JEE Main 2017 Maths Paper With Solutions (April 2)

Question 1: The function f : R → [(-1 / 2), (1 / 2)] is defined as f(x) = [x] / [1 + x^2], is:

(1) injective but not surjective.
(2) surjective but not injective.
(3) neither injective nor surjective.
(4) invertible.

Solution: (2)
f(x) = [x] / [1 + x^2]

$$f'(x) = \frac{(1+x^2)*1 - x*2x}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$

f'(x) changes sign at different intervals.
∴Not injective.
y = (x / (1 + x^2))
yx^2 - x + y = 0
For y ≠ 0
D = 1 - $4y^2 \geq 0$
y ∈ [(-1 / 2), (1 / 2)] - {0}
For, y = 0 ⇒ x = 0
∴Part of range
∴Range = [(-1 / 2), (1 / 2)]
∴Surjective but not injective.

Question 2: If, for a positive integer n, the quadratic equation,

$$x(x+1)+(x+1)(x+2)+\ldots.$$
$$+(x+\overline{n-1})\ (x+n)=10n$$

has two consecutive integral solutions, then n is equal to:
(1) 9

(2) 10
(3) 11
(4) 12

Solution: (3)
Rearranging equation, we get
$nx^2 + \{1 + 3 + 5 + ….. + (2n - 1)x$
$+ \{1.2 + 2.3 + ….. + (n - 1)n\} = 10n$
$nx^2 + n^2x + [(n - 1) n (n + 1)] / 3 = 10n$
$x^2 + nx + [n^2 - 31] / 3 = 0$
Given difference of roots = 1
$|\alpha - \beta| = 1$
$D = 1$
$n^2 - (4 / 3) (n^2 - 31) = 1$
$n = 11$

Question 3: Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k,$$

then k is equal to,

(1) z
(2) −1
(3) 1
(4) − z

Solution: (4)
$2\omega + 1 = z, z = \sqrt{3}i$
$\omega = (-1 + \sqrt{3}i) / 2$ [Cube root of unity]

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$$

$= 3 (\omega^2 - \omega^4)$

$= 3 [\{(-1 - \sqrt{3}i) / 2\} - \{(-1 + \sqrt{3}i) / 2\}]$

$= - 3\sqrt{3}i$

$= -3z$

$k = -z$

Question 4: If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to :

(1) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(2) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(3) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

(4) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Solution: (1)

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -3 \\ -4 & 1-\lambda \end{vmatrix}$$

$= (2 - 2\lambda - \lambda + \lambda^2) - 12$

$f(\lambda) = \lambda^2 - 3\lambda - 10$

$\because$ A satisfies $f(\lambda)$.

$A^2 - 3A - 10I = 0$

$A^2 - 3A = 10I$

$3A^2 - 9A = 30I$

$3A^2 + 12A = 30I + 21A$

$$= \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} + \begin{bmatrix} 42 & -63 \\ -84 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

Question 5: If S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is :

(1) an infinite set
(2) a finite set containing two or more elements
(3) a singleton
(4) an empty set

Solution: (3)

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$\Rightarrow - (1 - a^2) = 0$
$\Rightarrow a = 1$
For a = 1
Eq. (1) & (2) are identical i.e.,x + y + z = 1.
To have no solution with x + by + z = 0.
b = 1

Question 6: A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :

(1) 468
(2) 469
(3) 484
(4) 485

Solution: (4)

X(4 L 3 G)	Y(3 L 4 G)
3 L 0 G	0 L 3 G
2 L 1 G	1 L 2 G
1 L 2 G	2 L 1 G
0 L 3 G	3 L 0 G

Required number of ways

$= {}^4C_3 \cdot {}^4C_3 + ({}^4C_2 \cdot {}^3C_1)^2 + ({}^4C_1 \cdot {}^3C_2)^2 + ({}^3C_3)^2$

$= 16 + 324 + 144 + 1$

$= 485$

Question 7: The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots ({}^{21}C_{10} - {}^{10}C_{10})$ is

(1) $2^{21} – 2^{10}$
(2) $2^{20} – 2^{9}$
(3) $2^{20} – 2^{10}$
(4) $2^{21} – 2^{11}$

Solution: (3)

$${}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} = \frac{1}{2}\left\{{}^{21}C_0 + {}^{21}C_1 + \ldots + {}^{21}C_{21}\right\} - 1$$

$$= 2^{20} - 1$$

$$\left({}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10}\right) = 2^{10} - 1$$

Required sum $= (2^{20} – 1) – (2^{10} – 1)$

$= 2^{20} – 2^{10}$

Question 8: For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 – 3ac) = 15b(3a + c)$. Then :

(1) b, c and a are in A.P.

(2) a, b and c are in A.P.
(3) a, b and c are in G.P.
(4) b, c and a are in G.P.

Solution: (1)
$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$
$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 45b - 15b - 75ac = 0$
$(15a - 3b)^2 + (3b - 5c)^2 + (15a - 5c)^2 = 0$
It is possible when
$15a - 3b = 0$ and $3b - 5c = 0$ and $15a - 5c = 0$
$15a = 3b = 5c$
$(a / 1) = (b / 5) = (c / 3)$
∴ b, c, a are in A.P.

Question 9: Let a, b, c $\in$ R. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy$, $\forall$ x, y $\in$ R, then $\sum_{n=1}^{10} f(n) =$ is equal to :

(1) 165
(2) 190
(3) 255
(4) 330

Solution: (4)
As $f(x + y) = f(x) + f(y) + xy$
Given, $f(1) = 3$
Putting, $x = y = 1$
$\Rightarrow f(2) = 2f(1) + 1 = 7$
Similarly, $x = 1, y = 2,$
$\Rightarrow f(3) = f(1) + f(2) + 2 = 12$
Now, $\sum_{n=1}^{10} f(n) = f(1) + f(2) + \ldots . f(10)$
$= 3 + 7 + 12 + 18 + \ldots$

= S (let)
Now, $S_n = 3 + 7 + 12 + 18 \dots + t_n$
Again, $S_n = 3 + 7 + 12 + 18 \dots + t_{n-1} + t_n$
We get, $t_n = 3 + 4 + 5 + \dots$ n terms
= [n (n + 5)] / 2

$$i.e., S_n = \sum_{n=1}^{n} t_n = \frac{1}{2}\left\{\sum n^2 + 5\sum n\right\} = \frac{n(n+1)(n+8)}{6}$$

$$\text{So, } S_{10} = \frac{10 \times 11 \times 18}{6} = 330$$

Question 10: $\lim_{x \to \pi/2} \frac{cotx - cosx}{(\pi - 2x)^3}$ equals:

(1) 1 / 16
(2) 1 / 8
(3) 1 / 4
(4) 1 / 24

Solution: (1)

$$\lim_{x \to \pi/2} \frac{cotx - cosx}{(\pi - 2x)^3}$$

$$Put\ x = \frac{\pi}{2} - x = t$$

$$= \lim_{t=0} \frac{tant - sint}{8t^3}$$

$$= \lim_{t=0} \frac{sint * 2sin^2\frac{t}{2}}{8t^3}$$

$$= \frac{1}{16}$$

Question 11: If for x ∈ (0, (1 / 4)), the derivative of $\tan^{-1}(6x\sqrt{x} / (1 - 9x^3))$ is $\sqrt{x}$. g(x), then g(x) equals:

(1) $(3x\sqrt{x} / (1 - 9x^3))$
(2) $(3x / (1 - 9x^3))$
(3) $(3 / (1 + 9x^3))$
(4) $(9 / (1 + 9x^3))$

Solution: (4)
$f(x) = 2 \tan^{-1}(3x\sqrt{x})$ for $x \in (0, (1 / 4))$
$f'(x) = (9\sqrt{x} / (1 + 9x^3))$
$g(x) = (9 / (1 + 9x^3))$

Question 12: The normal to the curve $y (x − 2) (x − 3) = x + 6$ at the point where the curve intersects the y-axis passes through the point :

(1) (1 / 2, 1 / 2)
(2) (1 / 2, -1 / 3)
(3) (1 / 2, 1 / 3)
(4) (-1 / 2, -1 / 2)

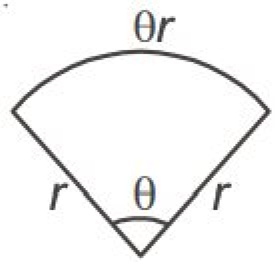
Solution: (1)
$y (x - 2) (x - 3) = x + 6$
At y-axis, $x = 0$, $y = 1$
Now, on differentiation,
$(dy / dx) (x - 2) (x - 3) + y (2x - 5) = 1$
$(dy / dx) (6) + 1 * (-5) = 1$
$(dy / dx) = 6 / 6 = 1$
Now slope of normal = –1
Equation of normal $y – 1 = –1 (x – 0)$
$y + x – 1 = 0$ ... (i)
Line (i) passes through (1 / 2, 1 / 2).

Question 13: Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :

(1) 10

(2) 25
(3) 30
(4) 12.5

Solution: (2)



$2r + \theta r = 20$ ---- (i)
$A = \text{Area} = (\theta / 2\pi) * (\pi r^2) = (\theta r^2 / 2)$ ---- (ii)
$A = (r^2 / 2) ([20 - 2r] / r)$
$A = [(20r - 2r^2) / 2] = 10r - r^2$
A to be maximum
$dA / dr = 10 - 2r = 0 \Rightarrow r = 5$
$d^2A / dr^2 = -2 < 0$
Hence for r = 5, A is maximum
Now, $10 + (\theta * 5) = 20 \Rightarrow \theta = 2$ (radian)
Area = $(2 / 2\pi) * (\pi) (5)^2 = 25$ sq m

Question 14: Let $I_n = \int \tan^n x \, dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (1 / 5, 0)
(2) (1 / 5, -1)
(3) (-1 / 5, 0)
(4) (-1 / 5, 1)

Solution: (1)
$I_n = \int \tan^n x \, dx, (n > 1)$
$I_4 + I_6 = \int (\tan^4 x + \tan^6 x) \, dx$
$= \int \tan^4 x \sec^2 x \, dx$
Let $\tan x = t$

$\sec^2 x\, dx = dt$
$= \int t^4\, dt$
$= (t^5 / 5) + C$
$= (1 / 5) \tan^5 x + C$
$a = (1 / 5), b = 0$

Question 15: The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + cosx}$ is equal to:

(1) 2
(2) 4
(3) −1
(4) −2

Solution: (1)

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{2\cos^2\frac{x}{2}}dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2\frac{x}{2}dx$$

$$= \frac{1}{2}\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \tan\frac{3\pi}{8} - \tan\frac{\pi}{8}$$

$$\left[\tan\frac{\pi}{8} = \sqrt{\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \frac{\sqrt{2}-1}{1}\right.$$

$$\left.\tan\frac{3\pi}{8} = \sqrt{\frac{1-\cos\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}} = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \sqrt{2}+1\right]$$

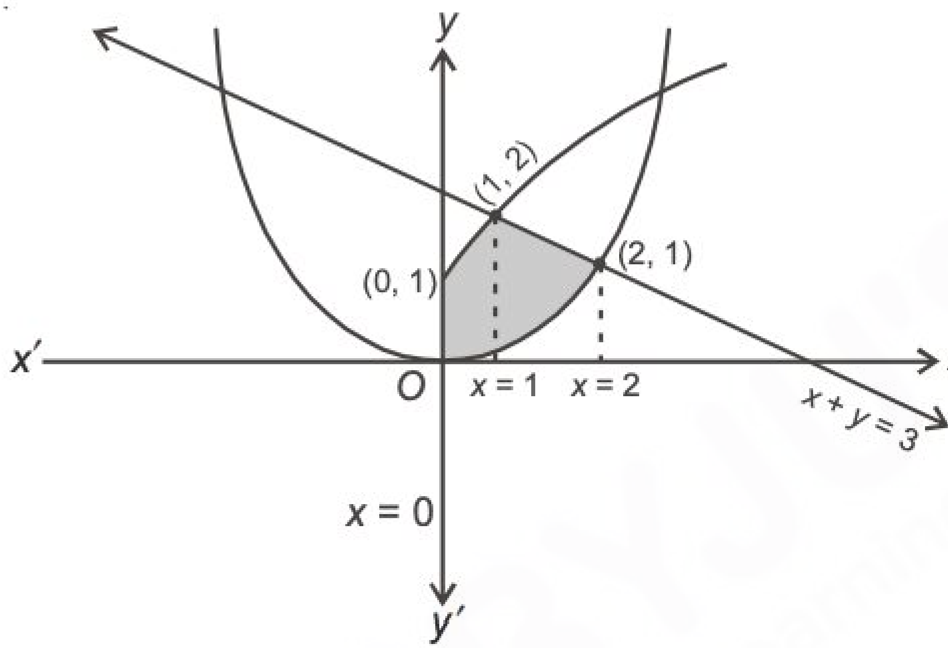
$$= (\sqrt{2}+1) - (\sqrt{2}-1)$$

$$= 2$$

Question 16: The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y$ and $y \leq 1 + x\}$ is :

(1) 3 / 2
(2) 7 / 3
(3) 5 / 2
(4) 59 / 12

Solution: (3)



Area of the shaded region

$$\int_0^1 (\sqrt{x} + 1 - \frac{x^2}{4})dx + \int_1^2 ((3 - x) - \frac{x^2}{4})dx$$

$$= \frac{5}{2}$$

sq units

Question 17: If (2 + sinx) (dy / dx) + (y + 1) cosx = 0 and y(0) = 1, then y(**π** / 2) is equal to:

(1) -2 / 3
(2) -1 / 3
(3) 4 / 3
(4) 1 / 3

Solution: (4)

$(2 + \sin x)(dy / dx) + (y + 1)\cos x = 0$

$y(0) = 1, y(\pi / 2) = ?$

$(1 / (y + 1))\, dy + (\cos x / [2 + \sin x])\, dx = 0$

$\ln|y + 1| + \ln(2 + \sin x) = \ln C$

$(y + 1)(2 + \sin x) = C$

Put $x = 0, y = 1$

$(1 + 1) . 2 = C \Rightarrow C = 4$

Now, $(y + 1)(2 + \sin x) = 4$

$(y + 1) = 4 / 3$

$y = (4 / 3) - 1 = 1 / 3$

Question 18: Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point :

(1) (1, 3 / 4)
(2) (1, -3 / 4)
(3) (2, 1 / 2)
(4) (2, -1 / 2)

Solution: (3)

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix}\right| = 28$$

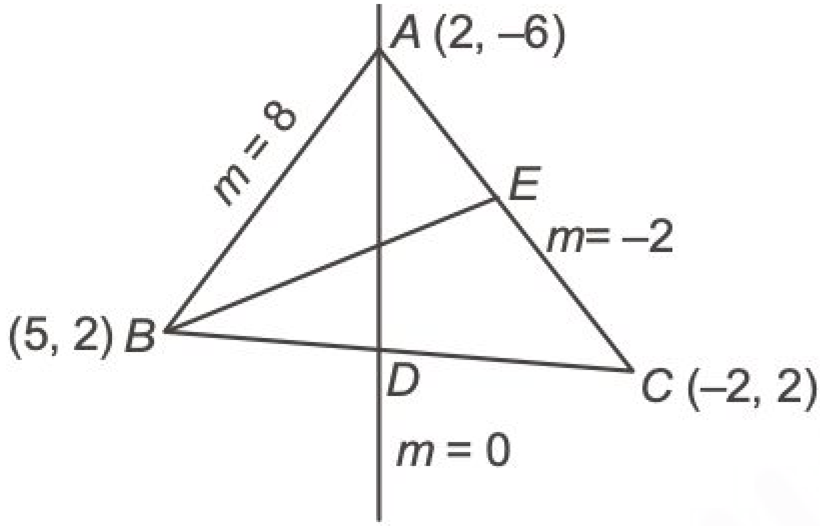
$$\begin{vmatrix} k-5 & -4k & 0 \\ 5+k & k-2 & 0 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$$

$(k^2 - 7k + 10) + 4k^2 + 20k = \pm 56$

$5k^2 + 13k - 46 = 0$

$5k^2 + 13k + 66 = 0$

$5k^2 + 13k - 46 = 0$
$k = [-13 \pm \sqrt{169 + 920}] / 10$
$= 2, – 4.6$ [reject]
For k = 2

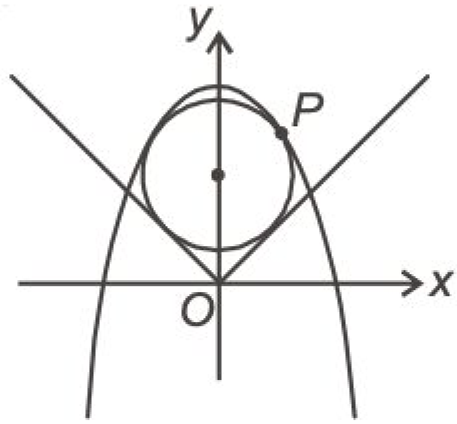


Equation of AD,
$x = 2$ ...(i)
Also equation of BE,
$(y - 2) = (1 / 2) (x - 5)$
$2y - 4 = x - 5$
$x - 2y - 1 = 0$ ...(ii)
Solving (i) & (ii), $2y = 1$
$y = 1 / 2$
Orthocentre is $(2, 1 / 2)$

Question 19: The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is :

(1) $2 (\sqrt{2} - 1)$
(2) $4 (\sqrt{2} - 1)$
(3) $4 (\sqrt{2} + 1)$
(4) $2 (\sqrt{2} + 1)$

Solution: (2)



$x^2 = - (y - 4)$

Let a point on the parabola P $[(t / 2) , (4 - (t^2 / 4))]$

Equation of normal at P is

$y + (t^2 / 4) - 4 = (1 / t) (x - [t / 2])$

$x - ty - (t^3 / 4) + (7 / 2)t = 0$

It passes through centre of circle, say (0, k)

$-tk - (t^3 / 4) + (7 / 2)t = 0$ ---- (i)

$t = 0, t^2 = 14 - 4k$

Radius $= r = |(0 - k) / \sqrt{2}|$ (Length of perpendicular from (0, k) to y = x)

$r = k / \sqrt{2}$

Equation of circle is $x^2 + (y - k)^2 = k^2 / 2$

It passes through point P

$(t^2 / 4) + [4 - (t^2 / 4) - k]^2 = k^2 / 2$

$t^4 + t^2 [8k - 28] + 8k^2 - 128k + 256 = 0$ ---- (ii)

For $t = 0 \Rightarrow k^2 - 16k + 32 = 0$

$k = 8 \pm 4\sqrt{2}$

$r = k / \sqrt{2} = 4 (\sqrt{2} - 1)$ (discarding $4 (\sqrt{2} + 1)$) ----- (iii)

For $t = \pm \sqrt{(14 - 4k)}$

$(14 - 4k)^2 + (14 - 4k) (8k - 28) + 8k^2 - 128k + 256 = 0$

$2k^2 + 4k - 15 = 0$

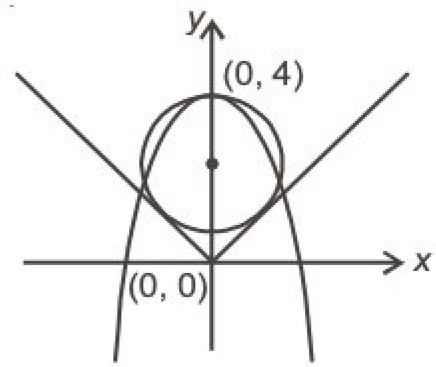
$k = [-2 \pm \sqrt{34}] /[2]$

$r = k / \sqrt{2} = [\sqrt{17} - \sqrt{2}] / 2$ (Ignoring negative value of r) ..(iv)

From (iii) & (iv),

$r_{min} = [\sqrt{17} - \sqrt{2}] / 2$

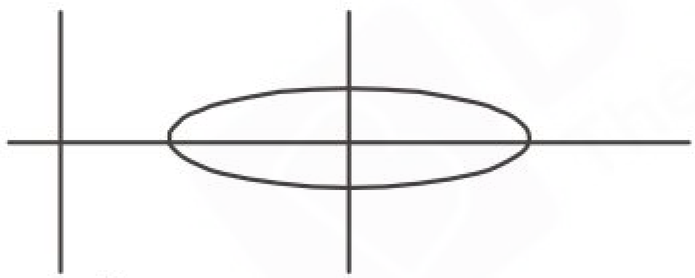
But from options, $4 (\sqrt{2} - 1)$



Question 20: The eccentricity of an ellipse whose centre is at the origin is 1 / 2. If one of its directrices is x = −4, then the equation of the normal to it at (1, (3 / 2)) is:

(1) 4x − 2y = 1
(2) 4x + 2y = 7
(3) x + 2y = 4
(4) 2y − x = 2

Solution: (1)



x = –4
e = 1 / 2
(-a / e) = -4
-a = -4 * e
a = 2
Now, $b^2 = a^2 (1 - e^2) = 3$
Equation to ellipse
$(x^2 / 4) + (y^2 / 3) = 1$
Equation of normal is

$$\frac{x-1}{\frac{1}{4}} = \frac{y-\frac{3}{2}}{\frac{3}{2\times 3}} \Rightarrow 4x-2y-1=0$$

Question 21: A hyperbola passes through the point P (√2, √3) and has foci at (±2, 0). Then the tangent to this hyperbola at P also passes through the point :

(1) (2√2 , 3√3)
(2) (√3, √2)
(3) (− √2 , –√3)
(4) (3√2 , 2√3)

Solution: (1)
$[x^2 / a^2] - [y^2 / b^2] = 1$
$a^2 + b^2 = 4$ and $[2 / a^2] - [3 / b^2] = 1$
$(2 / [4 - b^2]) - (3 / [b^2]) = 1$
$b^2 = 3$
$a^2 = 1$
$x^2 - (y^2 / 3) = 1$
∴ Tangent at P (√2 , √3) is √(2)x - (y / √3)) = 1
Clearly it passes through (2√2 , 3√3).

Question 22: The distance of the point (1, 3, −7) from the plane passing through the point (1, −1, −1), having normal perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$$
$$\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$$ is:

(1) 10 / √83
(2) 5 / √83
(3) 10 / √74

(4) $20 / \sqrt{74}$

Solution: (1)

Let the plane be a (x - 1) + b (y + 1) + c (z + 1) = 0.

It is perpendicular to the given lines

$a - 2b + 3c = 0$

$2a - b - c = 0$

Solving, a : b : c = 5 : 7 : 3

The plane is $5x + 7y + 3z + 5 = 0$

Distance of (1, 3, –7) from this plane = $10 / \sqrt{83}$

Question 23: If the image of the point P (1, −2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, (x / 1) = (y / 4) = (z / 5) is Q, then PQ is equal to :

(1) $2\sqrt{42}$

(2) $\sqrt{42}$

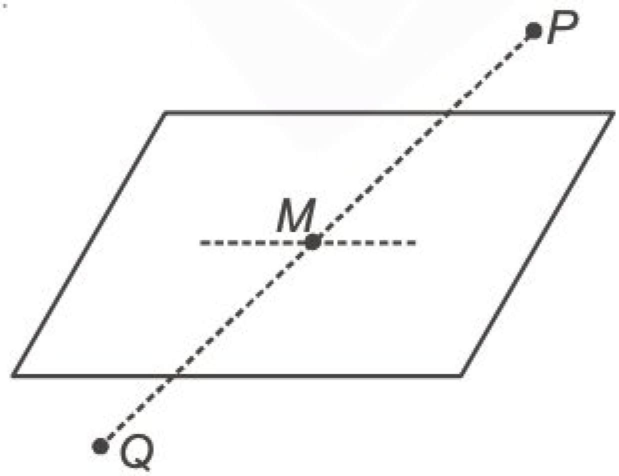
(3) $6\sqrt{5}$

(4) $3\sqrt{5}$

Solution: (1)

Equation of PQ, $\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5}$

Let M be $(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$



As it lies on $2x + 3y - 4z + 22 = 0$

$\lambda = 1$
For Q, $\lambda = 2$
Distance PQ = $2\sqrt{1^2 + 4^2 + 5^2} = 2\sqrt{42}$

Question 24: Let a = 2i + j - 2k and b = i + j. Let c be a vector such that |c - a| = 3, |(a x b) x c| = 3 and the angle between c and a × b be 30°. Then a · c is equal to :

(1) 2
(2) 5
(3) 1 / 8
(4) 25 / 8

Solution: (1)

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = 3 \qquad \vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow \quad |\vec{a} \times \vec{b}||\vec{c}| \sin 30^\circ = 3 \qquad |\vec{a}| = 3 = |\vec{a} \times \vec{b}|$$

$$\Rightarrow \quad |\vec{c}| = 2$$

$$|\vec{c} - \vec{a}| = 3$$

$$\Rightarrow \quad |\vec{c}|^2 + |\vec{a}|^2 - 2(\vec{a} \cdot \vec{c}) = 9$$

$$\vec{a} \cdot \vec{c} = \frac{9 - 3 - 2}{2} = 2$$

Question 25: A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is :

(1) 6
(2) 4

(3) 6 / 25
(4) 12 / 5

Solution: (4)
n = 10
p (Probability of drawing a green ball) = 15 / 25
p = 3 / 5, q = 2 / 5
var (X) = n.p.q
= 10 * (3 / 5) * (2 / 5)
= 10 * (6 / 25)
= 12 / 5

Question 26: For three events A, B and C, P (Exactly one of A or B occurs) = P (Exactly one of B or C occurs) = P (Exactly one of C or A occurs) = 1 / 4 and P (All the three events occur simultaneously) = 1 / 16. Then the probability that at least one of the events occurs is :

(1) 7 / 16
(2) 7 / 64
(3) 3 / 16
(4) 7 / 32

Solution: (1)
$P (A) + P (B) – P (A \cap B) = 1 / 4$
$P (B) + P (C) – P (B \cap C) = 1 / 4$
$P (C) + P (A) – P (A \cap C) = 1 / 4$
$P (A) + P (B) + P (C) – P (A \cap B) – P (B \cap C) – P (A \cap C = 3 / 8$
$\because P (A \cap B \cap C) = 1 / 16$
$\therefore P (A \cup B \cup C) = (3 / 8) + (1 / 16) = 7 / 16$

Question 27: If two different numbers are taken from the set {0, 1, 2, 3, ..., 10}; then the probability that their sum, as well as absolute difference, are both multiples of 4, is :

(1) 12 / 55

(2) 14 / 45
(3) 7 / 55
(4) 6 / 55

Solution: (4)
Total number of ways = $^{11}C_2 = 55$
Favourable ways are (0, 4), (0, 8), (4, 8), (2, 6), (2, 10), (6, 10)
Probability = 6 / 55

Question 28: If $5(\tan^2 x - \cos^2 x) = 2\cos^2 x + 9$, then the value of cos 4x is :

(1) 1 / 3
(2) 2 / 9
(3) −7 / 9
(4) −3 / 5

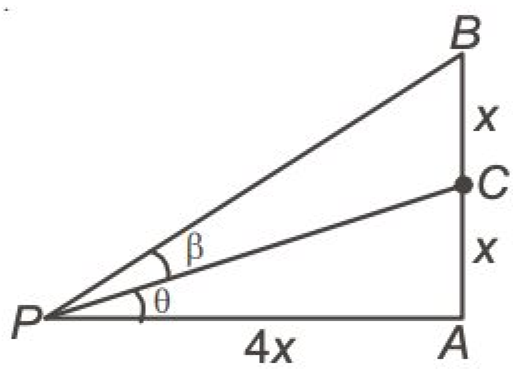
Solution: (3)
$5(\tan^2 x - \cos^2 x) = 2\cos^2 x + 9$
$5 \sec^2 x - 5 = 9 \cos^2 x + 7$
Let $\cos^2 x = t$
$(5 / t) = 9t + 12$
$9t^2 + 12t - 5 = 0$
$t = 1 / 3$ as $t \neq -5 / 3$
$\cos^2 x = 1 / 3$, $\cos 2x = 2\cos^2 x - 1 = -1 / 3$
$\cos 4x = 2\cos^2 2x - 1$
$= (2 / 9) - 1$
$= -7/ 9$

Question 29: Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If ∠BPC = β, then tan β is equal to :

(1) 1 / 4
(2) 2 / 9
(3) 4 / 9

(4) 6 / 7

Solution: (2)



tan θ = 1 / 4

tan (θ + β) = 1 / 2

$$\frac{\frac{1}{4} + tan\beta}{1 - \frac{1}{4}tan\beta} = \frac{1}{2}$$

$$tan\beta = \frac{2}{9}$$

Question 30: The following statement (p → q) → [(~p → q) → q] is :

(1) equivalent to ~p → q
(2) equivalent to p → ~q
(3) a fallacy
(4) a tautology

Solution: (4)

p	q	$p \rightarrow q$	$(\sim p \rightarrow q)$	$(\sim p \rightarrow q) \rightarrow q$	$(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

(a tautology)