21. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

a. 8 : 1
b. 1 : 8
c. 9 : 1
d. 1 : 9

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22. Let A,B be two distinct points on the parabola $y^2=4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, the slope of the line AB is

a. $-\frac{1}{r}$
b. $\frac{1}{r}$
c. $\frac{2}{r}$
d. $-\frac{2}{r}$

23. Let $P(at^2, 2at)$, $Q(ar^2, 2ar)$ be three points on a parabola $y^2 = 4ax$. If PQ is the focal chord and PK, QR are then the value of x areparallel where the co-ordinates of K is (2a, 0), then the value of r is

a. $\frac{t}{1-t^2}$
b. $\frac{1-t^2}{t}$
c. $\frac{t^2+1}{t}$
d. $\frac{t^2-1}{t}$

24. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{x^2}{4}=1$ and the line through P parallel to the y-axis meets the circle $x^2 + y^2 = 9$ at Q, where P, Q are on the same side of the x-axis. If R is a point on PQ such that $\frac{PR}{RQ}=\frac{1}{2}$, then the locus of R is

a. $\frac{x^2}{9}+\frac{9y^2}{49}=1$
b. $\frac{x^2}{49}+\frac{y^2}{9}=1$
c. $\frac{x^2}{9}+\frac{y^2}{49}=1$
d. $\frac{9x^2}{49}+\frac{y^2}{49}=1$

25. A point P lies on a line through Q(1, −2, 3) and is parallel to the line $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$. If P lies on the plane $2x + 3y - 4z + 22 = 0$, then segment PQ equals to

a. $\sqrt{42}$ units
b. $\sqrt{32}$ units
c. 4 unit
d. 5 units

26. The foot of the perpendicular drawn from the point (1, 8, 4) on the line joining the points (0, −11, 4) and (2, − 3, 1) is

a. (4, 5, 2)
b. (−4, 5, 2)
c. (4, −5, 2)
d. (4, 5, −2)

27. The approximate value of sin31º is

a. > 0.5
b. > 0.6
c. < 0.5
d. < 0.4

28. Let $f_1(x)=e^x, f_2(x)=e^{f_1(x)}$, …………, $f_{n+1}(x)= e^{f_n(x)}$ for all n ≥ 1. The for any fixed n, $\frac{d}{dx} f_n(x)$: is

a. $f_n(x)$
b. $f_n(x)f_{n-1}(x)$
c. $f_n(x)f_{n-1}(x)....f_1(x)$
d. $f_n(x)......... f_1(x)e^x$

29. The domain of definition of $f(x)=\sqrt{\frac{1-|x|}{2-|x|}}$ is

a. $(-\infty,-1) \cup (2, \infty)$
b. $[-1,1] \cup (2, \infty) \cup (-\infty,-2)$
c. $(-\infty,1) \cup (2, \infty)$
d. $[-1,1] \cup (2, \infty)$

30. Let f : [a, b] →R be differentiable on [a, b] and k ∈R. Let f(a) = 0 = f'(b). Also let J(x) = f ′(x) + kf(x). Then

a. J(x) > 0 for all x ∈ [a, b]
b. J(x) < 0 for all x ∈ [a, b]
c. J(x) = 0 has atleast one root in (a, b)
d. J(x) = 0 through (a, b)

31. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. Then $\frac{f(1-h)-f(1)}{h^3+3h}$

a. does not exist
b. is $\frac{50}{3}$
c. is $\frac{53}{3}$
d. is $\frac{22}{3}$

32. Let f : [a, b] → R be such that f is differentiable in (a, b), f is continuous at x = a and x = b and moreover f(a) = 0 = f(b). Then

a. there exists atleast one point c in (a, b) such that f ′(c) = f(c)
b. f ′(x) = f(x) does not hold at any point in (a, b)
c. at every point of (a, b), f ′(x) > f(x)
d. at every point of (a, b), f ′(x) < f(x)

33. Let f : R →R be a twice continuously differentiable function such that f(0) = f(1) = f ′(0) = 0. Then

a. $f''(0) = 0$

b. $f''(c) = 0$ for some $c \in R$

c. if $c \neq 0$, then $f''(c) \neq 0$

d. $f'(x) > 0$ for all $x \neq 0$

34. If $\int \left[\frac{x\cos^3 x - \sin x}{\cos^2 x}\right] dx = e^{\sin x} f(x) + c$ where c is constant of integration, then f(x) =

a. secx−x

b. x−secx

c. tanx−x

d. x−tanx

35. If $\int f(x)\sin x\cos x\,dx = \frac{1}{2(b^2-a^2)}\log f(x) + c$, where c is the constant of integration, then f(x) =

a. $\frac{2}{(b^2-a^2)\sin 2x}$

b. $\frac{2}{ab\sin 2x}$

c. $\frac{2}{(b^2-a^2)\cos 2x}$

d. $\frac{2}{ab\cos 2x}$

36. If $M = \int_0^{\pi/2} \frac{\cos x}{x+2}dx, N = \int_0^{\pi/4} \frac{\sin x\cos x}{(x+1)^2}dx$, then the value of M − N

a. π

b. $\frac{\pi}{4}$

c. $\frac{2}{\pi-4}$

d. $\frac{2}{\pi+4}$

37. The value of the integral $I = \int_{1/2014}^{2014} \frac{\tan^{-1}x}{x}dx$ is

a. $\frac{\pi}{4}\log 2014$

b. $\frac{\pi}{2}\log 2014$

c. $\pi \log 2014$

d. $\frac{1}{2}\log 2014$

38. Let $I=\int_{\pi/4}^{\pi/3}\frac{\sin x}{x}dx.$ Then

a. $\frac{1}{2}\le I\le 1$

b. $4\le I\le 2\sqrt{30}$

c. $\frac{\sqrt{3}}{8}\le I\le\frac{\sqrt{2}}{6}$

d. $1\le I\le\frac{2\sqrt{3}}{\sqrt{2}}$

39. The value of $I=\int_{\pi/2}^{5\pi/2}\frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)}+e^{\tan^{-1}(\sin x)}}dx,$ is

a. 1

b. π

c. e

d. $\pi/2$

40. The value of $\lim_{n\to\infty}\frac{1}{n}\left\{\sec^2\frac{\pi}{4n}+\sec^2\frac{2\pi}{4n}+\text{..........}+\sec^2\frac{n\pi}{4n}\right\}$ is

a. $\text{Log}_c 2$

b. $\frac{\pi}{2}$

c. $\frac{4}{\pi}$

d. e

41. The differential equation representing the family of curves $y^2=2d\left(x+\sqrt{d}\right)$ where d is a parameter, is of

a. order 2

b. degree 2

c. degree 3

d. degree 4

42. Let y(x) be a solution of $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ and y(0)=–1. Then y(1) is equal to

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{1}{6}$

d. –1

43. The law of motion of a body moving along a straight line is $x=\frac{1}{2}vt$, x being its distance from a fixed point on the line at time t and v is its velocity there.Then

a. acceleration f varies directly with x
b. acceleration f varies inversely with x
c. acceleration f is constant
d. acceleration f varies directly with t

44. Number of common tangents of $y = x^2$ and $y = -x^2 + 4x - 4$ is

a. 1
b. 2
c. 3
d. 4

45. Given that n numbers of A.Ms are inserted between two sets of numbers a, 2b and 2a, b where a, b $\in$R. Suppose further that the m^{th} means between these sets of numbers are same, then the ratio a : b equals

a. n – m + 1 : m
b. n – m + 1 : n
c. n : n – m + 1
d. m : n – m + 1

46. If $x + \log_{10}(1 + 2^x) = x \log_{10}5 + \log_{10}6$ then the value of x is

a. $\frac{1}{2}$
b. $\frac{1}{3}$
c. 1
d. 2

47. If $Z_r = \sin\frac{2\pi r}{11} - \cos\frac{2\pi r}{11}$ then $\sum_{r=0}^{10} Z_r =$

a. –1
b. 0
c. i
d. –i

48. If z_1 and z_2 be two non zero complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$, then the origin and the points represented by z_1 and z_2

a. lie on a straight line
b. form a right angled triangle
c. form an equilateral triangle
d. form an isosceles triangle

49. If $b_1b_2 = 2(c_1 + c_2)$ and b_1, b_2, c_1, c_2 are all real numbers, then at least one of the equations.$x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ has

a. real roots
b. purely imaginary roots
c. roots of the form a + ib (a, b $\in$R, ab$\neq$0)
d. rational roots

50. The number of selection of n objects from 2n objects of which n are identical and the rest are different is

a. 2^n
b. 2^{n-1}
c. $2^n - 1$
d. $2^{n-1} + 1$

51. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Let B(1, 7) and D(4, – 2) be two points on the circle such that tangents at B and D meet at C. The area of the quadrilateral ABCD is

a. 150 sq. units
b. 50 sq. units
c. 75 sq. units
d. 70 sq. units

52. Let $f(x) = \begin{cases} -2\sin x, & \text{If } x \le -\frac{\pi}{2} \\ A\sin x + B, & \text{If } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{If } x \ge \frac{\pi}{2} \end{cases}$. Then

a. f is discontinuous for all A and B
b. f is continuous for all A = – 1 and B = 1
c. f is continuous for all A = 1 and B = –1
d. f is continuous for all real values of A, B

53. The normal to the curve $y = x^2 - x + 1$, drawn at the points with the abscissa $x_1 = 0, x_2 = -1$ and $x_3 = \frac{5}{2}$

a. are parallel to each other
b. are pair wise perpendicular
c. are concurrent
d. are not concurrent

54. The equation x log x = 3 – x

a. has no root in (1, 3)
b. has exactly one root in (1, 3)
c. x log x – (3 – x) > 0 in [1, 3]
d. x log x – (3 – x) < 0 in [1, 3]

55. Consider the parabola $y^2 = 4x$. Let P and Q be points on the parabola where P(4, –4) & Q(9, 6). Let R be a point on the arc of the parabola between P & Q. Then the area of ΔPQR is largest when

a. $\angle PQR = 90°$

b. R(4, 4)

c. $R\left(\frac{1}{4}, 1\right)$

d. $R\left(1, \frac{1}{4}\right)$

56. A ladder 20 ft long leans against a vertical wall. The top end slides downwards at the rate of 2 ft per second. The rate at which the lower end moves on a horizontal floor when it is 12 ft from the wall is

a. $\frac{8}{3}$

b. $\frac{6}{5}$

c. $\frac{3}{2}$

d. $\frac{17}{4}$

57. For $0 \le p \le 1$ and for any positive a, b; let $I(p) = (a + b)^p$, $J(p) = a^p + b^p$, then

a. $I(p) > J(p)$

b. $I(p) < J(p)$

c. $I(P) < J(P) \text{ in } \left[0, \frac{p}{2}\right] \& I(p) > J(p) \text{ in } \left[\frac{p}{2}, \infty\right)$

d. $I(P) < J(P) \text{ in } \left[\frac{p}{2}, \infty\right) \& J(p) < I(p) \text{ in } \left[0, \frac{p}{2}\right)$

58. Let $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}, \vec{\beta} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$ be three vectors. A vector $\vec{\delta}$, in the plane of $\vec{\alpha}$ and $\vec{\beta}$, projection on $\vec{\gamma}$ is $\frac{1}{\sqrt{3}}$, is given by

a. $-\hat{i} - 3\hat{j} - 3\hat{k}$

b. $\hat{i} - 3\hat{j} - 3\hat{k}$

c. $-\hat{i} + 3\hat{j} + 3\hat{k}$

d. $\hat{i} + 3\hat{j} - 3\hat{k}$

59. Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be three unit vectors such that $\vec{\alpha}.\vec{\beta} = \vec{\alpha}.\vec{\gamma}$ and the angle between $\vec{\beta}$ and $\vec{\gamma}$ is 30°. Then $\vec{\alpha}$ is

a. $2(\vec{\beta} \times \vec{\gamma})$

b. $-2(\vec{\beta} \times \vec{\gamma})$

c. $\pm 2(\vec{\beta} \times \vec{\gamma})$

d. $(\vec{\beta} \times \vec{\gamma})$

60. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If $Re(z_1) > 0$ and $Im(z_2) < 0$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is

a. one

b. real and positive

c. real and negative

d. purely imaginary

61. From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

a. 284 × 17

b. 285 × 17

c. 284 × 16

d. 285 × 16

62. The least positive integer n such that $\begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}^n$ is an identity matrix of order 2 is

a. 4

b. 8

c. 12

d. 16

63. Let ρ be a relation defined on N, the set of natural numbers, as

$\rho = \{(x, y) \in N \times N : 2x + y = 41\}$ Then

a. ρ is an equivalence relation

b. ρ is only reflexive relation

c. ρ is only symmetric relation

d. ρ is not transitive

64. If the polynomial $f(x)=\begin{vmatrix}(1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a\end{vmatrix}$, then the constant term of f(x) is

a. $2 - 3.2^b + 2^{3b}$
b. $2 + 3.2^b + 2^{3b}$
c. $2 + 3.2^b - 2^{3b}$
d. $2 - 3.2^b - 2^{3b}$

65. A line cuts the x-axis at A(5, 0) and the y-axis at B(0, –3). A variable line PQ is drawn perpendicular to AB cutting the x-axis at P and the y-axis at Q. If AQ and BP meet at R, then the locus of R is

a. $x^2 + y^2 - 5x + 3y = 0$
b. $x^2 + y^2 + 5x + 3y = 0$
c. $x^2 + y^2 + 5x - 3y = 0$
d. $x^2 + y^2 - 5x - 3y = 0$

66. In a third order matrix A, a_{ij} denotes the element in the i-th row and j-th column. If
$a_{ij} = 0$ for i = j
= 1 for i> j
= –1 for i< j
Then the matrix is

a. skew symmetric
b. symmetric
c. not invertible
d. non-singula

67. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k), with the lines y = x and x + y = 2 is h^2. The locus of the point P is

a. x = y – 1
b. x = –(y – 1)
c. x = 1 + y
d. x = –(1 + y)

68. A hyperbola, having the transverse axis of length 2 sinθ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Its equation is

a. $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$
b. $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$
c. $(x^2 + y^2) \sin^2\theta = 1 + y^2$
δ. $x^2 \operatorname{cosec}^2\theta = x^2 + y^2 + \sin^2\theta$

69. Let $f(x) = \cos\left(\frac{\pi}{x}\right)$, $x \neq 0$ then assuming k as an integer,

a. f(x) increases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

b. f(x) decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$

c. f(x) decreases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

d. f(x) increases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

70. Consider the function $y = \log_a\left(x + \sqrt{x^2+1}\right), a > 0, a \neq 1$. The inverse of the function

a. does not exist

b. is $x = \log_{1/a}\left(y + \sqrt{y^2+1}\right)$

c. is x = sinh (y ln a)

d. is $x = \cosh\left(-y \ln\frac{1}{a}\right)$

71. Let $I = \int_0^1 \frac{x^3 \cos 3x}{2+x^2} dx$. Then

a. $-\frac{1}{2} < I < \frac{1}{2}$

b. $-\frac{1}{3} < I < \frac{1}{3}$

c. $-1 < I < 1$

d. $-\frac{3}{2} < I < \frac{3}{2}$

72. A particle is in motion along a curve $12y - x^3$. The rate of change of its ordinate exceeds that of abscissa in

a. $-2 < x < 2$

b. $x = \pm 2$

c. $x < -2$

d. $x > 2$

73. The area of the region lying above x-axis, and included between the circle $x^2 + y^2 = 2ax$ & the parabola $y^2 = ax$, $a > 0$ is

a. $8\pi a^2$

b. $a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$

c. $\frac{16\pi a^2}{9}$

d. $\pi\left(\frac{27}{8}+3a^2\right)$

74. If the equation x – cx + d = 0 has roots equal to the fourth powers of the roots of $x^2 + ax + b = 0$, where $a^2 > 4b$, then the roots of $x^2 - 4bx + 2b^2 - c = 0$ will be

a. both real

b. both negative

c. both positive

d. one positive and one negative

75. On the occasion of Diwali festival each student of a class sends greeting cards to others. If there are 20 students in the class, the number of cards send by students are

a. ${}^{20}C_2$

b. ${}^{20}P_2$

c. $2\times{}^{20}C_2$

d. $2\times{}^{20}P_2$

WBJEE-2018 (Mathematics)

ANSWER KEYS

1. (c)	2. (a)	3. (a)	4. (a)	5. (c)	6. (d)	7. (c)	8. (c)	9. (d)	10. (c)
11. (b)	12. (a)	13. (a)	14. (b)	15. (b)	16. (d)	17. (d)	18. (b)	19. (a)	20. (c)
21. (a)	22. (c,d)	23. (d)	24. (a)	25. (a)	26. (d)	27. (a)	28. (c)	29. (b)	30. (c)
31. (c)	32. (a)	33. (b)	34. (b)	35. (c)	36. (d)	37. (b)	38. (c)	39. (b)	40. (c)
41. (c)	42. (c)	43. (c)	44. (b)	45. (d)	46. (c)	47. (b)	48. (c)	49. (a)	50. (a)
51. (c)	52. (b)	53. (c)	54. (b)	55. (c)	56. (a)	57. (b)	58. (a,b,c)	59. (c)	60. (d)
61. (a)	62. (b)	63. (d)	64. (a)	65. (a)	66. (a,c)	67. (a,b)	68. (b)	69. (a,c)	70. (c)
71. (a,b,c,d)	72. (c,d)	73. (b)	74. (a,d)	75. (b,c)					

WBJEE-2018 (Mathematics)

Solution

1. (c)

$^nC_r + 2.\ ^nC_{r+1} + ^nC_{r+2}$

$\Rightarrow \quad ^nC_r + \ ^nC_{r+1} + ^nC_{r+1} + ^nC_{r+2}$

$\Rightarrow \quad ^{n+1}C_{r+1} + \ ^{n+1}C_{r+2}$

$\Rightarrow \quad ^{n+2}C_{r+2}$

2. (a)

$(101)^{100} - 1$

$\Rightarrow (100 + 1)^2 - 1$

$\Rightarrow [^{100}C_0 \cdot (100)^{100}\ (1)^0 + {}^{100}C_1\ (100)^{99}(1)^1 + \ldots\ldots + {}^{100}C_{99}(100)^1(1)^{99}$

$+ {}^{100}C_{100}\ (100)^0(1)^{100}] - 1$

$\Rightarrow [^{100}C_0\ (100)^{100} + {}^{100}C_1\ (100)^{99} + \ldots\ldots + {}^{100}C_{99}\ (100)^1 + 1 - 1]$

$\Rightarrow {}^{100}C_0\ (100)^{100} + {}^{100}C_1\ (100)^{99} + \ldots\ldots + (100)\ (100)^{100}$

$\Rightarrow 10^4\ [^{100}C_0\ (100)^{98} + {}^{100}C_1\ (100)^{97} + \ldots\ldots + 1]$

3. (a)

For greatest term we have

$$\frac{n}{2} < \frac{n+1}{1+|x|} \le \frac{n}{2} + 1$$

$$\Rightarrow \frac{n}{2} < \frac{n+1}{1+|x|} \text{ and } \frac{n+1}{|x|+1} \le \frac{n}{2} + 1$$

$$\Rightarrow \frac{1+|x|}{2} < \frac{n+1}{n} \text{ and } \frac{n+1}{n+2} \le \frac{|x|+1}{2}$$

$$\Rightarrow |x| < \frac{2n+2}{n} - 1 \text{ and } \frac{2n+2}{n+2} - 1 \le |x|$$

$$\Rightarrow x < \frac{n+2}{n} \text{ and } \frac{n}{n+2} \le x$$

$$\therefore \frac{n}{n+2} < x < \frac{n+2}{n}$$

4. (a)

Given

$$A = \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix}$$

$|A| = -1\,(1 + 12) - 7\,(2 + 9) + 0$

$|A| = -13 - 77$

$|A| = -90$

$$\text{Let } B = \begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$$

$$B = 5 \times 3 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ -7 & -1 & -1 \end{vmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$B = 15 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ 0 & 0 & -6 \end{vmatrix}$$

$B = 15\,[0 - 0 - 6\,(-90)]$

$B = (-90)\,(-90)$

$B = A.\,A$

$B = A^2$

5. (c)

Given,

$a_r = (\cos 2r\pi + i \sin 2r\pi)^{1/9}$

$a_r = e^{\frac{2r\pi i}{9}}$

Now,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} & e^{\frac{6\pi i}{9}} \\ e^{\frac{8\pi i}{9}} & e^{\frac{10\pi i}{9}} & e^{\frac{12\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$$

$$= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \begin{vmatrix} 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$$

$= 0$ { $\because$ two row are same}

6. (d)

$$S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} S_r = \begin{vmatrix} 2\sum_{r=1}^{n} r & x & n(n+1) \\ \sum_{r=1}^{n}(6r^2-1) & y & n^2(2n+3) \\ \sum_{r=1}^{n}(4r^3-2nr) & z & n^3(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} S_r = \begin{vmatrix} 2\left(\frac{n(n+1)}{2}\right) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix}$$

$\sum_{r=1}^{n} S_r = 0$ { ∵ two row are same}

7. (c)

System equations

$x + 4ay + az = 0$

$x + 3by + bz = 0$

$x + 2cy + cz = 0$

For non-trivial solution

$\Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$$

$\Rightarrow 1(3bc - 2bc) - 1\,(4ac - 2ac) + (4ab - 3\,ab) = 0$

$\Rightarrow bc - 2ac + ab = 0$

$\Rightarrow bc + ab = 2ac$

$b = \frac{2ac}{a+c}$

a, b, c are in H.P.

8. (c)

xRy$\Rightarrow$ x – y is zero or irrational

xRx$\Rightarrow$ 0 $\therefore$ reflexive

ifxRy$\Rightarrow$ x – y is zero or irrational

$\Rightarrow$ y – x is zero or irrational

$\therefore$ yRx symmetric

xRy$\Rightarrow$ x –y is 0 or irrational

yRz$\Rightarrow$ y –z is 0 or irrational

Then (x – y) + (y – z) = x – z may be rational

$\therefore$ It is not transitive

9. (d)

$(x, x) \in R \Rightarrow x > |x|$ {false}

$\therefore$ not reflexive

If $(x, y) \in R \Rightarrow x > |y| \Rightarrow y > |x|$

$\therefore$ not symmetric

If $(x, y) \in R \Rightarrow x > |y|$; $(y, z) \in R \not\Rightarrow y > |z|$

$\Rightarrow x > |z| \Rightarrow (x, z) \in R$

$\therefore$ transitive

10. (c)

Given

$f(x) = e^x$

$g(x) = x^2$

$$(g\,of)(x) = g[f(x)] = g[e^x] = (e^x)^2 = e^{2x} : x \in R$$

Clearly g(f(x)) is injective and g(x) is not injective.

11. (b)

$p(H) = \frac{1}{2}$

$p(T) = \frac{1}{2}$

$\Rightarrow P = 1 - \frac{1}{2^n} \geq 0.9$

$\Rightarrow 1 - \frac{9}{10} \geq \frac{1}{2^n}$

$\Rightarrow \frac{1}{10} \geq \frac{1}{2^n}$

$\Rightarrow 10 \leq 2^n$

n = 4

12. (a)

Let A, B and C be the events that the student is successful in tests I, II and III respectively. Then p (The student is successful)

$= P\,[(I \cap II \cap III') \cup (I \cap II' \cap III) \cup (I \cap II \cap III)]$

$\Rightarrow \frac{1}{2} = P(I \cap II \cap III') + P(I \cap II' \cap III) + P(I \cap II \cap III)$

$\Rightarrow \frac{1}{2} = P(I)P(II)P(III') + P(I)\,P(II')\,P(III) + P(I)P(II)P(III)$

[$\because$ I, II and III are independent]

$\Rightarrow \frac{1}{2} = p.q.\left(1 - \frac{1}{2}\right) + p \cdot (1-q) \cdot \frac{1}{2} + p.q.\frac{1}{2}$

$\Rightarrow 1 = pq + p$

$\Rightarrow p(q + 1) = 1$

13. (a)

$\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow \sin 4\theta + 2\sin \frac{8\theta}{2} \cos \frac{4\theta}{2} = 0$

$\Rightarrow \sin 4\theta + 2\sin 4\theta \cos 2\theta = 0$

$\Rightarrow \sin 4\theta\,(1 + 2\cos 2\theta) = 0$

$\text{Sin } 4\theta = 0$ or $1 + 2\cos 2\theta = 0$

$4\theta = n\pi$ or $\cos 2\theta = \frac{-1}{2} = \cos \frac{2\pi}{3}$

$\theta = \frac{n\pi}{4}$ or $2\theta = 2n\pi \pm \frac{2\pi}{3}$

$\theta = \frac{n\pi}{4}$ or $\theta = n\pi \pm \frac{\pi}{3}$

14. (b)

$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$

$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\frac{\cot A + \cot^3 A}{1 - \cot^4 A}\right)$

$= \tan^{-1}\left(\frac{1}{2}\frac{2\tan A}{1 - \tan^2 A}\right) + \tan^{-1}\left(\frac{\cot A}{1 - \cot^2 A}\right)$

$= \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) + \tan^{-1}\left(\frac{\tan A}{\tan^2 A - 1}\right)$

$= \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) - \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right)$

$= 0$

15. (b)

By replacing

$X \to x + 2, y \to y + 3$

Given equation of circle is $x^2 + y^2 - 4x + 6y + 9 = 0$

$\therefore (x + 2)^2 + (y +3)^2 - 4 (x + 2) - 6 (y + 3) +9 = 0$

$\Rightarrow x^2 +4x + 4 + y^2 + 6x + 9 - 4x - 8 - 6y - 18 + 9 =0$

$\Rightarrow x^2 +y^2 -4 =0$

16. (d)

Let the centre be 0, points on circle from where tangents are drawn is A, B and point of intersection of tangent is P.

$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$

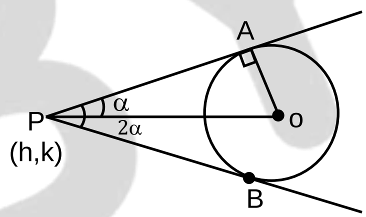
centre of circle O = (–2, 3)

$r = \sqrt{4+9-9\sin^2\alpha-13\cos^2\alpha}$

$r = \sqrt{13-9\sin^2\alpha-13(1-\sin^2\alpha)}$

$r = \sqrt{13\sin^2\alpha-9\sin^2\alpha}$

$r = 2\sin\alpha$



2α is the angle between tangents

$\sin x = \dfrac{OA}{OP}$

$\Rightarrow \sin x = \dfrac{2\sin x}{\sqrt{(h+2)^2+(k-3)^2}}$

$\Rightarrow (h +2)^2+(k-3)^2 = 4$

$\Rightarrow h^2 + k^2 + 4h - 6k + 9 = 0$

Focus of point p is

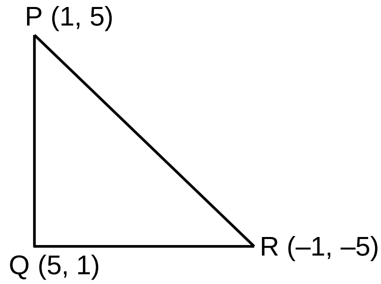
$x^2 + y^2 + 4x - 6y + 9 = 0$

17. (d)

Clearly P (1, 5)

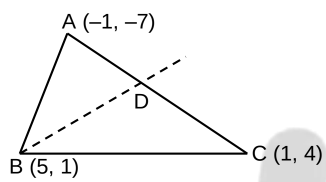
Q = (5, 1) {∵ y = x}

R = (–1, –5) { ∵ y = –x}



∴ Circum center of PQR is $\left(\frac{1-1}{2}, \frac{5-5}{2}\right)$ = (0, 0)

18. (b)



AB = $\sqrt{(-1-5)^2 + (-7-1)^2} = 10$

BC = $\sqrt{(1-5)^2 + (4-1)^2} = 5$

BD divides AC in ratio 2 : 1

D = $\left(\frac{-1+2}{2+1}, \frac{-7+8}{2+1}\right)$

D = $\left(\frac{1}{3}, \frac{1}{3}\right)$

∴ Equation of BD is

⇒ y – 1 = $\frac{1-\frac{1}{3}}{5-\frac{1}{3}}(x-5)$

⇒ (y – 1) = $\frac{2}{14}(x-5)$

⇒ x – 7 y + 2 = 0

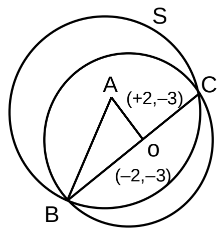
⇒ 7 y = x + 2

19. (a)

Given equation of circle is $x^2 + y^2 + 4x + 6y - 12 = 0$

Whose centre is (2, –3) and radius = $\sqrt{2^2 + (-3)^2 + 12}$ = 5

Now, according to given information, we have the following figure.



$x^2 + y^2 - 4x + 6y - 12 = 0$

Clearly, $AO \perp BC$, as O is mid point of the chord.

Now, in ΔAOB we have

OA = $\sqrt{(2+2)^2 + (-3+3)^2} = \sqrt{16} = 4$

And OB = 5

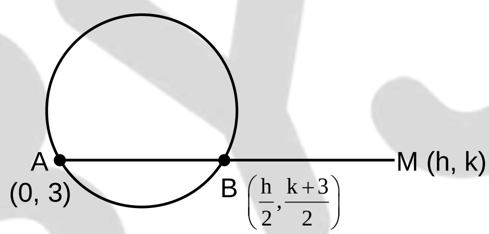
$\therefore$ AB = $\sqrt{OA^2 + OB^2}$ AB = $\sqrt{160+25}$

AB = $\sqrt{41}$

20. (c)

Given equation of circle is $x^2 + 4x + (y - 3)^2 = 0$

AM = 2AB



B is the mid point of AM

$\Rightarrow$ B = $\left(\frac{h}{2}, \frac{k+3}{2}\right)$ lies on the circle

Equation of circle is $x^2 + 4x + (y - 3)^2 = 0$

Let x = $\frac{h}{2}$, y = $\frac{k+3}{2}$

$\therefore$ $\frac{h^2}{4} + 2h + \left(\frac{k+3}{2} - 3\right)^2 = 0$

$\frac{h^2}{4} + 2h + \frac{k^2 - 6K + 9}{4} = 0$

$\therefore$ $k^2 + h^2 + 8h - 6k + 9 = 0$

$\therefore$ Locus of m is $x^2 + y^2 + 8x - 6y + 9 = 0$

21. (a)

Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Ellipse: $x^2 + 9y^2 = 9$

$$\frac{x^2}{9} + \frac{y^2}{1} = 1$$

Eccentricity of ellipse e = $\sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}}$

$\therefore$ the eccentricity of the hyperbola be reciprocal to the ellipse

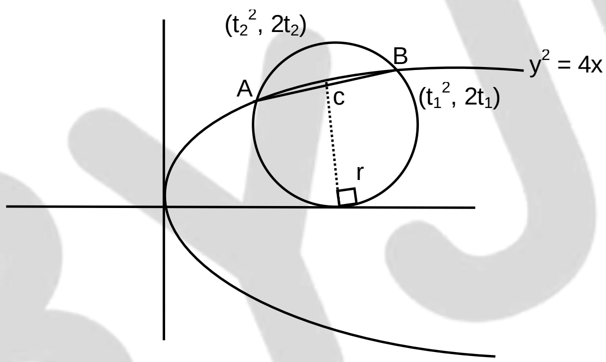
Eccentricity of hyperbola = $\sqrt{\frac{9}{8}}$

$1 + \frac{b^2}{a^2} = \frac{9}{8}$

$\frac{b^2}{a^2} = \frac{1}{8}$

$a^2 : b^2 = 8 : 1$

22. (c, d)



Let, A $\left(t_2^2, 2t_2\right)$, B$(t_1^2, 2t_1)$ be the two points on the parabola.

AB is the diameter of the circle.

Let, c be the centre of the circle.

$\Rightarrow$ C = $\left(\frac{t_1^2 + t_2^2}{2}, t_1 + t_2\right)$

This axis of the parabola is x-axis

So, the circle touches the x-axis

Hence, distance of c from x-axis = radius of circle.

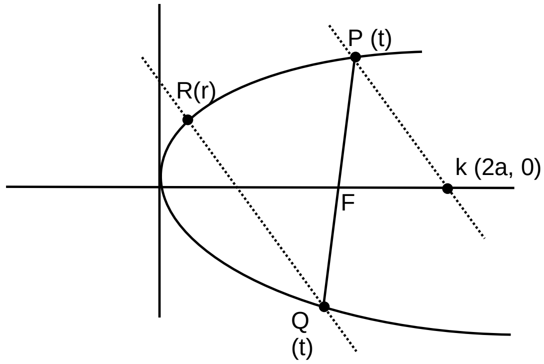
$\Rightarrow |t_1 + t_2| = r$

$\Rightarrow (t_1 + t_2) = \pm r$

Slope of AB = $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{2t_1 - 2t_2}{t_1^2 - t_2^2} = \frac{2}{t_1 + t_2} = \pm\frac{2}{r}$$

23. (d)



Slope of line Pk = slope of line QR

$m_{Pk} = m_{QR}$

$\Rightarrow \frac{2at-0}{at^2-2a} = \frac{2at'-2ar}{a(t')^2-ar^2}$

$\Rightarrow \frac{t}{t^2-2} = \frac{t'-r}{(t')^2-r^2}$

$\Rightarrow -t' - tr^2 = -t - rt^2 - 2t' + 2r$ $\{tt' = -1\}$

$\Rightarrow t' - tr^2 = -t + 2r - rt^2$

$\Rightarrow -tr^2 + r(t^2-2) + t' + t = 0$

$\lambda = \frac{(2-t^2) \pm \sqrt{(t^2-2)^2 + 4(-1+t^2)}}{-2t}$

$= \frac{(2-t^2) \pm \sqrt{t^4}}{-2t} = \frac{2-t^2 \pm t^2}{-2t}$

$r = -\frac{1}{t}$

It is not possible as the R & Q will be one and same.

Or $r = \frac{t^2-1}{2}$

24. (a)

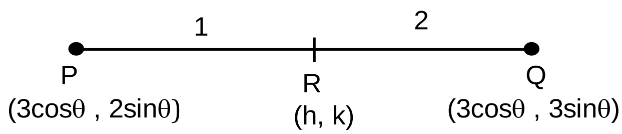
Given

Ellipse: $\frac{x^2}{9} + \frac{y^2}{4} = 1$

P = (3cosθ, 2sinθ)

Circle: $x^2 + y^2 = 9$

Q = (3cosθ, 3sinθ)



$h = \frac{3\cos\theta + 6\cos\theta}{3}$; $k = \frac{3\sin\theta + 4\sin\theta}{3}$

h = 3cosθ ; $k = \frac{7}{3}\sin\theta$

∴ $\sin^2\theta + \cos^2\theta = 1$

$\frac{h^2}{9} + \frac{9k^2}{49} = 1$

Locus is $\frac{x^2}{9} + \frac{9y^2}{49} = 1$

25. (a)

Equation line : $\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$ (let)

Point P (λ + 1, 4λ – 2, 5λ + 3) .

Point p lies on 2x + 3y – 4z + 22 = 0

⇒ 2(λ + 1) + 3 (4λ – 2) – 4(5λ + 3) + 22 = 0

⇒ –6λ + 6 = 0

⇒ λ = 1

Point p = (2, 2, 8), q = (1, –2, 3)

Distance PQ $= \sqrt{1^2 + 4^2 + 5^2}$

$= \sqrt{1 + 16 + 25}$

$= \sqrt{42}$

26. (d)

Equation of line joining point (0, –11, 4) and (2, –3, 1)

$\frac{x-2}{2} = \frac{y+3}{8} = \frac{z-1}{-3} = \lambda$ (let)

DR's of PQ (2λ + 1 , 8λ – 11, –3λ – 3)

Now, (2λ + 1) 2 + (8λ – 11)8 +(–3λ – 3)(–3)=0

⇒ 77λ – 77 =0

⇒ λ =1

Q = (4, 5, –2)

27. (a)

$\because \sin 30 = \frac{1}{2}$

$\therefore \sin 31 > \frac{1}{2}$ { $\because$ sinx is increasing function}

28. (c)

$\frac{d}{dx} f_r(x)$

$= \frac{d}{dx} e^{f_{r-1}(x)}$

$= e^{f_{r-1}(x)} \frac{d}{dx} f_{r-1}(x)$

$= f_r(x) \frac{d}{dx} f_{r-1}(x) \quad \forall r \in N > 1$

$\therefore \frac{d}{dx} f_n(x) = f_n(x) \frac{d}{dx} f_{n-1}(x)$

$= f_n(x)\, f_{n-1}(x) \frac{d}{dx} f_{n-2}(x)$

$= f_n(x)\, f_{n-1}(x)\, f_{n-2}(x) \frac{d}{dx} f_{n-3}(x)$

⋮

$= f_n(x) f_{n-1}(x)\, f_{n-2}(x) \ldots\ldots. f_3(x) f_2(x) \frac{d}{dx} f_1(x)$

$= f_n(x) f_{n-1}(x)\, f_{n-2}(x) \ldots\ldots. f_3(x) f_2(x) \frac{d}{dx} e^x$

$= f_n(x)\, f_{n-1}(x) f_{n-2}(x) \ldots\ldots\ldots\ldots f_3(x) f_2(x) e^x$

$= f_n(x)\, f_{n-1}(x) f_{n-2}(x) \ldots\ldots\ldots\ldots f_3(x) f_2(x) f_1(x)$

29. (b)

$f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$

$\therefore \frac{1-|x|}{2-|x|} \geq 0 \Rightarrow |x| \leq 1$ or $|x| \geq 2$

$\Rightarrow x \in [-1, 1]$ or $x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow x \in [-1,1] \cup (-\infty,-2) \cup (2,\infty)$

30. (c)

Let $g(x) = e^{kx}f(x)$

$f(a) = 0 = f(b)$

by Rolle's theorem

$g'(c) = 0 \quad , \quad c \in (a,b)$

$g'(x) = e^{kx}f'(x) + ke^{kx}f(x)$

$g'(c) = 0$

$\Rightarrow e^{kc}(f'(c) + kf(c)) = 0$

$f'(c) + kf(c) = 0$

foratleast one c in (a, b)

31. (c)

Given

$f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

Now,

$$\lim_{h\to 0} \frac{f(1-h)-f(1)}{h^3+3h}$$

$\Rightarrow \frac{0}{0}$ form, using L' hospital rule

$$\lim_{h\to 0} \frac{-f'(1-h)}{3h^2+3}$$

$$\Rightarrow \frac{-f'(1)}{3}$$

$$\Rightarrow \frac{-[30(1)^9 - 56(1)^7 + 30(1)^5 - 63(1)^2 + 6(1)]}{3}$$

$$\Rightarrow \frac{-[30-56+30-63+6]}{3}$$

$$\Rightarrow \frac{53}{3}$$

32. (a)

Let, $h(x) = e^{-x} f(x)$

$h(a) = 0, h(b) = 0$

h(x) is continuous and differentiable function

by Rolle's theorem

$h'(c) = 0 \quad , \quad c \in (a,b)$

$e^{-x}f'(c) + (-e^{-x})f(c) = 0$

$e^{-x}f'(c) = e^{-x}f(c)$

$f'(c) = f(c)$

33. (b)

f(x) is continuous and differentiable function

$f(0) = f(1) = 0 \Rightarrow$ by Rolle's theorem

$f'(a) = 0, \quad a \in (0,1)$

Given $f'(0) = 0$

By Rolle's theorem $f''(0) = 0$ for some c, $c \in (0, a)$

34. (b)

$$I = \int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right)dx$$

$$I = \int e^{\sin x}(x\cos x - \tan x \sec x)dx$$

$$I = \left(xe^{\sin x} - \int e^{\sin x}\right) - \left[e^{\sin x}\sec x - \int e^{\sin x}dx\right] + c$$

$I = xe^{\sin x} - e^{\sin x}\sec x + c$

$I = e^{\sin x}(x - \sec x) + c$

35. (c)

$$\int f(x)\sin x\cos x\,dx = \log(f(x))\frac{1}{2(b^2-a^2)} + C$$

Differentiate with respect to x

$$f(x)\sin x\cos x = \frac{f'(x)}{f(x)}\frac{1}{2(b^2-a^2)} + C$$

$$\Rightarrow \sin 2x\,(b^2 - a^2) = \frac{f'(x)}{(f(x))^2}$$

On integrating

$$\frac{-1}{f(x)} = \frac{-(b^2-a^2)\cos 2x}{2}$$

$f(x) = \dfrac{2}{(b^2 - a^2)\cos 2x}$

36. (d)

$N = \int_0^{\frac{\pi}{4}} \dfrac{\sin x \cos x}{(x+1)^2} dx$

$N = \int_0^{\frac{\pi}{4}} \dfrac{\sin 2x}{2(x+1)^2} dx$

Let $2x = t \Rightarrow 2dx = dt$ — $\frac{\pi}{2}$ (U.L), 0 (L.L)

$N = \int_0^{\frac{\pi}{2}} \dfrac{\sin t}{4\left(\frac{t}{2}+1\right)^2} dt$

$N = \int_0^{\frac{\pi}{2}} \dfrac{\sin t}{(t+2)^2} dt$

Apply by part method

$N = \sin t\left(\dfrac{-1}{t+2}\right) + \int_0^{\frac{\pi}{2}} \dfrac{\cos t}{t+2} dt$

$N = \left(-\sin\left(\dfrac{1}{t+2}\right)\right)_0^{\frac{\pi}{2}} + M$

$M - N = \dfrac{2}{\pi + 4}$

37. (b)

$I = \int_{\frac{1}{2014}}^{2014} \dfrac{\tan^{-1} x}{x}\, dx$ ....(1)

Let $x = \dfrac{1}{t} \Rightarrow dx = -\dfrac{1}{t^2} dt$

$I = \int_{2014}^{\frac{1}{2014}} \dfrac{\tan^{-1}\left(\frac{1}{t}\right)}{\left(\frac{1}{t}\right)}\left(-\dfrac{1}{t^2}\right) dt$

$$I = \int_{\frac{1}{2014}}^{2014} \frac{\cot^{-1} t}{t} dt \quad \ldots\ldots(2)$$

From eq(1) + eq(2)

$$2I = \int_{\frac{1}{2014}}^{2014} \frac{\tan^{-1} t + \cot^{-1} t}{t} dt$$

$$2I = \int_{\frac{1}{2014}}^{2014} \frac{\pi/2}{t} dt$$

$$I = \frac{\pi}{4}(\ell nt)_{\frac{1}{2014}}^{2014}$$

$$I = \frac{\pi}{4}(\ell n2014 - \ell n\frac{1}{2014})$$

$$I = \frac{\pi}{4}(2\ell n2014)$$

$$I = \frac{\pi}{2}\ell n2014$$

38. (c)

$$I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$$

$\frac{\sin x}{x}$ is a decreasing function

$$\text{So } \frac{\pi}{12} \times \frac{\sin\frac{\pi}{3}}{\frac{\pi}{3}} \le I \le \frac{\pi}{12} \times \frac{\sin\frac{\pi}{4}}{\frac{\pi}{4}}$$

$$\Rightarrow \frac{1}{4} \times \frac{\sqrt{3}}{2} \le I \le \frac{1}{3} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{3}}{8} \le I \le \frac{\sqrt{2}}{6}$$

39. (b)

$$I = \int_{\pi/2}^{\frac{5\pi}{2}} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$$

$$I = \int_{\pi/2}^{\pi} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{\frac{5\pi}{2}} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx \text{ ....(1)}$$

$$I = \int_{\pi/2}^{\pi} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{\frac{5\pi}{2}} \frac{e^{-\tan^{-1}(\sin x)}}{e^{-\tan^{-1}(\sin x)} + e^{-\tan^{-1}(\cos x)}} dx$$

$$\left\{ \text{by} \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx \right\}$$

$$I = \int_{\frac{\pi}{2}}^{\pi} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx + \int_{\pi}^{\frac{5\pi}{2}} \frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx \text{ ....(2)}$$

From eq (1) + eq(2)

$$2I = \int_{\pi/2}^{\pi} 1dx + \int_{\pi}^{\frac{5\pi}{2}} 1dx$$

$$2I = (x)_{\frac{\pi}{2}}^{\frac{5\pi}{2}} = 2\pi$$

$$I = \pi$$

40. (c)

$$L = \lim_{n\to\infty} \frac{1}{n}\left\{ \sec^2\frac{\pi}{4n} + \sec^2\frac{2\pi}{4n} + \ldots\ldots + \sec^2\frac{n\pi}{4n} \right\}$$

$$L = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \sec^2\left(r\cdot\frac{\pi}{4n}\right)$$

$$\text{Let } \frac{r}{n} = x \Rightarrow \frac{1}{n} = dx$$

$$L = \int_0^1 \sec^2\left(\frac{\pi}{4}x\right)dx$$

$$L = \left[\tan\left(\frac{\pi}{4}x\right)\right]_0^1 \times \frac{4}{\pi}$$

$L = \frac{4}{\pi}\left(\tan\frac{\pi}{4} - \tan 0\right)$

$L = \frac{4}{\pi}$

41. (c)

$y^2 = 2d\left(x + \sqrt{d}\right)$ .....(i)

Differentiate with respect to x

$2y\frac{dy}{dx} = 2d \qquad \Rightarrow d = y\frac{dy}{dx}$

Put in equation (i)

$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{y\frac{dy}{dx}}\right)$

$y^2 = 2y\frac{dy}{dx} + 2y^{3/2}\left(\frac{dy}{dx}\right)^{3/2}$

$\left(y^2 - 2xy\frac{dy}{dx}\right)^2 = 4y^3\left(\frac{dy}{dx}\right)^3$

Degree three.

42. (c)

$\left(1+x^2\right)\frac{dy}{dx} + 2xy - 4x^2 = 0$

$\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{4x^2}{1+x^2}$

$\text{I.F.} = e^{\int Pdx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\ell n\left|1+x^2\right|} = \left(1+x^2\right)$

$\Rightarrow y\left(1+x^2\right) = \int \frac{4x^2}{\left(1+x^2\right)} \times \left(1+x^2\right)dx + c$

$\Rightarrow y\left(1+x^2\right) = \int 4x^2 dx + c$

$\Rightarrow y\left(1+x^2\right) = \frac{4x^3}{3} + c$

Put $y(0) = -1 \Rightarrow -1 = c$

$\therefore y\left(1+x^2\right) = \frac{4x^3}{3} - 1$

$y(1) \Rightarrow y(1 + 1) = \frac{4(1)}{3} - 1$

$$2y = \frac{1}{3}$$

$$y = \frac{1}{6}$$

43. (c)

$x = \frac{1}{2}vt$

Differentiate with respect to x

$\Rightarrow x = \frac{1}{2}.\frac{dx}{dt}.t$

$\Rightarrow \int \frac{2dt}{t} = \int \frac{dx}{x}$

$\Rightarrow \ell nc + 2\ell nt = \ell nx$

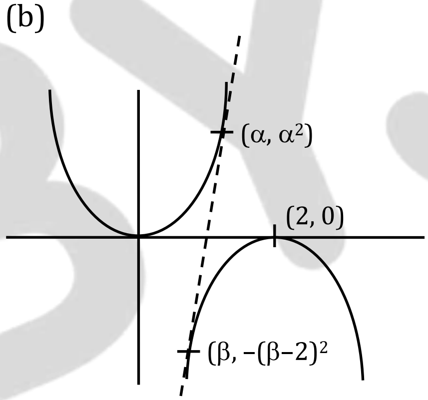
$\Rightarrow x = t^2c$

$\Rightarrow \frac{dx}{dt} = 2tc$

$\Rightarrow \frac{d^2x}{dt^2} = 2c$

Hence, acceleration is constant.

44. (b)



$y = x^2$; $y = -(x-2)^2$

$\frac{\alpha^2 + (\beta-2)^2}{\alpha-\beta} = 2\alpha = -2(\beta-2)$

$\Rightarrow \alpha = 2 - \beta$ $\Rightarrow \beta = 2 - \alpha$

$\frac{\alpha^2+\alpha^2}{\alpha-2+\alpha} = 2\alpha \Rightarrow \frac{2\alpha^2}{2\alpha-2} = 2\alpha$

$\Rightarrow \alpha^2 = \alpha(2\alpha - 2)$

$\Rightarrow \alpha^2 = 2\alpha^2 - 2\alpha$

$\Rightarrow \alpha^2 = 2\alpha \quad \Rightarrow \alpha = 0, 2$

$\Rightarrow \alpha = 0 \quad \Rightarrow \beta = 2$

$\Rightarrow \alpha = 2 \quad \Rightarrow \beta = 0$

Hence, two common tangent.

45. (d)

a..........n A.Ms...........2b

(Difference)d = $\dfrac{2b-a}{n+1}$

$A_m = a + m\left(\dfrac{2b-a}{n+1}\right)$ .....(i)

2a..........n A.Ms...........b

$d = \dfrac{b-2a}{n+1}$

$A_m = 2a + m\left(\dfrac{b-2a}{n+1}\right)$ .....(ii)

Equating equation (i) & (ii)

$a = \dfrac{m}{n+1}(b+a)$

$\Rightarrow \dfrac{a}{b} = \dfrac{m}{n-m+1}$

46. (c)

$x + \log_{10}(1 + 2^x) = x \log_{10}5 + \log_{10}6$

$\Rightarrow \quad x \log_{10}10 + \log_{10}(1 + 2^x) = \log_{10}5^x + \log_{10}6$

$\Rightarrow \quad \log_{10}(1 + 2^x) = \log_{10}5^x + \log_{10}6 - \log_{10} 10^x$

$\Rightarrow \quad \log_{10}(1 + 2^x) = \log_{10}\left(\dfrac{5^x.6}{10^x}\right)$

$\Rightarrow \quad 1 + 2^x = \dfrac{6\,.\,5^x}{2^x\,.\,5^x}$

$\Rightarrow \quad 1 + 2^x = \dfrac{6}{2^x}$

let $2^x = t$

$\Rightarrow \quad 1 + t = \dfrac{6}{t}$

$\Rightarrow \quad t^2 + t - 6 = 0$

$\Rightarrow \quad (t + 3)(t - 2) = 0$

$t = -3$	$t = 2$
$2^x = -3$	$2^x = 2$
(not possible)	$x = 1$

47. (b)

Given

$$Z_r = \sin\frac{2\pi r}{11} - i\cos\frac{2\pi r}{11}$$

$$Z_r = -i\left(\cos\frac{2\pi r}{11} + i\sin\frac{2r\pi}{11}\right)$$

$$Z_r = -ie^{\frac{i2r\pi}{11}}$$

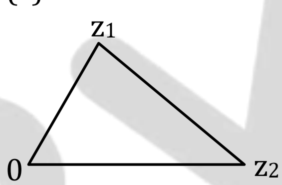
Now

$$\sum_{r=0}^{10} Z_r = -i\sum_{r=0}^{10}\left(e^{\frac{i2r\pi}{11}}\right)$$

$= -i(0)$

$= 0$

48. (c)



$$\Rightarrow z_1 = z_2 e^{\frac{i\pi}{3}}$$

$$\Rightarrow 2z_1 = z_2\left(1 + i\sqrt{3}\right)$$

$$\Rightarrow 2z_1 = z_2 + i\sqrt{3}z_2$$

$$\Rightarrow 2z_1 - z_2 = i\sqrt{3}z_2$$

Squaring both side

$$\Rightarrow 4z_1^2 + z_2^2 - 4z_1z_2 = -3z_2^2$$

$$\Rightarrow 4z_1^2 + z_2^2 = 4z_1z_2$$

Hence from equilateral triangle.

49. (a)

Suppose the equations $x^2 + b_1x + c_1 = 0$ & $x^2 + b_2x + c_2 = 0$ have real roots.

then $b_1^2 \geq 4c_1$ .....(i)

$b_2^2 \geq 4c_2$ .....(ii)

Given that $b_1b_2 = 2(c_1 + c_2)$

On squaring $b_1^2 b_2^2 = 4\left(c_1^2 + c_2^2 + 2c_1c_2\right) = 4\left[\left(c_1 - c_2\right)^2 + 4c_1c_2\right]$

$\Rightarrow b_1^2 b_2^2 - 16c_1c_2 = 4\left(c_1 - c_2\right)^2 \geq 0$

Multiplying (i) & (ii), we get

$b_1^2 b_2^2 \geq 16c_1c_2$

Therefore, at least one equation have real roots.

50. (a)

Total no. of ways = ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots\dots\dots + {}^nC_n$

$= 2^n$

51. (c)

s : $x^2 + y^2 - 2x - 4y - 20 = 0$

Centre of circle A = (1, 2)

Equation of tangent at B(1, 7)

$\Rightarrow x + 7y - (x + 1) - 2(y + 7) - 20 = 0$

$\Rightarrow 5y = 35$

$\Rightarrow y = 7$

Equation of tangent at D(4, –2)

$\Rightarrow 4x - 2y - (x + 4) - 2(y - 2) - 20 = 0$

$\Rightarrow 3x - 4y = 20$

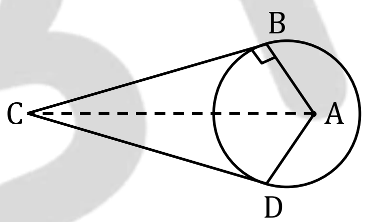
$\therefore$ coordinates of C are (16, 7)

Length of AB = $\sqrt{\left(1-1\right)^2 + \left(7-2\right)^2} = 5$

Length of BC = $\sqrt{\left(16-1\right)^2 + \left(7-7\right)^2} = 15$

$\therefore$ The area of quadrilateral ABCD = $2 \times \frac{1}{2} \times 5 \times 15$

= 75 sq. units.



52. (B)

Given,

$$f(x) = \begin{cases} -2\sin x & \text{if } x \leq -\frac{\pi}{2} \\ A\sin x + b & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & \text{if } x \geq \frac{\pi}{2} \end{cases}$$

From above conditions function f(x) is continuous throughout the real line, when function f(x) is continuous at $x=-\frac{\pi}{2}$ and $\frac{\pi}{2}$ for continuity at $x=-\frac{\pi}{2}$

$\lim_{x\to\frac{\pi^-}{2}} f(x)=\lim_{x\to\frac{\pi^+}{2}} f(x)=f\left(-\frac{\pi}{2}\right)$ .....(ii)

$\lim_{x\to\frac{\pi^-}{2}} f(x)=2$

$\therefore$ from equation (ii) we get

–A + B = 2 .....(iii)

For continuity at $x=\frac{\pi}{2}$

$\lim_{x\to\frac{\pi^-}{2}} f(x)=\lim_{x\to\frac{\pi^+}{2}} f(x)=f\left(\frac{\pi}{2}\right)$ .....(iv)

Here $\lim_{x\to\frac{\pi^-}{2}} f(x)=A+B$

$\Rightarrow \lim_{x\to\frac{\pi^+}{2}} f(x)=0$

and $f\left(\frac{\pi}{2}\right)=0$

$\therefore$ from equation (iv)

A + B = 0 .....(v)

From equation (iii) & (iv)

A = –1, B = 1

53. (c)

$y = x^2 - x + 1$

$\frac{dy}{dx}=2x-1=m_T$

$\because m_N=\frac{-1}{m_T}\Rightarrow m_N=\frac{1}{1-2x}$

For tangent

$m_{x_1}=1$ Point (0, 1)

(y – 1) = 1(x – 0) $\Rightarrow$ x – y + 1 = 0 .....(i)

For tangent

$m_{x_2}=\frac{1}{3}$ point (–1, 3)

(y – 3) = $\frac{1}{3}$(x + 1) $\Rightarrow$ 3y – 9 = x + 1

$\Rightarrow$ x – 3y + 10 = 0 .....(ii)

For normal

$m_{x_3}=-\frac{1}{4}$ point $\left(\frac{5}{2},\frac{19}{4}\right)$

$$\left(y-\frac{19}{4}\right)=-\frac{1}{4}\left(x-\frac{5}{2}\right) \Rightarrow x+4y=\frac{43}{2} \quad .....(iii)$$

To find intersection point tangent (1) & tangent (2)

$\Rightarrow y - 1 - 3y + 10 = 0$

$\Rightarrow -2y = -9 \Rightarrow y = \frac{9}{2}$

Intersection point is $\left(\frac{7}{2}, \frac{9}{2}\right)$ passes (3)

Hence, normal are concurrent.

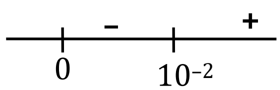
54. (b)

$f(x) = x \log x - 3 + x$

Differentiate with respect to x

$$f'(x) = x.\frac{1}{x} + \log.x + 1$$

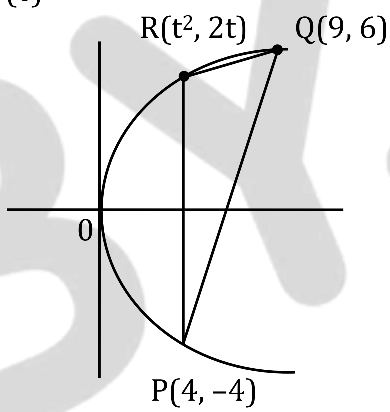
$f'(x) = 2 + \log x$



$f(1)f(3) = -2(3\log 3) = -ve$

Hence, one root is (1, 3).

55. (c)



$$\text{Area} = \frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[t^2(10) - 2t(5) + 1(-60)\right]$$

$$= \frac{10}{2}\left(t^2 - t - 6\right)$$

$f(t) = 5t^2 - 5t - 30$

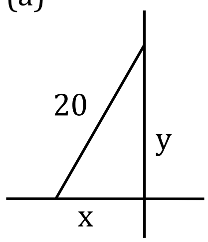
$f'(t) = 10t - 5 = 0$

$t = \frac{1}{2}$

Point $R = \left(\left(\frac{1}{2}\right)^2, 2\left(\frac{1}{2}\right)\right)$

$= \left(\frac{1}{4}, 1\right)$

56. (a)



Using right angle triangle concept

$x^2 + y^2 = 400$

Differentiate with respect to t

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Given

$\frac{dy}{dt} = 2ft/sec$

$x = 12 \Rightarrow 16$

$\Rightarrow 2(12)\frac{dx}{dt} + 2(16)\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{-8}{3}$

57. (b)

a = 9

b = 16

I(P) = 5 and J(P) = 7

J(P) > I(P)

Now,

$a = \frac{1}{9}$ and $b = \frac{1}{16}$

$I(P) = \frac{5}{12}$ & $J(P) = \frac{7}{12}$

J(P) > I(P)

58. (a,b,c)

Given that

$\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$

$\vec{\beta} = \hat{i} - \hat{j} - \hat{k}$

$\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k} \qquad \Rightarrow |\vec{\gamma}| = \sqrt{3}$

$\vec{\delta} = \vec{\alpha} + n\vec{\beta} = (\hat{i} + \hat{j} + \hat{k}) + n(\hat{i} - \hat{j} - \hat{k})$

$\vec{\delta} = (1+n)\hat{i} + (1-n)\hat{j} + (1-n)\hat{k}$

Now,

Projection on $\vec{\delta} = \frac{1}{\sqrt{3}} = \frac{\vec{\delta}.\vec{\gamma}}{|\vec{\gamma}|}$

$\Rightarrow \frac{\left[(1+n)\hat{i} + (1-n)\hat{j} + (1-n)\hat{k}\right].(-\hat{i} + \hat{j} - \hat{k})}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{|-1-n+1-n-1+n|}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow |n + 1| = 1$

$\Rightarrow n = 0$ or $n = -2$

$\vec{\delta} = -\hat{i} + 3\hat{j} + 3\hat{k}$

59. (c)

$\vec{\alpha}.\vec{\beta} = \vec{\alpha}.\vec{\gamma}$

Thus, α is perpendicular to b and c.

A unit vector perpendicular to b and c

$= \pm \frac{(\vec{\beta} \times \vec{\gamma})}{|\vec{\beta} \times \vec{\gamma}|}$

$= \pm \frac{(\vec{\beta} \times \vec{\gamma})}{|\vec{\beta}|\,|\vec{\gamma}|\sin\frac{\pi}{6}}$

$= \pm \frac{(\vec{\beta} \times \vec{\gamma})}{1/2}$

$= \pm 2(\vec{\beta} \times \vec{\gamma})$

60. (d)

$z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$R_e(z_1) > 0 \Rightarrow x_1 > 0$ and $I_m(z_2) < 0 \Rightarrow y_2 < 0$

$|z_1| = |z_2| \Rightarrow |z_1|^2 = |z_2|^2 \Rightarrow \; z_1\bar{z}_1 = z_2\bar{z}_2$

Now,

$\left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \left(\overline{\frac{z_1 + z_2}{z_1 - z_2}}\right)$

$= \left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \left(\frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 - \bar{z}_2}\right)$

$$= \frac{z_1\bar{z} + z_2\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_2 + z_1\bar{z}_1 + z_1\bar{z}_2 - z_2\bar{z}_1 - z_2\bar{z}_1}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}$$

$$= \frac{2\left(|z_1|^2 - |z_2|^2\right)}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)} = 0 \qquad \{\because |z_1|^2 = |z_2|^2\}$$

$$\Rightarrow \frac{z_1 + z_2}{z_1 - z_2} \text{ is purely imaginary}$$

61. (a)

1, 2, 3, 4, 5, ..., 19, 20

There are 17 ways for four consecutive number

Number ways = $^{20}C_4 - 17$

$$= \frac{20\times19\times18\times17}{1\times2\times3\times4} - 17$$

$$= 285 \times 17 - 17$$

$$= 284 \times 17$$

62. (b)

$$\text{Let } A = \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}^n$$

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^n$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

63. (d)

P = {(x, y) $\leftarrow$ N × N : 2x + y = 41}

For reflexive relation

$\Rightarrow xRx \Rightarrow 2x + x = 41 \Rightarrow x = \frac{41}{3} \notin N$ (not reflexive)

For symmetric

$\Rightarrow xRy \Rightarrow 2x + y = 41 \neq yRx$ (not symmetric)

For transitive

$\Rightarrow xRy \Rightarrow 2x + y = 41$ and $yRz \Rightarrow 2y + z = 41$ $x \not R z$ (not transitive)

64. (a)

$$f(x)=\begin{vmatrix}(1+x)^a & (2+x)^b & 1\\ 1 & (1+x)^a & (2+x)^b\\ (2+x)^b & 1 & (1+x)^a\end{vmatrix}$$

For constant term [put x = 0]

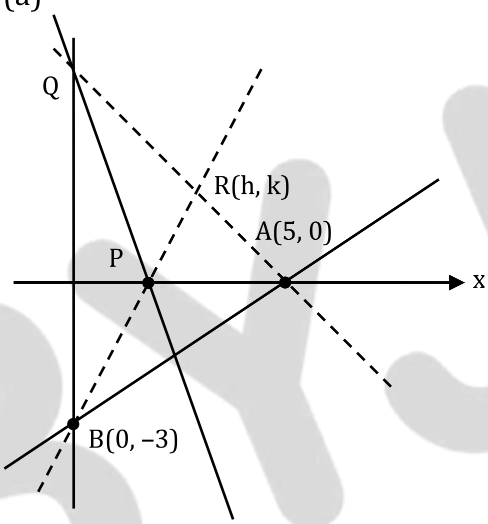
$$f(x)=\begin{vmatrix}1 & 2^b & 1\\ 1 & 1 & 2^b\\ 2^b & 1 & 1\end{vmatrix}$$

$f(x) = 1(1 - 2^b) - 2^b(1 - 2^{2b}) + 1(1 - 2^b)$

$f(x) = 1 - 2^b - 2^b + 2^{3b} + 1 - 2^b$

$f(x) = 2 - 3.2^b + 2^{3b}$

65. (a)



Line AB is $\frac{x}{5}+\frac{y}{-3}=1 \Rightarrow 3x - 5y = 15$

Any perpendicular line to AB

$5x + 3y = \lambda$ So $P\left(\frac{\lambda}{5},0\right),\ Q\left(0,\frac{\lambda}{3}\right)$

AQ is $\frac{x}{5}+\frac{y}{\lambda/3}=1$

$\Rightarrow \frac{3y}{\lambda}=1-\frac{x}{5}$

$\Rightarrow \frac{1}{\lambda}=\frac{1}{3y}\left(1-\frac{x}{5}\right)$ .....(i)

And BP is $\frac{x}{\lambda/5}-\frac{y}{3}=1$

$\Rightarrow \frac{5x}{\lambda}=1+\frac{y}{3}$

$\Rightarrow \frac{1}{\lambda}=\frac{1}{5x}\left(1+\frac{y}{3}\right)$ .....(ii)

Equation (i) = equation (ii)

$\Rightarrow \frac{1}{3y}\left(1-\frac{x}{5}\right)=\frac{1}{5x}\left(1+\frac{y}{3}\right)$

$\Rightarrow 5x\left(1-\frac{x}{5}\right)=3y\left(1+\frac{y}{3}\right)$

$\Rightarrow 5x - x^2 = 3y + y^2$

$\Rightarrow x^2 + y^2 - 5x + 3y = 0$

66. (a,c)

Given

$a_{ij}=\begin{cases}0, & \text{for } i=j\\ 1, & \text{for } i>j\\ -1, & \text{for } i<j\end{cases}$

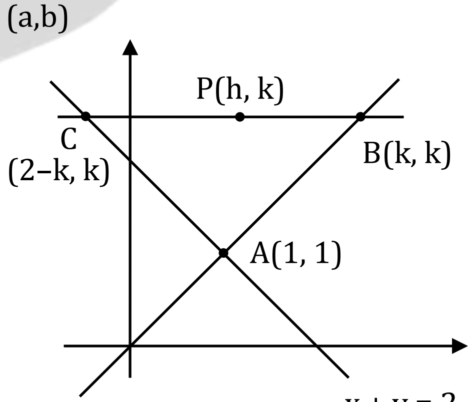
$A=\begin{bmatrix}0 & -1 & -1\\ 1 & 0 & -1\\ 1 & 1 & 0\end{bmatrix}\Rightarrow$skew symmetric matrix

$|A|=\begin{vmatrix}0 & -1 & -1\\ 1 & 0 & -1\\ 1 & 1 & 0\end{vmatrix}$

$|A| = 0 + 1 - 1$

$|A| = 0 \Rightarrow$ non invertible.

67. (a,b)



Area of triangle = h^2

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & k & k \\ 1 & 2-k & k \end{vmatrix} = \pm h^2$$

$\Rightarrow 1(k^2 - (2 - k)) - 1(k - k) + 1(2 - k - k) = \pm 2h^2$

$\Rightarrow k^2 - 2k + k^2 + 2 - 2k = \pm 2h^2$

$\Rightarrow 2k^2 - 4k + 2 = \pm 2h^2$

$\Rightarrow k^2 - 2k + 1 = \pm h^2$

Locus is $(k - 1)^2 = h^2 \Rightarrow y - 1 = \pm x$

$x - y + 1 = 0$ or $x + y = 1$

$x = y - 1$ or $x = -(y - 1)$

68. (b)

The length of transverse axis = $2\sin\theta = 2a$

$\therefore\ a = \sin\theta$

Also for ellipse, $3x^2 + 4y^2 = 12$

i.e. $\left(\frac{x^2}{4}\right) + \left(\frac{y^2}{3}\right) = 1$

$\therefore\ a^2 = 4$ & $b^2 = 3$

Now,

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{3}{4}} = \pm\frac{1}{2}$$

$\therefore$ focus of ellipse is = $(ae, 0)$ & $(-ae, 0)$

$\therefore$ focus = $(1, 0)$ and $(-1, 0)$

As the hyperbola if confocal

$\Rightarrow$ focus is same

And length of transverse axis = $2\sin\theta$

$\therefore$ length of semi transverse axis = $\sin\theta$

i.e. $A = \sin\theta$

And $C = 1$ where A, B, C are parameters in hyperbola similar to ellipse

$\therefore\ C^2 = A^2 + B^2$

$\therefore\ B^2 = 1 - \sin^2\theta = \cos^2\theta$

$\therefore$ Equation of hyperbola is $\left(\frac{x^2}{A^2}\right) - \left(\frac{y^2}{B^2}\right) = 1$

$$\Rightarrow \frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

$$x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$$

69. (a,c)

$f(x) = \cos\left(\frac{\pi}{x}\right)$

Differentiate with respect to x

$f'(x) = -\sin\left(\frac{\pi}{x}\right).-\left(\frac{\pi}{x}\right)$

$f'(x) = \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right) > 0$

For increasing function f'(x) > 0

$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0$

$\Rightarrow (2k\pi) < \frac{\pi}{x} < (2k+1)\pi \qquad \Rightarrow \frac{1}{2k} > x > \frac{1}{2k+1}$

For decreasing function f'(x) < 0

$\sin\left(\frac{\pi}{x}\right) < 0$

$\Rightarrow \frac{\pi}{x} \in ((2k+1)\pi, (2k+2)\pi)$

$\Rightarrow \frac{\pi}{x} \in \left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

70. (c)

Given

$y = \log_a\left(x + \sqrt{x^2+1}\right)$

$a^y = x + \sqrt{x^2+1}$ .....(i)

Now,

$a^{-y} = \frac{1}{x+\sqrt{x^2+1}} \times \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}-x}$

$a^{-y} = \frac{\sqrt{x^2+1}-x}{x^2+1-x^2}$

$a^{-y} = \sqrt{x^2+1}-x$

Equation (i) – equation (ii)

$\Rightarrow a^y - a^{-y} = 2x$

$\Rightarrow x = \frac{a^y - a^{-y}}{2} \qquad \Rightarrow f^{-1}(y) = x = \frac{a^y - a^{-y}}{2}$

$f^{-1}(y) = \frac{e^{y\ell na} - e^{-y\ell na}}{2} \qquad \left\{\because \sin h x = \frac{e^x - e^{-x}}{2}\right\}$

$f^{-1}(y) = \sin h\,(y\ell na)$

71. (a,b,c,d)

We know that

$-1 < \cos 3x < 1$

$-x^3 < x^3\cos 3x < x^3$

$$\frac{-x^3}{2+x^2} < \frac{x^3\cos 3x}{2+x^2} < \frac{x^3}{2+x^2}$$

Taking integration from 0 to 1

$$\Rightarrow \int_0^1 -x^2 dx < I < \int_0^1 x^2 dx$$

$$\Rightarrow \left(\frac{-x^3}{3}\right)_0^1 < I < \left(\frac{x^3}{3}\right)_0^1$$

$$\Rightarrow -\frac{1}{3} < I < \frac{1}{3}$$

72. (c,d)

Given

$$\frac{dy}{dt} > \frac{dx}{dt} \quad \text{.....(i)}$$

$12y = x^3$

Differentiate with respect to

$$\Rightarrow 12\frac{dy}{dt} = 3x^2\frac{dx}{dt} \quad \text{.....(ii)}$$

From equation (i)

$$3x^2\frac{dx}{dt} > 12\frac{dx}{dt}$$

$\Rightarrow \quad x^2 - 4 > 0$

$\Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty)$

73. (b)

Given

C_1: $x^2 + y^2 = 2ax$

C_2: $y^2 = ax$

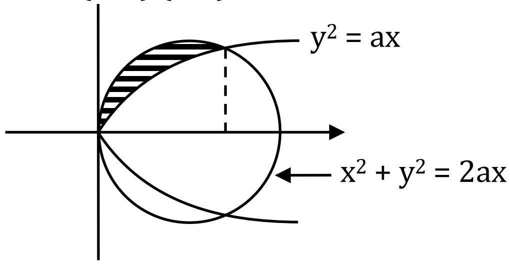
To find intersection points

$x^2 + (ax) = 2ax$

$x^2 = ax$

$x(x - a) = 0 \quad \Rightarrow \quad x = 0, a$

$\therefore \quad (0, 0)\ (a, a)$



$$\text{Area} = \frac{1}{4}(\text{area of circle}) - \int_0^a \sqrt{ax}\,dx$$

$$= \frac{1}{4}(\pi a^2) - \sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a$$

$$= \frac{\pi a^2}{4} - \frac{2a^2}{3}$$

$$= a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$$

74. (a,d)

Let $x^2 + ax + b = 0$ has roots α and β

$x^2 - cx + d = 0$ roots are α^4 and β^4

$\alpha + \beta = -a$ .....(i)

$\alpha\beta = b$ .....(ii)

$\alpha^4 + \beta^4 = c$ .....(iii)

$(\alpha\beta)^4 = d$ .....(iv)

From equation (ii) & (iv)

$b^4 = d$

And $\alpha^4 + \beta^4 = c$

$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = c$

$((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2 = c$

$\Rightarrow (a^2 - 2b)^2 - 2b^2 = c \Rightarrow 2b^2 + c = (a^2 - 2b)^2$

$\Rightarrow 2b^2 - c = 4a^2b - a^2$

$\Rightarrow 2b^2 - c = a^2(4b - a^2)$

Now for equation

$x^2 - 4bx + 2b^2 - c = 0$

$D = (4b)^2 - 4(1)(2b^2 - c)$

$D = 16b^2 - 8b^2 + 4c$

$D = 4(2b^2 + c)$

$D = 4(a^2 - 2b)^2 > 0 \Rightarrow$ real roots

Now,

$f(0) = 2b^2 - c$

$f(0) = a^2(4b - a^2) < 0$ {since $a^2 > 4b$}

Roots are opposite in sign

75. (b,c)

Total students = 20

Number of ways = ${}^{20}C_2 \times 21$

$$= \frac{20 \times 19}{2} \times 2$$

$= 20 \times 19$

$= {}^{20}P_2$