

- P is the extremity of the latus rectum of ellipse $3x^2 + 4y^2 = 48$ in the first quadrant. The eccentric angle of P is
 - $\frac{\pi}{8}$
 - $\frac{3\pi}{4}$
 - $\frac{\pi}{3}$
 - $\frac{3\pi}{3}$
- The direction ratios of the normal of the plane through the points $(1,2,3)$, $(-1,-2,1)$ and parallel to $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4}$ is
 - $(2, 3, 4)$
 - $(14, -8, -1)$
 - $(-2, 0, -3)$
 - $(1, -2, -3)$
- The equation of the plane. which bisects the line joining the points $(1, 2, 3)$ and $(3, 4, 5)$ at right angles is
 - $x + y + z = 0$
 - $x + y - z = 9$
 - $x + y + z = 9$
 - $x + y - z + 9 = 0$
- The limit of the interior angle of a regular polygon of n sides as $n \rightarrow \infty$ is
 - π
 - $\frac{\pi}{3}$
 - $\frac{3\pi}{2}$
 - $\frac{2\pi}{3}$

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5. Let $f(x) > 0$ for all x and $f'(x)$ exists for all x . If f is the inverse function of h and $h'(x) = \frac{1}{1 + \log x}$. Then $f'(x)$ will be
- $1 + \log (f(x))$
 - $1 + f(x)$
 - $1 - \log (f(x))$
 - $\log f(x)$
6. Consider the function $f(x) = \cos x^2$. Then
- f is of period 2π
 - f is of period $\sqrt{2\pi}$
 - f is not periodic
 - f is of periodic π
7. $\lim_{x \rightarrow 0^+} (e^x + x)^{1/x}$
- Does or exist finitely
 - is 1
 - is e^2
 - is 2
8. Let $f(x)$ be a derivable function, $f'(x) > f(x)$ and $f(0) = 0$. Then
- $f(x) > 0$ for all $x > 0$
 - $f(x) < 0$ for all $x > 0$
 - no sign of $f(x)$ can be ascertained
 - $f(x)$ is a constant function
9. Let $f: [1, 3] \rightarrow \mathbb{R}$ be a continuous function that is differentiable in $(1, 3)$ and $f'(x) = |f(x)|^2 + 4$ for all $x \in (1, 3)$. Then
- $f(3) - f(1) = 5$ is true
 - $f(3) - f(1) = 5$ is false
 - $f(3) - f(1) = 7$ is false
 - $f(3) - f(1) < 0$ only at one point $(1, 3)$

10. $\lim_{x \rightarrow 0^+} (x^n \ln x), n > 0$

- a. does not exist
- b. exists and is zero
- c. exists and is 1
- d. exists and is e^{-1}

11. If $\int \cos x \log\left(\tan \frac{x}{2}\right) dx = \sin x \log\left(\tan \frac{x}{2}\right) + f(x)$ then $f(x)$ is equal to, (assuming c is a arbitrary real constant)

- a. c
- b. $c - x$
- c. $c + x$
- d. $2x + c$

12. $y = \int \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\} dx$ is an equation of a family of

- a. straight lines
- b. circles
- c. ellipses
- d. parabolas

13. The value of the integration $\int_{-\pi/4}^{\pi/4} \left(\lambda |\sin x| + \frac{\mu \sin x}{1 + \cos x} + \gamma \right) dx$

- a. is independent of λ only
- b. is independent of μ only
- c. is independent of γ only
- d. depends on γ, μ and λ

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14. The value of $\lim_{x \rightarrow 0} \frac{1}{x} \left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt \right]$ is equal to

- a. $e^{\sin^2 y}$
- b. $e^{2\sin y}$
- c. $e^{|\sin y|}$
- d. $e^{\operatorname{cosec}^2 y}$

15. If $\int 2^{2^x} \cdot 2^x dx = A \cdot 2^x + c$, then $A =$

- a. $\frac{1}{\log 2}$
- b. $\log 2$
- c. $(\log 2)^2$
- d. $\frac{1}{(\log 2)^2}$

16. The value of the integral $\int_{-1}^1 \left\{ \frac{x^{2015}}{e^{|x|}(x^2 + \cos x)} + \frac{1}{e^{|x|}} \right\} dx$ is equal to

- a. 0
- b. $1 - e^{-1}$
- c. $2e^{-1}$
- d. $2(1 - e^{-1})$

17. $\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ 1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \dots + \sqrt{\frac{n}{n+3(n-1)}} \right\}$

- a. does not exist
- b. is 1
- c. is 2
- d. is 3

18. The general solution of the differential equation $\left(1 + e^{\frac{x}{y}}\right)dx + \left(1 - \frac{x}{y}\right)e^{\frac{x}{y}}dy = 0$ is (c is an

arbitrary constant)

a. $x - ye^{\frac{x}{y}} = c$

b. $y - xe^{\frac{x}{y}} = c$

c. $x + ye^{\frac{x}{y}} = c$

d. $y + xe^{\frac{x}{y}} = c$

19. General solution of $(x+y)^2 \frac{dy}{dx} = a^2$, $a \neq 0$ is (c is arbitrary constant)

a. $\frac{x}{a} = \tan \frac{y}{a} + c$

b. $\tan xy = c$

c. $\tan (x + y) = c$

d. $\tan \frac{y+c}{a} = \frac{x+y}{a}$

20. Let P(4, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at P intersects the x-

axis at (16, 0), then the eccentricity of the hyperbola is

a. $\frac{\sqrt{5}}{2}$

b. 2

c. $\sqrt{2}$

d. $\sqrt{3}$

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21. If the radius of a spherical balloon increases by 0.1% then its volume increases approximately by

- a. 0.2%
- b. 0.3%
- c. 0.4%
- d. 0.05%

22. The three sides of a right-angled triangle are in G.P. (Geometrical Progression). If the two acute angles be α and β , then $\tan\alpha$ and $\tan\beta$ are:

- a. $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$
- b. $\sqrt{\frac{\sqrt{5}+1}{2}}$ and $\sqrt{\frac{\sqrt{5}-1}{2}}$
- c. $\sqrt{5}$ and $\frac{1}{\sqrt{5}}$
- d. $\frac{\sqrt{5}}{2}$ and $\frac{2}{\sqrt{5}}$

23. If $\log_2 6 + \frac{1}{2x} = \log_2 \left(2^{\frac{1}{x}} + 8 \right)$, then the value of x are

- a. $\frac{1}{4}, \frac{1}{3}$
- b. $\frac{1}{4}, \frac{1}{2}$
- c. $-\frac{1}{4}, \frac{1}{2}$
- d. $\frac{1}{3}, -\frac{1}{2}$

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24. Let z be a complex number such that the principal value of argument, $\arg z > 0$. Then $\arg z - \arg(-z)$ is

- a. $\frac{\pi}{2}$
- b. $\pm\pi$
- c. π
- d. $-\pi$

25. The general value of the real angle θ , which satisfies the equation $(\cos\theta + i \sin\theta)(\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = 1$ is given by (assuming k is an integer)

- a. $\frac{2k\pi}{n+2}$
- b. $\frac{4k\pi}{n(n+1)}$
- c. $\frac{4k\pi}{n+1}$
- d. $\frac{6k\pi}{n(n+1)}$

26. Let a, b, c be real numbers such $a + b + c < 0$ and the quadratic equation $ax^2 + bx + c = 0$ has imaginary roots. Then

- a. $a > 0, c > 0$
- b. $a > 0, c < 0$
- c. $a < 0, c > 0$
- d. $a < 0, c < 0$

27. A candidate is required to answer 6 out of 12 questions which are divided into two parts A and B each containing 6 questions and he/she is not permitted to attempt more than 4 questions from any part. In how many different ways can he/she make up his/her choice of 6 questions?
- 850
 - 800
 - 750
 - 700
28. There are 7 greetings cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour is:
- 7C_3
 - $2 \cdot {}^7C_3$
 - $3! \cdot {}^4C_4$
 - $3! \cdot {}^7C_3 \cdot {}^4C_3$
29. $7^{2n} + 16n - 1$ ($n \in \mathbb{N}$) is divisible by
- 65
 - 63
 - 61
 - 64
30. The number of irrational terms in the expansion of $(3^{1/8} + 5^{1/4})^{84}$ is
- 73
 - 74
 - 75
 - 76

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31. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then the matrix $A - 3I_3$ is
- invertible
 - orthogonal
 - non-invertible
 - real skew symmetric matrix

32. If M is any square matrix of order 3 over R and If M' be the transpose of M , then $\text{adj}(M') - \text{adj}(M)'$ is equal to
- M
 - M'
 - null matrix
 - identity matrix

33. If $A = \begin{pmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{pmatrix}$ and $|A^2| = 25$, then $|x|$ is equal to

- $\frac{1}{5}$
 - 5
 - 5^2
 - 1
34. Let A and B be two square matrices of order 3 and $AB = O_3$, where O_3 denotes the null matrix of order 3. Then
- must be $A = O_3, B = O_3$
 - if $A \neq O_3$, must be $B \neq O_3$
 - if $A = O_3$, must be $B \neq O_3$
 - may be $A \neq O_3, B \neq O_3$

35. Let P and T be the subsets of X-Y plane defined by

$$P = \{(x,y): x > 0, y > 0 \text{ and } x^2 + y^2 = 1\}$$

$$T = \{(x,y): x > 0, y > 0 \text{ and } x^8 + y^8 < 1\}$$

Then $P \cap T$ is

- a. the void set ϕ
- b. P
- c. T
- d. $P - T^c$

36. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 - \frac{x^2}{1+x^2}$ for all $x \in \mathbb{R}$. Then

- a. f is one-one but not onto mapping
- b. f is onto but not one-one mapping
- c. f is both one-one and onto
- d. f is neither one-one nor onto

37. Let the relation ρ be defined on \mathbb{R} as $a\rho b$ if $1 + ab > 0$. Then

- a. ρ is reflexive only
- b. ρ is equivalence relation
- c. ρ is reflexive and transitive but not symmetric
- d. ρ is reflexive and symmetric but not transitive

38. A problem in mathematics is given to 4 students whose chances of solving individually are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. Then probability that the problem will be solved at least by one student is

- a. $\frac{2}{3}$
- b. $\frac{3}{5}$
- c. $\frac{4}{5}$
- d. $\frac{3}{4}$

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39. If X is a random variable such that $\sigma(X) = 2.6$, then $\sigma(1 - 4X)$ is equal to
- 7.8
 - 10.4
 - 13
 - 10.4
40. If $e^{\sin x} - e^{-\sin x} - 4 = 0$, then the number of real values of x is
- 0
 - 1
 - 2
 - 3
41. The angles of a triangle are in the ratio $2 : 3 : 7$ and the radius of the circumscribed circle is 10 cm. The length of the smallest side is
- 2 cm
 - 5 cm
 - 7 cm
 - 10 cm
42. A variable line passes through a fixed point (x_1, y_1) and meets the axes at A and B . If the rectangle $OAPB$ be completed, the locus of P is, (O being the origin of the system of axes)
- $(y - y_1)^2 = 4(x - x_1)$
 - $\frac{x_1}{x} + \frac{y_1}{y} = 1$
 - $x^2 + y^2 = x_1^2 + y_1^2$
 - $\frac{x^2}{2x_1^2} + \frac{y^2}{2y_1^2} = 1$

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43. A straight line through the point $(3, -2)$ is inclined at an angle 60° to the line $\sqrt{3}x + y =$

1. If it intersects the X-axis, then its equation will be

a. $y + x\sqrt{3} + 2 + 3\sqrt{3} = 0$

b. $y - x\sqrt{3} + 2 + 3\sqrt{3} = 0$

c. $y - x\sqrt{3} - 2 - 2\sqrt{3} = 0$

d. $x - x\sqrt{3} + 2 - 3\sqrt{3} = 0$

44. A variable line passes through the fixed point (α, β) . The locus of the foot of the perpendicular from the origin on the line is

a. $x^2 + y^2 - \alpha x - \beta y = 0$

b. $x^2 - y^2 + 2\alpha x + 2\beta y = 0$

c. $ax + by \pm \sqrt{(\alpha^2 + \beta^2)} = 0$

d. $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

45. If the point of intersection of the lines $2ax + 4ay + c = 0$ and $7bx + 3by - d = 0$ lies in the 4th quadrant and is equidistant from the two axes, where a, b, c and d are non-zero numbers, then $ad : bc$ equals to

a. $2 : 3$

b. $2 : 1$

c. $1 : 1$

d. $3 : 2$

46. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is

a. $(x - p)^2 = 4qy$

b. $(x - q)^2 = 4py$

c. $(y - p)^2 = 4qx$

d. $(y - q)^2 = 4px$

47. If $P(0, 0)$, $Q(1, 0)$ and $R\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are three given points, then the centre of the circle for

which the lines PQ , QR and RP are the tangents is

- a. $\left(\frac{1}{2}, \frac{1}{4}\right)$
- b. $\left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$
- c. $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$
- d. $\left(\frac{1}{2}, \frac{-1}{\sqrt{3}}\right)$

48. For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$, which of the following remains fixed when α varies?

- a. Directrix
- b. Vertices
- c. foci
- d. Eccentricity

49. S and T are the foci of an ellipse and B is the end point of the minor axis. If STB is equilateral triangle, the eccentricity of the ellipse is

- a. $\frac{1}{4}$
- b. $\frac{1}{3}$
- c. $\frac{1}{2}$
- d. $\frac{2}{3}$

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50. The equation of the directrices of the hyperbola $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ is

a. $x = 3 \pm \sqrt{\frac{13}{6}}$

b. $x = 3 \pm \sqrt{\frac{6}{13}}$

c. $x = 6 \pm \sqrt{\frac{13}{3}}$

d. $x = 6 \pm \sqrt{\frac{3}{13}}$

51. The graphs of the polynomial $x^2 - 1$ and $\cos x$ intersect

- a. at exactly two points
- b. at exactly 3 points
- c. at least 4 but at finitely many points
- d. at infinitely many points

52. A point is in motion along a hyperbola $y = \frac{10}{x}$ so that its abscissa x increases uniformly

at a rate of 1 unit per second. Then, the rate of change of its ordinate, when the point passes through (5, 2)

- a. increases at the rate of $\frac{1}{2}$ unit per second
- b. decreases at the rate of $\frac{1}{2}$ unit per second
- c. decreases at the rate of $\frac{2}{5}$ unit per second
- d. increases at the rate of $\frac{2}{5}$ unit per second

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53. Let $a = \min\{x^2 + 2x + 3 : x \in \mathbb{R}\}$ and $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2}$. Then $\sum_{r=0}^n a^r b^{n-r}$ is

- a. $\frac{2^{n+1} - 1}{3 \cdot 2^n}$
- b. $\frac{2^{n+1} + 1}{3 \cdot 2^n}$
- c. $\frac{4^{n+1} - 1}{3 \cdot 2^n}$
- d. $\frac{1}{2}(2^n - 1)$

54. Let $a > b > 0$ and $I(n) = a^{1/n} - b^{1/n}$, $J(n) = ((a - b)^{1/n})$ for all $n \geq 2$. then

- a. $I(n) < J(n)$
- b. $I(n) > J(n)$
- c. $I(n) = J(n)$
- d. $I(n) + J(n) = 0$

55. Let $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ be three unit vectors such that $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} \times \hat{\gamma})$ where $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = (\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma}$. If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is

- a. $\frac{5\pi}{6}$
- b. $\frac{\pi}{6}$
- c. $\frac{\pi}{3}$
- d. $\frac{2\pi}{3}$

56. The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$, $5\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + \lambda\hat{k}$ respectively. If the points A, B, C and D lie on a plane, the value of λ is
- 0
 - 1
 - 2
 - 4

57. A particle starts at the origin and moves 1 unit horizontally to the right and reaches P_1 , then it moves $\frac{1}{2}$ unit vertically up and reaches P_2 , then it moves $\frac{1}{4}$ unit horizontally to right and reaches P_3 , then it moves $\frac{1}{8}$ unit vertically down and reaches P_4 , then it moves $\frac{1}{16}$ unit horizontally to right and reaches P_5 and so on. Let $P_n = (x_n, y_n)$ and $\lim_{n \rightarrow \infty} x_n = \alpha$ and $\lim_{n \rightarrow \infty} y_n = \beta$. Then (α, β) is

- $(2, 3)$
 - $\left(\frac{4}{3}, \frac{2}{5}\right)$
 - $\left(\frac{2}{5}, 1\right)$
 - $\left(\frac{4}{3}, 3\right)$
58. For any non-zero complex number z , the minimum value of $|z| + |z - 1|$ is
- 1
 - $\frac{1}{2}$
 - 0
 - $\frac{3}{2}$

59. The system of equations

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$$\lambda x + y + 3z = 0$$

$$2x + \mu y - z = 0$$

$$5x + 7y + z = 0$$

Has infinitely many solutions in \mathbb{R} . Then,

a. $\lambda = 2, \mu = 3$

b. $\lambda = 1, \mu = 2$

c. $\lambda = 1, \mu = 3$

d. $\lambda = 3, \mu = 1$

60. Let $f : X \longrightarrow Y$ and A, B are non-void subsets of Y , then (where the symbols have their usual interpretation)

a. $f^{-1}(A) - f^{-1}(B) \supset f^{-1}(A - B)$ but the opposite does not hold

b. $f^{-1}(A) - f^{-1}(B) \subset f^{-1}(A - B)$ but the opposite does not hold

c. $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$

d. $f^{-1}(A - B) = f^{-1}(A) \cup f^{-1}(B)$

61. Let S, T, U be three non-void sets and $f : S \rightarrow T, g : T \rightarrow U$ be so that $g \circ f : S \rightarrow U$ is surjective. Then

a. g and f are both surjective

b. g is surjective, f may not be so

c. f is surjective, g may not be so

d. f and g both may not be surjective

62. The polar coordinate of a point P is $\left(2, -\frac{\pi}{4}\right)$. The polar coordinate of the point Q, which

is such that the line joining PQ is bisected perpendicularly by the initial line, is

- a. $\left(2, \frac{\pi}{4}\right)$
- b. $\left(2, \frac{\pi}{6}\right)$
- c. $\left(-2, \frac{\pi}{4}\right)$
- d. $\left(-2, \frac{\pi}{6}\right)$

63. The length of conjugate axis of a hyperbola is greater than the length of transverse axis.

Then the eccentricity e is

- a. $=\sqrt{2}$
- b. $>\sqrt{2}$
- c. $<\sqrt{2}$
- d. $\frac{1}{\sqrt{2}}$

64. The value of $\lim_{x \rightarrow 0^+} \frac{x}{p} \left[\frac{q}{x} \right]$ is

- a. $\frac{[q]}{p}$
- b. 0
- c. 1
- d. ∞

65. Let $f(x) = x^4 - 4x^3 + 4x^2 + c$, $c \in \mathbb{R}$. Then

- a. $f(x)$ has infinitely many zeroes in $(1, 2)$ for all c
- b. $f(x)$ has exactly one zero in $(1, 2)$ if $-1 < c < 0$
- c. $f(x)$ has double zeroes in $(1, 2)$ if $-1 < c < 0$
- d. whatever be the value of c , $f(x)$ has no zero in $(1, 2)$

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Category-III (Q. 66 to Q. 75)

Carry 2 marks each and on or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked + actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. Let f and g be differentiable on the interval I and let $a, b \in I, a < b$. Then

- a. If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ is solvable in (a, b)
- b. If $f(a) = 0 = f(b)$, the equation $f'(x) + f(x)g'(x) = 0$ may not be solvable in (a, b)
- c. If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ is solvable in $(a, b), k \in \mathbb{R}$
- d. If $g(a) = 0 = g(b)$, the equation $g'(x) + kg(x) = 0$ may not be solvable in $(a, b), k \in \mathbb{R}$

67. Consider the function $f(x) = \frac{x^3}{4} - \sin \pi x + 3$

- a. $f(x)$ does not attain value within the interval $[-2, 2]$
- b. $f(x)$ takes on the value $2\frac{1}{3}$ in the interval $[-2, 2]$
- c. $f(x)$ takes on the value $3\frac{1}{4}$ in the interval $[-2, 2]$
- d. $f(x)$ takes no value $p, 1 < p < 5$ in the interval $[-2, 2]$

68. Let $I_n = \int_0^1 x^n \tan^{-1} x \, dx$. If $a_n I_{n+2} + b_n I_n = c_n$ for all $n \geq 1$, then

- a. a_1, a_2, a_3 are in G.P.
- b. b_1, b_2, b_3 are in A.P.
- c. c_1, c_2, c_3 are in H.P.
- d. a_1, a_2, a_3 are in A.P.

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69. Two particles A and B move from rest along a straight line with constant accelerations f and h respectively. If A takes m seconds more than B and describes n units more than that of B acquiring the same speed, then

- a. $(f + h)m^2 = fhn$
- b. $(f - fh)m^2 = fhn$
- c. $(h - f)n = \frac{1}{2} fhm^2$
- d. $\frac{1}{2} (f + h)n = fhm^2$

70. The area bounded by $y = x + 1$ and $y = \cos x$ and the x -axis, is

- a. 1 sq. unit
- b. $\frac{3}{2}$ sq. unit
- c. $\frac{1}{4}$ sq. unit
- d. $\frac{1}{8}$ sq. unit

71. Let x_1, x_2 be the roots of $x^2 - 3x + a = 0$ and x_3, x_4 be the roots of $x^2 - 12x + b = 0$. If $x_1 < x_2 < x_3 < x_4$ and x_1, x_2, x_3, x_4 are in G.P. then ab equals

- a. $\frac{24}{5}$
- b. 64
- c. 16
- d. 8

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72. If $q \in \mathbb{R}$ and $\frac{1 - i\cos\theta}{1 + 2i\cos\theta}$ is real number, then θ will be (when I : Set of integers)

- a. $(2n + 1)\frac{\pi}{2}, n \in I$
- b. $\frac{3n\pi}{2}, n \in I$
- c. $n\pi, n \in I$
- d. $2n\pi, n \in I$

73. Let $A = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}$. Then the roots of the equation $\det(A - \lambda I_3) = 0$

(Where I_3 is the identity matrix of order 3) are

- a. 3, 0, 3
- b. 0, 3, 6
- c. 1, 0, -6
- d. 3, 3, 6

74. Straight lines $x - y = 7$ and $x + 4y = 2$ intersect at B. Points A and C are so chosen on these two lines such that $AB = AC$. The equation of line AC passing through $(2, -7)$ is:

- a. $x - y - 9 = 0$
- b. $23x + 7y + 3 = 0$
- c. $2x - y - 11 = 0$
- d. $7x - 6y - 56 = 0$

75. Equation of a tangent to the hyperbola $5x^2 - y^2 = 5$ and which passes through an external point $(2, 8)$ is

- a. $3x - y + 2 = 0$
- b. $3x + y - 14 = 0$
- c. $23x - 3y - 22 = 0$
- d. $3x - 23y + 178 = 0$

WBJEE -2019 (Mathematics)



ANSWER KEYS

1. (c)	2. (b)	3. (c)	4. (a)	5. (a)	6. (c)	7. (c)	8. (a)	9. (b,c)	10. (b)
11. (b)	12. (d)	13. (b)	14. (a)	15. (d)	16. (d)	17. (c)	18. (c)	19. (d)	20. (b)
21. (b)	22. (b)	23. (b)	24. (c)	25. (b)	26. (d)	27. (a)	28. (b)	29. (d)	30. (b)
31. (c)	32. (c)	33. (a)	34. (d)	35. (b)	36. (d)	37. (d)	38. (c)	39. (d)	40. (a)
41. (d)	42. (b)	43. (b)	44. (a)	45. (b)	46. (a)	47. (c)	48. (c)	49. (c)	50. (a)
51. (a)	52. (c)	53. (c)	54. (a)	55. (d)	56. (d)	57. (b)	58. (a)	59. (c)	60. (c)
61. (b)	62. (a)	63. (b)	64. (a)	65. (b)	66. (a,c)	67. (b,c)	68. (b,d)	69. (c)	70. (b)
71. (b)	72. (a)	73. (b)	74. (b)	75. (a,c)					

Solution

1. (c)
Given

$$\text{Equation of ellipse is } \frac{x^2}{16} + \frac{y^2}{12} = 1$$

Let point P on curve $(4\cos\theta, 2\sqrt{3}\sin\theta)$

But we know point P is extremity of latus rectum of ellipse

So, P is $(2, 3)$

On comparing point P

$$4\cos\theta = 2 \quad \text{and} \quad 2\sqrt{3}\sin\theta = 3$$

$$\cos\theta = \frac{1}{2} \quad \text{and} \quad \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

2. (b)
Normal of the plane passing through the points $(1, 2, 3)$, $(-1, -2, 1)$ and parallel to given line

$$\begin{aligned} \text{is } & \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1+1 & 2+2 & -3-1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2 & 4 & -4 \end{vmatrix} \\ & = -28\hat{i} + 16\hat{j} + 2\hat{k} \end{aligned}$$

Direction ratio's of the normal to plane is $\langle 14, -8, -1 \rangle$

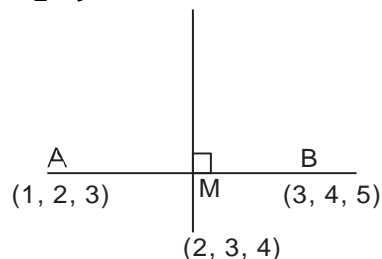
3. (c)
Equation of plane is $(3-1)x + (4-2)y + (5-3)z = k$ (1)

$$\text{This plane passes through } m \equiv \left(\frac{3+1}{2}, \frac{2+4}{2}, \frac{5+3}{2} \right)$$

$$\equiv (2, 3, 4)$$

$$\Rightarrow k = 2 \times 2 + 2 \times 3 + 2 \times 4 = 18 \quad (\text{put in eq (1)})$$

$$\Rightarrow \text{Eq. of plane is } x + y + z = 9$$



4.

(a)

We know,

$$\text{One interior angle} = \frac{(n-2)\pi}{n}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{(n-2)\pi}{n} = \pi$$

5.

(a)

Given

$$\Rightarrow f(x) = h^{-1}(x)$$

$$\Rightarrow h(f(x)) = x$$

On differentiating with respect to x

$$\Rightarrow h'(f(x)) \cdot f'(x) = 1$$

$$\Rightarrow f'(x) = \frac{1}{h'(f(x))} = 1 + \log(f(x))$$

6.

(c)

Given

$$f(x) = \cos x^2$$

Let, f(x) is periodic with period T

$$\text{We know, } f(x+T) = f(x)$$

$$\Rightarrow \cos(x+T)^2 = \cos x^2$$

$$\Rightarrow \cos(x+T)^2 - \cos x^2 = 0$$

$$\Rightarrow -2 \sin\left(\frac{(x+T)^2 - x^2}{2}\right) \sin\left(\frac{(x+T)^2 + x^2}{2}\right) = 0$$

$$\Rightarrow (x+T)^2 - x^2 = n\pi \quad \text{or} \quad (x+T)^2 + x^2 = n\pi$$

Which is not possible because these equation are quadratic equation not identity

$$\Rightarrow f(x) \text{ is not periodic}$$

7.

(c)

$$\lim_{x \rightarrow 0^+} (e^x + x)^{1/x} (1^\infty) \text{ form}$$

$$\lim_{e^x \rightarrow 0^+} \left(\frac{e^x + x - 1}{x} \right)$$

$$\text{Using binomial expansion of } e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$$\Rightarrow e^{1+1} = e^2$$

8. (a)

Given

$$f'(x) > f(x)$$

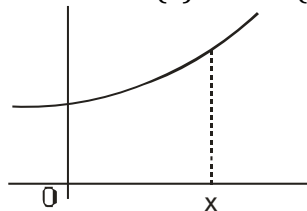
Multiply e^{-x} both sides

$$\Rightarrow f'(x) \cdot e^{-x} - f(x) \cdot e^{-x} > 0$$

$$\Rightarrow (f(x) \cdot e^{-x})' > 0$$

$\Rightarrow e^{-x} \cdot f(x)$ is increasing function

$$\Rightarrow e^{-x} \cdot f(x) > e^{-0} \cdot f(0) \quad \forall x > 0$$



$$\Rightarrow e^{-x} f(x) > 0 \quad \because f(0) = 0 \text{ (given)}$$

$$\Rightarrow f(x) > 0 \quad \forall x > 0$$

9. (b,c)

$$f'(x) = |f(x)|^2 + 4$$

Using LMVT theorem

$$\Rightarrow \frac{f(3) - f(1)}{3 - 1} = f'(c) \text{ for at least one } c \in (1, 3)$$

$$\Rightarrow f(3) - f(1) = 2(f(c))^2 + 8$$

$$\Rightarrow f(3) - f(1) \geq 8$$

$$\Rightarrow f(3) - f(1) = 5 \text{ (false)}$$

$$\Rightarrow \text{Similarly } f(3) - f(1) = 7 \text{ (false)}$$

10. (b)

$$\lim_{x \rightarrow 0^+} (x^n \ell nx)$$

$$= \lim_{x \rightarrow 0^+} \left(\frac{\ell nx}{1/x^n} \right) \left(\frac{\infty}{\infty} \right) \text{ form}$$

Using L-Hospital Rule

$$= \lim_{x \rightarrow 0^+} \frac{1/x}{\frac{-n}{x^{n+1}}}$$

$$= \lim_{x \rightarrow 0^+} \left(\frac{x^n}{-n} \right) = 0$$

11. (b)

$$\int \cos x \log \left(\tan \frac{x}{2} \right) dx = \sin x \log \left(\tan \frac{x}{2} \right) + f(x) \text{ (given)}$$

Using integration by parts in R.H.S.

$$\Rightarrow \log \left(\tan \frac{x}{2} \right) \cdot \sin x - \int \sin x \cdot \frac{\sec^2 \frac{x}{2}}{\tan \frac{x}{2}} \cdot \frac{1}{2} dx$$

$$\Rightarrow \log \left(\tan \frac{x}{2} \right) \cdot \sin x - \int \sin x \cdot \frac{1}{\sin x} dx$$

$$\Rightarrow \sin x \cdot \log \left(\tan \frac{x}{2} \right) - x + c \text{ on comparing with given solution}$$

$$\Rightarrow f(x) = c - x$$

12. (d)

$$y = \int \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\} dx$$

$$\text{put } x = \cos 2\theta \Rightarrow dx = -2 \sin 2\theta d\theta$$

$$y = \int \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} \right\} (-2 \sin 2\theta) d\theta$$

$$y = \int \cos(2 \tan^{-1} \tan \theta) (-2 \sin 2\theta) d\theta$$

$$y = -\int 2 \sin 2\theta \cdot \cos 2\theta d\theta$$

$$y = -\int \sin 4\theta d\theta$$

$$y = \frac{\cos 4\theta}{4} + c'$$

$$y = \frac{2 \cos^2 2\theta - 1}{4} + c' \quad \therefore x = \cos 2\theta$$

$$y = \frac{x^2}{2} + C$$

\Rightarrow Curve represents family of parabola

13. (b)

$$\int_{-\pi/4}^{\pi/4} \left(\lambda |\sin x| + \frac{\mu \sin x}{1 + \cos x} + \gamma \right) dx$$

$$\Rightarrow \lambda \int_{-\pi/4}^{\pi/4} |\sin x| dx + \underbrace{\mu \int_{-\pi/4}^{\pi/4} \frac{\sin x dx}{1 + \cos x}}_{\text{(odd function)}} + \int_{-\pi/4}^{\pi/4} \gamma dx$$

$$\Rightarrow 2\lambda \int_0^{\pi/4} \sin x + 0 + \frac{\gamma\pi}{2}$$

$$\Rightarrow 2\lambda \left(1 - \frac{1}{\sqrt{2}} \right) + \frac{\gamma\pi}{2}$$

$$\Rightarrow \text{Independent of } \mu$$

14. (a)

$$\lim_{x \rightarrow 0} \frac{1}{x} \left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt \right]$$

$$= \lim_{x \rightarrow 0} \frac{1}{x} \left[\int_y^a e^{\sin^2 t} dt + \int_a^{x+y} e^{\sin^2 t} dt \right]$$

$$= \lim_{x \rightarrow 0} \frac{\int_y^{x+y} e^{\sin^2 t} dt}{x} \quad (\text{using L- Hospital rule})$$

$$= \lim_{x \rightarrow 0} \frac{e^{\sin^2(x+y)}}{1} = e^{\sin^2 y}$$

15. (d)

$$\int 2^{2^x} \cdot 2^x dx = A \cdot 2^{2^x} + C \quad \dots\dots(1)$$

$$\text{Put } 2^{2^x} = t \Rightarrow (2^{2^x} \cdot \ln 2) (2^x \cdot \ln 2) dx = dt$$

$$\Rightarrow \int \frac{dt}{(\ln 2)^2}$$

$$\Rightarrow \frac{t}{(\ln 2)^2} + C \Rightarrow \frac{2^{2^x}}{(\ln 2)^2} + C \quad (\text{on comparing with equation (1)})$$

$$\Rightarrow A = \frac{1}{(\ln 2)^2}$$

16. (d)

$$I = \int_{-1}^1 \left(\frac{x^{2015}}{e^{|x|}(x^2 + \cos x)} + \frac{1}{e^{|x|}} \right) dx$$

$$= \int_{-1}^1 \underbrace{\left(\frac{x^{2015}}{e^{|x|}(x^2 + \cos x)} \right)}_{\text{odd function}} dx + \int_{-1}^1 \underbrace{\frac{1}{e^{|x|}}}_{\text{even function}} dx$$

$$I = 0 + 2 \int_0^1 e^{-x} dx = 2(-e^{-x}) \Big|_0^1 = 2 \left(\frac{-1}{e} + 1 \right)$$

$$I = 2(1 - e^{-1})$$

17. (c)

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left\{ 1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \dots + \sqrt{\frac{n}{n+3(n-1)}} \right\}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{3}{n} \left\{ \sqrt{\frac{1}{1+3\left(\frac{0}{n}\right)}} + \sqrt{\frac{1}{1+3\left(\frac{1}{n}\right)}} + \sqrt{\frac{1}{1+3\left(\frac{2}{n}\right)}} + \dots + \sqrt{\frac{1}{1+3\left(\frac{n-1}{n}\right)}} \right\}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{r=0}^{n-1} \sqrt{\frac{1}{1+3\left(\frac{r}{n}\right)}}$$

Put $\frac{1}{n} \rightarrow dx$

$\frac{r}{n} \rightarrow x$

Lower limit $\Rightarrow x = 0$

Upper limit $\Rightarrow x = 1$

$$\Rightarrow 3 \int_0^1 \frac{1}{\sqrt{1+3x}} dx = \frac{3(1+3x)^{1/2}}{\frac{1}{2} \times 3} \Big|_0^1 = 3 \times \frac{2}{3} \times (4^{1/2} - 1^{1/2})$$

$$= \frac{2}{3} \times 3 = 2$$

18. (c)

$$\left(1 + e^{\frac{x}{y}}\right) dx + \left(1 - \frac{x}{y}\right) e^{x/y} dy = 0$$

$$\frac{dy}{dx} = \frac{-(1 + e^{x/y})}{\left(1 - \frac{x}{y}\right) e^{x/y}} \quad \dots\dots(1)$$

$$\text{Put } \frac{x}{y} = t \Rightarrow y = \frac{x}{t} \Rightarrow \frac{dy}{dx} = \frac{1}{t} - \frac{x}{t^2} \frac{dt}{dx} \quad \dots\dots(2)$$

From equation (1) & (2)

$$\Rightarrow \frac{1}{t} - \frac{x}{t^2} \frac{dt}{dx} = \frac{-(1 + e^t)}{(1-t)e^t}$$

$$\Rightarrow \frac{x dt}{dx} = \frac{t(t + e^t)}{(1-t)e^t}$$

$$\frac{dx}{x} = \frac{e^t(1-t)}{t(t + e^t)} dt$$

$$\Rightarrow \frac{dx}{x} = \frac{(e^t + t) - t(e^t + 1)}{t(t + e^t)} dt$$

$$\Rightarrow \frac{dx}{x} = \left(\frac{1}{t} - \frac{(e^t + 1)}{e^t + t} \right) dt$$

On integrating both sides

$$\Rightarrow \ln x = \ln t - \ln(e^t + t) + \ln c \quad \because \frac{x}{y} = t$$

$$\Rightarrow \ln x = \ln \left(\frac{x/y}{e^{x/y} + x/y} \right) + \ln c$$

$$\Rightarrow x = \left(\frac{x/y \cdot c}{e^{x/y} + x/y} \right)$$

$$\Rightarrow ye^{x/y} + x = c$$

19. (d)

$$(x+y)^2 \frac{dy}{dx} = a^2 \Rightarrow \frac{dy}{dx} = \frac{a^2}{(x+y)^2} \quad \dots\dots(1)$$

$$\text{Put } x+y=t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \quad \dots\dots(2)$$

\Rightarrow From (1) & (2)

$$\Rightarrow 1 + \frac{a^2}{t^2} = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{t^2}{t^2 + a^2} dt$$

$$\Rightarrow dx = \left(1 - \frac{a^2}{t^2 + a^2}\right) dt$$

Integrating both sides

$$\Rightarrow x + c = t - a \tan^{-1} t/a \quad \because t = x + y$$

$$\Rightarrow x = x + y - a \tan^{-1} \left(\frac{x+y}{a}\right) + c$$

$$\Rightarrow a \tan^{-1} \left(\frac{x+y}{a}\right) = y + c \Rightarrow \frac{x+y}{a} = \tan \left(\frac{y+c}{a}\right)$$

20. (b)

$$\left. \frac{dy}{dx} \right|_{(4,3)} = \frac{-1}{m_N}$$

$$\Rightarrow \left. \frac{b^2 x}{a^2 y} \right|_{(4,3)} = - \left(\frac{4-16}{3-0} \right)$$

$$\Rightarrow \frac{4b^2}{3a^2} = \frac{12}{3}$$

$$\Rightarrow \frac{b^2}{a^2} = 3$$

We know

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1+3} = 2$$

21. (b)

$$\text{Volume of spherical balloon } V = \frac{4}{3} \pi r^3$$

$$\frac{\Delta V}{V} \times 100 = \frac{\left(\frac{4}{3} \pi \left(r + \frac{r}{10} \right)^3 - \frac{4}{3} \pi r^3 \right)}{\frac{4}{3} \pi r^3} \times 100$$

$$= \left(1 + \frac{1}{10} \right)^3 - 1$$

$$= 1 + \frac{3}{10} + \frac{3}{100} + \frac{1}{1000} - 1 \simeq 0.3\% \text{ approximately}$$

22. (b)

Let sides are a, ar, ar^2 ($r > 1$)

Using Pythagoras theorem

$$\Rightarrow a^2 + a^2 r^2 = a^2 r^4$$

$$\Rightarrow r^4 + r^2 - 1 = 0$$

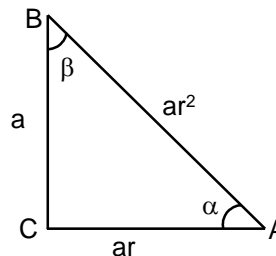
$$\Rightarrow r^2 = \frac{-1 + \sqrt{5}}{2}, \frac{-1 - \sqrt{5}}{2} \text{ (not possible)}$$

$$\Rightarrow r = \sqrt{\frac{-1 + \sqrt{5}}{2}}$$

$$\tan \alpha = \frac{1}{r}$$

$$= \sqrt{\frac{2}{\sqrt{5} - 1}} = \sqrt{\frac{\sqrt{5} + 1}{2}}$$

$$\tan \beta = r = \sqrt{\frac{\sqrt{5} - 1}{2}}$$



23. (b)

$$\log_2 6 + \frac{1}{2x} = \log_2(2^{1/x} + 8)$$

$$\Rightarrow \log_2 \left(\frac{2^{1/x} + 8}{6} \right) = \frac{1}{2x}$$

On taking anti log

$$\Rightarrow \frac{2^{1/x} + 8}{6} = 2^{1/2x}$$

$$\Rightarrow \left(2^{\frac{1}{2x}} \right)^2 - 6 \cdot 2^{1/2x} + 8 = 0$$

$$\Rightarrow t^2 - 6t + 8 = 0$$

Let $2^{\frac{1}{2x}} = t$

$$\Rightarrow t = 4, 2$$

$$\Rightarrow 2^{\frac{1}{2x}} = 4, 2$$

$$\Rightarrow 2x = 1, \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{2}, \frac{1}{4}$$

24. (c)

$$\Rightarrow \arg z - \arg(-z)$$

$$\Rightarrow \arg z - [\arg z - \arg(-1)]$$

$$\because \arg \left(\frac{z_1}{z_2} \right) = \arg z_1 - \arg z_2$$

$$\Rightarrow \arg z - [\arg z - \arg(-1)]$$

$$\because \arg(-1) = \pi$$

$$\Rightarrow \pi$$

25. (b)

$$\Rightarrow (\cos\theta + i \sin\theta) (\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = 1$$

$$\Rightarrow e^{i\theta} \cdot e^{i2\theta} \dots e^{in\theta} = 1 \quad (\text{using Euler's formula})$$

$$\Rightarrow e^{i\theta(1+2+\dots+n)} = 1$$

$$\Rightarrow e^{i\theta \left(\frac{n(n+1)}{2} \right)} = 1$$

$$\Rightarrow \frac{n(n+1)\theta}{2} = 2k\pi$$

$$\Rightarrow \theta = \frac{4k\pi}{n(n+1)}$$

26. (d)

Given

$$\Rightarrow ax^2 + bx + c = 0 \text{ has imaginary roots} \Rightarrow D < 0$$

$$\Rightarrow a + b + c < 0 \Rightarrow f(1) < 0$$

$$\Rightarrow f(x) < 0 \quad \forall x \in \mathbb{R} \text{ and } a < 0$$

$$\Rightarrow f(0) < 0 \Rightarrow c < 0$$

27. (a)
 $\Rightarrow {}^6C_2 \times {}^6C_4 + {}^6C_3 \times {}^6C_3 + {}^6C_4 \times {}^6C_2$
 $\Rightarrow (15)^2 + (20)^2 + (15)^2$
 $\Rightarrow 225 + 400 + 225 = 850$
28. (b)
 $\Rightarrow {}^7C_4 \times (\text{De-arrangement of 3 things})$
 $\Rightarrow 35 \times 2 = 70 = {}^7C_3 \times 2$
29. (d)
 $\Rightarrow 7^{2n} + 16n - 1$
 $\Rightarrow (8 - 1)^{2n} + 16n - 1$
 $\Rightarrow [{}^{2n}C_0 8^{2n} - {}^{2n}C_1 8^{2n-1} + \dots - {}^{2n}C_{2n-1} 8^1 + {}^{2n}C_{2n}] + 16n - 1$
 $\Rightarrow [64\lambda - 2n \cdot 8 + 1] + 16n - 1$
 $\Rightarrow 64\lambda - 16n + 1 + 16n - 1$
 $\Rightarrow 64\lambda$
30. (b)
 $\Rightarrow \frac{84-n}{8} = \text{rational} \Rightarrow n = 4, 12, \dots, 76$
 $\Rightarrow \frac{n}{4} = \text{rational} \Rightarrow n = 4, 8, 12, \dots, 84$
 $\Rightarrow n$ can take total 11 terms
 \Rightarrow Total number of rational terms = 11
 Irrational terms = total terms - rational terms = $85 - 11 = 74$
31. (c)
 $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}; I_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$
 $\Rightarrow A - 3I_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$
 $\Rightarrow \det. (A - 3I_3) = -2(3) - 1(-3) + 1(3) = 0$
 $\Rightarrow A - 3I_3$ is non invertible matrix
32. (c)
 We know, for square matrix $\text{adj}(M') = (\text{adj } M)'$
 So, $\text{adj}(M') - (\text{adj } M)' = 0 = \text{null matrix}$

33. (a)

$$A = \begin{vmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{vmatrix} \text{ (given)}$$

$$\Rightarrow |A^2| = 25(\text{given})$$

$$\Rightarrow |A| = \pm 5$$

$$\Rightarrow |A| = \begin{vmatrix} 5 & 5x & x \\ 0 & x & 5x \\ 0 & 0 & 5 \end{vmatrix} = \pm 5$$

$$\Rightarrow 25x = \pm 5$$

$$\Rightarrow x = \pm \frac{1}{5}$$

$$\Rightarrow |x| = \frac{1}{5}$$

34. (d)

$$\text{Let } A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \text{ \& } B = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Hence $AB = 0$ but $A \neq 0, B \neq 0$

So, if $AB = 0$ then may be $A \neq 0, B = 0$

35. (b)

Let (h, k) satisfies $x^2 + y^2 = 1$ then $h^2 + k^2 = 1$

Now $h^8 + k^8 = h^8 + (1 - h^2)^4 = 2h^8 - 4h^6 - 4h^2 + 1$

$= 2h^2(h^2 - 1)(h^4 - h^2 + 2) + 1$

$= -2h^2k^2(h^4 - h^2 + 2) + 1 < 1 \quad \forall \quad h > 0, k > 0$

\Rightarrow All solution of $x^2 + y^2 = 1$ satisfies $x^8 + y^8 < 1$

$\Rightarrow P \cap T = P$

36. (d)

$$f(x) = x^2 - \frac{x^2}{1+x^2}$$

$$\Rightarrow f(x) = \frac{x^4}{1+x^2}$$

$$\therefore f(-x) = f(x)$$

$\therefore f(x)$ is an even function, hence it is many one.

Also, $f(x) \geq 0 \quad \forall \quad x \in \mathbb{R}$, hence it is into function

$\Rightarrow f(x)$ is neither one -one nor onto

37. (d)
 $(a, a) \in \rho$ because $1 + a^2 > 0 \Rightarrow \rho$ is reflexive
If $1 + ab > 0$ then $1 + ba < 0 \Rightarrow$ if $(a, b) \in \rho$ then $(b, a) \in \rho$
 $\Rightarrow \rho$ is symmetric
Now $\left(-4, \frac{1}{8}\right) \in \rho$ and $\left(\frac{1}{8}, 5\right) \in \rho$
But $(-4, 5) \notin \rho$, hence ρ is not transitive

38. (c)
Probability that no student solves problem is
 $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}$

\Rightarrow Probability that the problem will be solved by at least one student is $= 1 - \frac{1}{5} = \frac{4}{5}$

39. (d)
Given
 $\sigma(x) = 2.6$
We know
 $\sigma(ax + b) = |a|(\sigma(x))$
So $s(-4x + 1) = |-4|(\sigma(x)) = 4 \times 2.6 = 10.4$

40. (a)
 $e^{\sin x} - e^{-\sin x} = 4$
Let $e^{\sin x} = t$
 $\Rightarrow t^2 - 4t - 1 = 0$
 $\Rightarrow t = 2 + \sqrt{5}, 2 - \sqrt{5}$
We know that t is real positive number
 $\Rightarrow t = 2 + \sqrt{5}$
 $\Rightarrow e^{\sin x} = 2 + \sqrt{5}$
 $\Rightarrow [e^{-1}, e^1] \notin 2 + \sqrt{5}$
 \Rightarrow hence no solution exist

41. (d)
Let, angles are $-2x, 3x, 7x$
We know
 $\Rightarrow 2x + 3x + 7x = 180^\circ \Rightarrow 12x = 180^\circ \Rightarrow x = 15^\circ$
 \Rightarrow angles are $-30^\circ, 45^\circ, 75^\circ$
 \Rightarrow length of smallest side $a \Rightarrow \frac{a}{\sin A} = 2R$
 $\Rightarrow a = 2R \sin A$
 $\Rightarrow a = 2 \times 10 \times \sin 30^\circ = 10$

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42. (b)

Let P(h, k) then A is (h, 0) & B is (0, k)

Equation of AB is $\frac{x}{h} + \frac{y}{k} = 1$ which passes through (x_1, y_1) is

$$\Rightarrow \frac{x_1}{h} + \frac{y_1}{k} = 1$$

$$\Rightarrow \text{Required locus is } \frac{x_1}{x} + \frac{y_1}{y} = 1$$

43. (b)

$$L_1 : \sqrt{3}x + y = 1 \Rightarrow m_1 = -\sqrt{3}$$

$\theta = 60^\circ$ (given)

$$\Rightarrow \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow \pm \sqrt{3} = \frac{-\sqrt{3} - m_2}{1 - \sqrt{3} m_2}$$

$$\Rightarrow m_2 = \sqrt{3}, 0 \text{ (not possible because lines are not parallel)}$$

$$\Rightarrow m_2 = \sqrt{3}$$

So, equation of line passing through (3, -2) and her slope $\sqrt{3}$ is

$$\Rightarrow y + 2 = \sqrt{3} (x - 3)$$

$$\Rightarrow y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

44. (a)

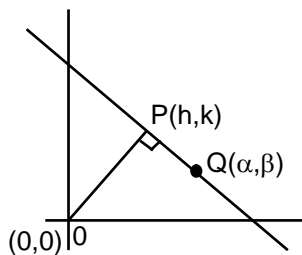
$$M_{PQ} \cdot M_{OP} = -1$$

$$\Rightarrow \left(\frac{k - \beta}{h - \alpha} \right) \cdot \left(\frac{k}{h} \right) = -1$$

$$\Rightarrow k^2 - k\beta = -(h^2 - h\alpha)$$

$$\Rightarrow h^2 + k^2 - h\alpha - k\beta = 0$$

$$\Rightarrow \text{Required locus is } -x^2 + y^2 - \alpha x - \beta y = 0$$



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45. (b)

Let point on 4th quadrant which is equidistant from both the axis is $(\alpha, -\alpha)$

$$\Rightarrow L_1 : 2a\alpha - 4a\alpha + c = 0 \Rightarrow \alpha = \frac{c}{2a} \dots\dots\dots(1)$$

$$\Rightarrow L_2 : 7b\alpha - 3b\alpha - d = 0 \Rightarrow \alpha = \frac{d}{4b} \dots\dots\dots(2)$$

From equation (1) & (2)

$$\Rightarrow \frac{c}{2a} = \frac{d}{4b}$$

$$\Rightarrow \frac{4}{2} = \frac{ad}{cb}$$

$$\Rightarrow ad : bc = 2 : 1$$

46. (a)

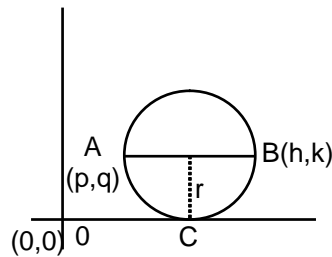
Let other end is (h, k) then centre equal to $\left(\frac{p+h}{2}, \frac{q+k}{2}\right)$

Because circle touches x-axis hence radius = $\left|\frac{q+k}{2}\right|$

We know

$$\Rightarrow (AB) \text{ Diameter} = 2r$$

$$\Rightarrow \sqrt{(h-p)^2 + (k-q)^2} = 2 \left|\frac{q+k}{2}\right|$$



On squaring both sides

$$\Rightarrow (h-p)^2 + (k-q)^2 = (q+k)^2$$

$$\Rightarrow (h-p)^2 = (q+k)^2 - (k-q)^2$$

$$\Rightarrow \text{Required locus is } (x-p)^2 = 4qy$$

47. (c)

ΔPQR is equilateral triangle

So, incentre is same as centroid

$$\Rightarrow \text{incentre} = \text{centroid} = \text{centre of circle} = \left(\frac{1 + \frac{1}{2} + 0}{3}, \frac{0 + 0 + \frac{\sqrt{3}}{2}}{3}\right) = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$$

48. (c)

$$\text{Focus are } (\pm\sqrt{a^2 + b^2}, 0) = (\pm\sqrt{\cos^2 \alpha + \sin^2 \alpha}, 0)$$

\Rightarrow focus are $(\pm 1, 0)$ which is independent of α

\Rightarrow focus are fixed

49. (c)

$$S = (ae, 0)$$

$$T = (-ae, 0)$$

$$B = (0, b)$$

$$\Rightarrow (ST)^2 = (SB)^2$$

$$\Rightarrow (2ae)^2 = (ae)^2 + b^2$$

$$\Rightarrow 3(ae)^2 = b^2$$

$$\Rightarrow b^2 = 3(a^2 - b^2) \Rightarrow \frac{b^2}{a^2} = \frac{3}{4}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{3}{4}}$$

$$e = \frac{1}{2}$$

50. (a)

$$\text{Equation of hyperbola is } 3(x^2 - 6x) - 3(y^2 - 4y) + 2 = 0$$

$$\Rightarrow (x - 3)^2 - (y - 2)^2 = \frac{13}{3}$$

$$\Rightarrow \frac{(x-3)^2}{\sqrt{\frac{13}{3}}} - \frac{(y-2)^2}{\sqrt{\frac{13}{3}}} = 1 \Rightarrow e = \sqrt{2}$$

We know equation of directrix is $X = \pm \frac{a}{e}$

$$\Rightarrow x - 3 = \pm \frac{\sqrt{13/3}}{\sqrt{2}}$$

$$\Rightarrow x = 3 \pm \frac{\sqrt{13/3}}{\sqrt{2}}$$

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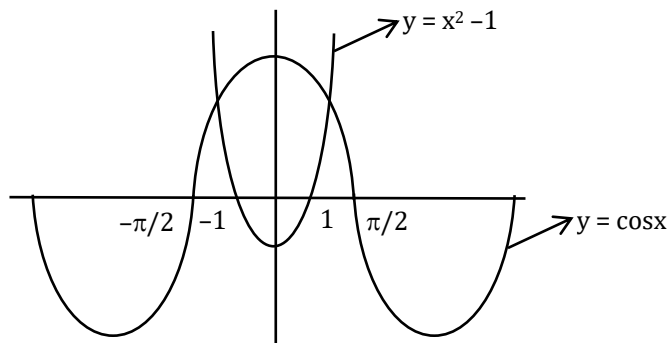


51. (a)

$$y = x^2 - 1 \text{ \& } y = \cos x$$

intersect at exactly

two points



52. (c)

Given

$$\frac{dx}{dt} = 1 \text{ unit per second}$$

$$y = \frac{10}{x} \quad \dots\dots\dots(1)$$

differentiating with respect to x

$$\Rightarrow \left. \frac{dy}{dx} \right|_{(5,2)} = \frac{-10}{x^2} \frac{dx}{dt}$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{(5,2)} = \frac{-10}{25} \times 1$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{(5,2)} = \frac{-2}{5}$$

\Rightarrow Ordinate decreases at rate $\frac{2}{5}$ unit per second.

53. (c)

$$a = \min [x^2 + 2x + 3; x \in \mathbb{R}]$$

$$a = \min [(x+1)^2 + 2]$$

$$\Rightarrow a = 2$$

$$b = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2} = \frac{1}{2^2} \cdot \frac{2 \sin^2 \frac{\theta}{2}}{\frac{\theta^2}{2^2}} = \frac{1}{2}$$

$$\Rightarrow b = \frac{1}{2}$$

Now,

$$\Rightarrow \sum_{r=0}^n a^r b^{n-r} = a^0 b^n + a b^{n-1} + a^2 b^{n-2} + \dots + a^n b^0$$

$$= \left(\frac{1}{2}\right)^n + \frac{2}{2^{n-1}} + \frac{2^2}{2^{n-2}} + \dots + 2^n$$

$$= \left(\frac{1}{2}\right)^n [1 + 4 + 4^2 + \dots + 4^n]$$

$$= \frac{1}{2^n} \left(\frac{4^{n+1} - 1}{4 - 1}\right)$$

$$= \frac{4^{n+1} - 1}{3 \cdot 2^n}$$

54. (a)

Given

$$\Rightarrow a > b > 0$$

$$\Rightarrow a > b$$

$$\Rightarrow \frac{b}{a} < 1$$

Now,

$$I_{(n)} = a^{1/n} - b^{1/n}$$

$$J_{(n)} = (a - b)^{1/n}$$

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B

$$\frac{I_{(n)}}{J_{(n)}} = \frac{a^{1/n} - b^{1/n}}{(a-b)^{1/n}} = \frac{\frac{a^{1/n} - b^{1/n}}{a^{1/n}}}{\frac{(a-b)^{1/n}}{a^{1/n}}}$$
$$= \frac{1 - \left(\frac{b}{a}\right)^{1/n}}{\left(1 - \frac{b}{a}\right)^{1/n}}$$

Let $b = 16$
 $a = 625$
 $n = 4$

$$= \frac{1 - \left(\frac{16}{625}\right)^{1/4}}{\left(1 - \frac{16}{625}\right)^{1/4}} = \frac{1 - \frac{2}{5}}{\left(\frac{609}{625}\right)^{1/4}} = \frac{3}{4.96} < 1$$

$$= \frac{I_{(n)}}{J_{(n)}} < 1$$

$$= I_{(n)} < J_{(n)}$$

55. (d)

$$|\hat{\alpha}| = |\hat{\beta}| = |\hat{\gamma}| = 1 \quad (\text{given})$$

Now,

$$\Rightarrow \hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2} (\hat{\beta} + \hat{\gamma})$$

On comparing both sides

$$\Rightarrow -\hat{\alpha} \cdot \hat{\beta} = \frac{1}{2}$$

$$\Rightarrow \hat{\alpha} \cdot \hat{\beta} = \frac{-1}{2}$$

$$\Rightarrow |\hat{\alpha}| |\hat{\beta}| \cos \theta = \frac{-1}{2}$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

56. (d)

Given

A, B, C, & D are on a plane

$\therefore \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar

$$\therefore [\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ -1 & -1 & -1-\lambda \end{vmatrix} = 0$$

$R_3 \rightarrow R_3 + R_1$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ 0 & 0 & -4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-4-\lambda)(-1+2) = 0$$

$$\Rightarrow \lambda = -4$$

57. (b)

$$\Rightarrow x_n = 1 + \frac{1}{4} + \frac{1}{16} + \dots$$

$$\Rightarrow \lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{4} + \frac{1}{16} + \dots \right)$$

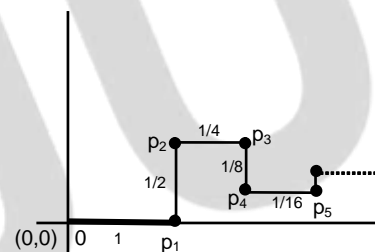
$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3} = \alpha$$

$$\Rightarrow y_n = \frac{1}{2} - \frac{1}{8} + \frac{1}{32} + \dots$$

$$\Rightarrow \lim_{n \rightarrow \infty} y_n = \lim_{n \rightarrow \infty} \left(\frac{1}{2} - \frac{1}{8} + \frac{1}{32} + \dots \right)$$

$$= \frac{1/2}{1 + \frac{1}{4}} = \frac{2}{5} = \beta$$

$$= (\alpha, \beta) = \left(\frac{4}{3}, \frac{2}{5} \right)$$



58. (a)

Using inequality $|A| + |B| \geq |A - B|$

$$\Rightarrow |z| + |z - 1| \geq |z - (z - 1)|$$

$$\Rightarrow |z| + |z - 1| \geq 1$$

\Rightarrow minimum value of $|z| + |z - 1|$ is 1

59. (c)
For infinitely many solution $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda & 1 & 3 \\ 2 & \mu & -1 \\ 5 & 7 & 1 \end{vmatrix} = 0$$

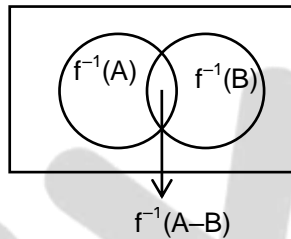
$$\Rightarrow \lambda\mu + 7\lambda - 7 + 42 - 15\mu = 0$$

$$\Rightarrow (\lambda - 15)(\mu + 7) + 140 = 0$$

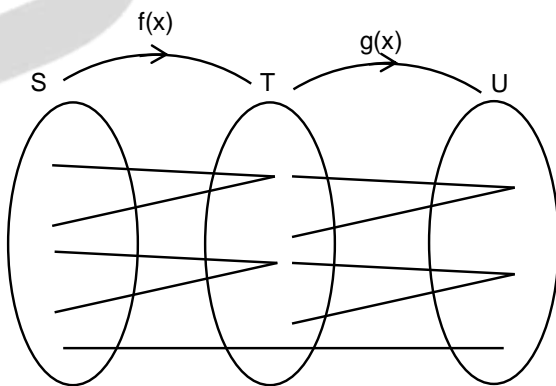
Now check from options

$$\Rightarrow (\lambda, \mu) = (1, 3)$$

60. (c)
We can see that pre image of $A - B$ i.e. $f^{-1}(A - B)$ will be $f^{-1}(A) - f^{-1}(B)$
Therefore, $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$



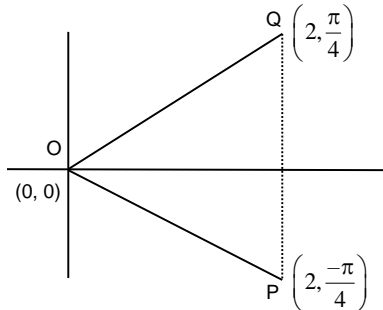
61. (b)
Obvious g is surjective other wise $g \circ f$ cannot be surjective but there is no need of f to be surjective.
Example.



Hence $f(x)$ is not surjective still $g \circ f$ is surjective

62. (a)
line joining PQ is bisected
by x-axis

So point Q is $\left(2, \frac{\pi}{4}\right)$



63. (b)
 $\Rightarrow b > a$ (given)
On squaring both sides

$$\Rightarrow \frac{b^2}{a^2} > 1$$

On adding 1 in both sides

$$\Rightarrow 1 + \frac{b^2}{a^2} > 2$$

Taking root both sides

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} > \sqrt{2}$$

$$\Rightarrow e > \sqrt{2}$$

64. (a)
 $\Rightarrow \lim_{x \rightarrow 0^+} \frac{x}{p} \left[\frac{q}{x} \right]$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{x}{p} \left(\frac{q}{x} - \left\{ \frac{q}{x} \right\} \right)$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{x}{p} \cdot \frac{q}{x} - \lim_{x \rightarrow 0^+} \frac{x}{p} \left\{ \frac{q}{x} \right\} \quad \therefore 0 \leq \left\{ \frac{q}{x} \right\} < 1$$

$$\Rightarrow \frac{q}{p} - \lim_{x \rightarrow 0^+} \frac{x}{p} (\text{finite})$$

$$\Rightarrow \frac{q}{p} - 0 \times (\text{finite})$$

$$\Rightarrow \frac{q}{p} - 0 \Rightarrow \frac{q}{p}$$

65. (b)

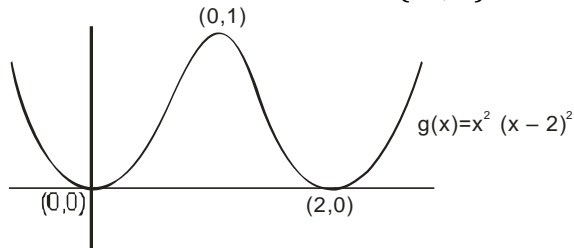
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$$f(x) = x^4 - 4x^3 + 4x^2 + c, c \in \mathbb{R}$$

Using IVT theorem

$$\begin{aligned} \text{For atleast one root/zero} &= f(1) \cdot f(2) < 0 \\ &= (1 + c) \cdot c < 0 \\ &= c \in (-1, 0) \end{aligned}$$



$$f(x) = x^2(x - 2)^2 + c$$

$\Rightarrow f(x)$ has exactly one zero in $(1, 2)$ if $c \in (-1, 0)$

66. (a,c)

$$f(a) = 0 = f(b) \quad \Rightarrow f'(a)f'(b) < 0$$

$$\text{Let } h(x) = f'(x) + f(x)g'(x) \dots\dots\dots(1)$$

Put $x = a$ in equation (1)

$$h(a) = f'(a) \quad \because f(a) = 0$$

Put $x = b$ in equation (1)

$$h(b) = f'(b) \quad \because f(b) = 0$$

$\therefore h(a) \cdot h(b) < 0 \Rightarrow h(x) = 0$ has roots between (a, b)

Similarly $g'(x) + kg(x) = 0$ has roots between (a, b) as $g(a) = 0 = g(b)$

67. (b,c)

$$f(x) = \frac{x^3}{4} - \sin \pi x + 3$$

$$f(-2) = 1$$

$$f(2) = 5$$

\because function is continuous

By intermediate value theorem, $f(x)$ takes all values between 1 to 5

68. (b,d)

$$I_n = \int_0^1 x^n \tan^{-1} x dx$$

Using Integration by parts

$$\Rightarrow I_n = \tan^{-1} x \frac{x^{n+1}}{n+1} \Big|_0^1 - \int_0^1 \frac{x^{n+1}}{n+1} \cdot \left(\frac{1}{1+x^2} \right) dx$$

$$\Rightarrow (n+1) I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{1+x^2} dx \quad \dots\dots(1)$$

Put $n \rightarrow n+2$ in equation (1)

$$\Rightarrow (n+3) I_{n+2} = \frac{\pi}{4} - \int_0^1 \frac{x^{n+3}}{1+x^2} dx \quad \dots\dots(2)$$

From equation(1) + (2)

$$\Rightarrow (n+1)I_n + (n+3) I_{n+2} = \frac{\pi}{2} - \frac{1}{n+2}$$

\Rightarrow on comparing with given equation

$$\Rightarrow a_n = n+1, b_n = n+3, c_n = \frac{\pi}{2} - \frac{1}{n+2}$$

69. (c)

$$S + n = \frac{1}{2} f(t+m^2) \text{ and } S = \frac{1}{2} ht^2, V = ht$$

$$\therefore \frac{1}{2} ht^2 + n = \frac{1}{2} f(t+m^2) \quad \dots\dots(1)$$

$$\text{Also } V = 0 + ht = 0 + f(t+m) \Rightarrow t+m = \frac{ht}{f} \text{ (put in equation (1))}$$

from equation (1),

$$\Rightarrow \frac{1}{2} ht^2 + n = \frac{1}{2} f \left(\frac{ht}{f} \right)^2$$

$$\Rightarrow t^2 = \frac{2hf}{h(h-f)}$$

Also,

$$ht = f(t+m) \Rightarrow t^2 = \frac{m^2 f^2}{(h-f)^2}$$

$$\therefore \frac{2nf}{h(h-f)} = \frac{m^2 f^2}{(h-f)^2}$$

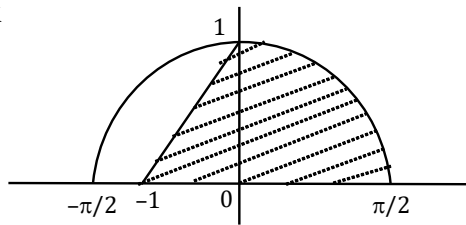
$$\Rightarrow n(h-f) = \frac{1}{2} f hm^2$$

70. (b)

$$\text{Area} = \frac{1}{2} \times 1 \times 1 + \int_0^{\pi/2} \cos x dx$$

$$= \frac{1}{2} + \sin x \Big|_0^{\pi/2}$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$



71. (b)

$$x^2 - 3x + a = 0 \begin{cases} x_1 \\ x_2 \end{cases} \Rightarrow x_1 + x_2 = 3; x_1 \cdot x_2 = a$$

$$x^2 - 12x + b = 0 \begin{cases} x_3 \\ x_4 \end{cases} \Rightarrow x_3 + x_4 = 12; x_3 \cdot x_4 = b$$

$\therefore x_1, x_2, x_3, x_4$ are in G.P, then

$$\Rightarrow \frac{x_3 + x_4}{x_1 + x_2} = \frac{12}{3}$$

$$\Rightarrow \frac{Ar^2 + Ar^3}{A + Ar} = 4$$

$$\Rightarrow r^2 = 4 \Rightarrow x = 2 \quad (\because \text{G.P is increasing})$$

$$\therefore x_1 + x_2 = 3$$

$$\Rightarrow A + Ar = 3$$

$$\Rightarrow A(3) = 3$$

$$\Rightarrow A = 1$$

$$\therefore ab = x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 1 \cdot 2 \cdot 4 \cdot 8 = 64$$

72. (a)

$$\Rightarrow \frac{1 - i \cos \theta}{1 + 2i \cos \theta}$$

Using rationalization

$$\Rightarrow \frac{1 - i \cos \theta}{1 + 2i \cos \theta} \times \frac{1 - 2i \cos \theta}{1 - 2i \cos \theta}$$

$$\Rightarrow \frac{(1 - 2 \cos^2 \theta) - 3i \cos \theta}{1 + 4 \cos^2 \theta} \text{ is real}$$

$$\Rightarrow \therefore \cos \theta = 0$$

$$\Rightarrow \theta = 2n\pi + \frac{\pi}{2}, n \in \mathbb{I}$$

73. (b)

$$\text{Let } (A - \lambda I_3) = 0$$

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$$\Rightarrow \begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)^3 - 9(3-\lambda) = 0$$

$$\Rightarrow (3-\lambda)[(3-\lambda)^2 - 3^2] = 0$$

$$\Rightarrow 3-\lambda = 0 \text{ or } 3-\lambda-3 = 0 \text{ or } 3-\lambda+3 = 0$$

$$\Rightarrow \lambda = 0, 3, 6$$

74. (b)

AB = AC (given)

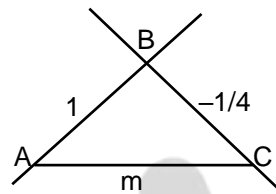
$\Rightarrow \angle ABC = \angle BCA$

Let slope of AC is m.

$$\therefore \left| \frac{m + \frac{1}{4}}{1 - \frac{m}{4}} \right| = \left| \frac{-1 - 1}{4 - 1} \right|$$

$$\Rightarrow m = \frac{-23}{7}, 1 \text{ (rejected)}$$

\therefore equation of line is $23x + 7y + 3 = 0$



75. (a,c)

$$\frac{x^2}{1} - \frac{y^2}{5} = 1$$

Let the tangent of hyperbola $y = mx \pm \sqrt{a^2 m^2 - b^2}$

$$\Rightarrow y = mx \pm \sqrt{m^2 - 5} \quad \dots\dots(1)$$

\downarrow Passes through (2, 8)

$$\Rightarrow (8 - 2m) = \pm \sqrt{m^2 - 5}$$

On squaring both sides

$$\Rightarrow (8 - 2m)^2 = (m^2 - 5)$$

$$\Rightarrow m = 3 \text{ or } \frac{23}{3} \text{ (put in equation (1))}$$

So, equation of tangent is $\Rightarrow y = 3x \pm 2$ or $y = \frac{23x}{3} \pm \frac{22}{3}$

$$\Rightarrow y = 3x + 2, y = 3x - 2, 3y = 23x + 22, 3y = 23x - 22$$